MODERN ELEMENTARY MATHEMATICS.

(A Text-Book for use in Forms IV, V & VI)
(Adapted to the S. S. L. C. syllabus.)

BY

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PREFACE
TO
THE SECOND EDITION.

"Much of Mathematics teaching in the earlier stages, must be based on intuition and experiment." We have taken this for our motto in the preparation of this Second Edition. Of course, the book is based on the First Edition, but, while much of the old matter retained, has been rearranged, much new matter has also been added, especially in the form of prefaratory note to each section and oral questions. The *reading matter* that prefaces each section is not a mere statement of facts and solutions of a few examples. Presumptuous as it may appear, we have made bold to insert teaching notes and hints to teachers here and there, in the hope that more experienced and successful masters will criticise them, improve them and will thus enable us to revise them, that beginners in the profession may have a sure guidance in the future.

The abundance of problems and their practical nature, the correlation that it is our belief we have effected between Arithmetic, Algebra and Geometry, and the three appendices, especially the first two—on Artifices and the note on Framing of Problems by boys—will, we believe, enhance the popularity which the first edition has gained.

It is our honest conviction that we have, however imperfectly, succeeded in placing before the public a book of Elementary Mathematics with the Heuristic vein running through it.

In spite of the great care taken, errors and obscurities might have crept in, and we shall esteem it a favour if those using this book will kindly send us a note of them.

It is almost impossible for us to adequately express our gratitude to Rev. Br. Joseph S.J., Superintendent, St. Joseph's

Industrial Press. Had it not been for his unremitting zeal and the special attention he paid to this work, it would have been impossible to publish this book in the short space of about a month and a half. The nice get up of this volume, the neatness and promptness of execution are a few of the special features which have won for the press a wide reputation.

PUDUKOTAH,
6th June, 1925.

P. R. SUBRAHMANIAN.

PREFACE TO THE THIRD EDITION.

This editon is only a reprint of the previous one, but for the following improvements which, as will be seen, do not stand in the way of the book being used side by side with copies of the Second Edition.

The improvements are :—

1. The introduction of a few articles and examples on 'The Language of Algebra,' 'Simple Equations' and 'Algebraic Fundamental Processes.'

2. The enhancement of the number of Revision Papers of the Third Series to Thirty-six.

3. The insertion of instructions to teachers as to when, in the High School course, a particular series of examples may conveniently be taken up.

It is hoped that the book with these additions, made at the suggestion of masters who have been using the book, will meet with a warm reception.

PUDUKOTAH,
10—8—26.

P. R. SUBRAHMANIAN.

PREFACE TO THE FOURTH EDITION.

This Fourth Edition is only a reprint of the Third: A few errors that had crept in the Third Edition have however been corrected.

PUDUKOTAH,
30th June, 1928.

P. R. SUBRAHMANIAN.

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Modern Elementary Mathematics.

SECTION I. A

The Language of Algebra.

1. The pupil has learnt in the lower forms that 1, 3, 5, 7, 9, 11 etc. are odd numbers and that 2, 4, 6, 8, 10, 12 etc. are even numbers. If the pupil is asked to state the odd numbers he can only say that every alternate number beginning with 1 is an odd number; but we can include all odd numbers, in one statement '$2n+1$' where n is any number. Give values to n and the value of $2n+1$ is an odd number.

Similarly $2n$ represents even numbers.

We have included all odd numbers in one statement $2n+1$. This *general* statement is the purpose of Algebra.

The price of mangoes at Re. 1-8 as. a doz. may be stated as $24x$ annas where x represents the number of mangoes.

2. How do we express the statement, "Eighteen is increased by seven"? *Ans.* $18-7$.

Similarly the statement, "A certain number is increased by 5" can be expressed by $x+5$; Instead of 'a certain number' we put x.

The statement, "A certain number is increased by 4 and from the result 2 is subtracted" may be represented thus: $x+4-2$, which when simplified gives $x+2$.

3. (*a*) We know that $3+3+3+3+3=5\times 3$.

Similarly $a+a+a+a+a=5\times a$ or $5a$

$p+p+p+p+p+p=6\times p$ or $6p$

$$p+2p+5p+6p+p$$

$$=\underbrace{p+p}+p+\underbrace{p+p+p+p+p}$$

$$+\underbrace{p+p+p+p+p+p}+p=15\times p=15p.$$

(*b*) (i) If $a=2$ what is the value of $5a$?

$$5a=a+a+a+a+a$$
$$=2+2+2+2+2=5\times2=10.$$

(ii) If $x=1\frac{1}{2}$ what is the value of $7x$?

$$7x=7\times1\frac{1}{2}=10\frac{1}{2}.$$

4. If a boy has five apples and he eats away three apples, how many will remain?

Symbolically $5a-3a=2a$ (a stands for apples)
Similarly $6a-4a=2a$
Hence $6a-2a+3a-a=6a$.

5. If two be multiplied by itself we have $2\times2=2^2=4$
Similarly $a\times a=a^2$.
Hence $2p\times p=2\times p\times p=2\times p^2$.

(i) Find the value of $2p^2$ when $p=3$
$2p^2=2\times3\times3=18$.

(ii) Find the value of $3a^2b$ when $a=2$ and $b=3$
$3a^2b=3\times a\times a\times b=3\times2\times2\times3=36$.

(iii) Find the value of $3a\times2b$
$3a\times2b=3\times a\times2\times b=6ab$.

(iv) $3a^2\times2b\times a=3a^3\times2b=6a^3b$.

(v) If $a=2$ and $b=\frac{2}{3}$ find the value of $6a^3b$.
$6a^3b=6\times2\times2\times2\times\frac{2}{3}=32$.

(vi) Find the sum of $3a^2$, $2a+3b$, $3a-b-a^2$
The sum is $3a^2+2a+3b+3a-b-a^2=2a^2+5a+2b$.

6. What does $3a^2+2a-b$ express?

"To three times the square of a number, twice that number is added and from that result another number is subtracted." This long statement is *expressed* in shortened form by $3a^2+2a-b$.

Hence $3a^2+2a-b$ which *expresses* the statement is called an Algebraical Expression.

The parts of the expression $3a^2$, $2a$, b are called the *terms*.

Therefore we have the definition:—A number of terms connected by the signs + or − constitute an algebraical expression.

Again taking the term $3a^2$, 3 is called the numerical co-efficient and 2 is called the power or index; for it indicates the power which a has.

Test Exercise I.

1. Express the following as statements in words:—

(i) $8-3$ (ii) $7-\frac{2}{3}$ (iii) $x-6$ (iv) $x-1\frac{1}{2}$ (v) $2x-1$

(vi) $\frac{a}{2}-\frac{1}{2}$ (vii) $a+b$ (viii) $3a^2+5$ (ix) $5b^2-b$ (x) $8a^2-3ab^2$.

2. Translate the following into algebraic symbols:—

(i) Seven is subtracted from twice the sum of four and three.
(ii) Two is added to half the difference between seven and three.
(iii) A certain number is added to the sum of seven and eight.
(iv) Seven is added to half the difference between a certain number and three.
(v) A certain number is added to the difference between itself and three.

3. The side of a square is a inches. What is its perimeter?

4. The length of a rectangle is twice its breadth. Express this in symbols.

5. Express symbolically the statement, "The perimeter of a rectangle is six inches more than the permiter of a square whose side is a inches.

6. Represent by means of letters the following statements:—

(i) Three times the sum of two numbers is added to half their difference.
(ii) A number is multiplied by itself and to three times this result, four times another number is added.
(iii) Three times a number is multiplied by itself and from this result twice the number is subtracted.

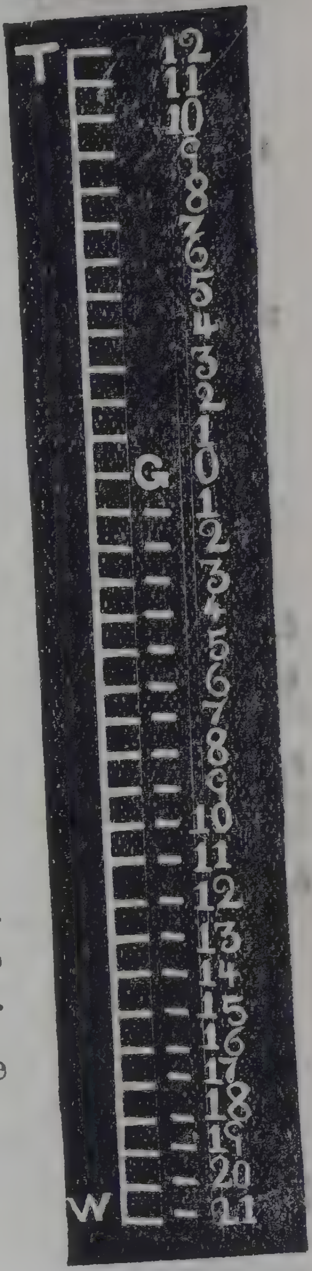
The Rule of signs.

7. The student knows that + (read *plus*) is the sign of addition and that − (read *minus*) is the sign of subtraction.

We generally assign some meaning to the signs:—If the sign

+ denotes increase,	− denotes decrease
+ ,, gain,	− ,, loss
+ ,, motion forward,	− ,, motion backward
+ ,, climbing up,	− ,, climbing down.

8. If G denote ground level and GW a well 20 feet deep and GT a tree on the brink of the well 12 feet high, then we may mark the height by $+1$, $+2$, $+3$ etc. $+12$ and the depth by -1, -2, -3 etc. -20 and G itself is 0.



(i) $4-2$ means 'Take 2 from 4'. In the scale proceed from 2 to 4. You have to go two feet up...Ans. $4-2=2$.

$7-4$ means 'Take 4 from 7'. Proceed from 4 to 7. You have to go three feet up $\therefore 7-4=3$.

(ii) Take 3 from -2; Proceed from 3 to -2 you have to go 5 feet down $\therefore -2-3 = -5$.

(iii) Take -11 from -18; Proceed from -11 to -18; you have to go 7 feet down $-18-(-11)= -7$.

i.e $-18+11= -7$.

(iv) Take -11 from -2; Proceed from -11 to -2; you have to go 9 feet up; $\therefore -2-(-11)= -2+11=9$.

Hence the following rules are deduced.

$$\left.\begin{aligned} a+(-b) &= a-b \\ a-(+b) &= a-b \\ a-(-b) &= a+b \\ a+(+b) &= a+b \end{aligned}\right\}$$

Therefore the following working rules are deduced.

(i) If the minus sign precedes a parenthesis, remove the parenthesis and write down the terms inside with their signs changed $a-(b-c) = a-b+c$.

Also $a-3(b-c) = a-3b+3c$

and $a-3b(c-d) = a-3bc+3bd$.

(ii) If a parenthesis is preceded by the plus sign, remove the parenthesis writing each term with its original sign $a+(b-c)=a+b-c$.

Also $a+3(b-c)=a+3b-3c$

and $a+3b\,(c-d)=a+3bc-3bd$.

The different pairs of parenthesis of brackets are

( )	called	the simple bracket
{ }	,,	the double bracket
[ ]	,,	the square bracket

Sometimes a bar—— placed above a certain number of terms shows that those terms must be taken together. The——is called a *vinculum.*

9. *To Teachers.*

To drill in algebraical principles, the kind of work to be done in Form IV, is mainly the generalisation of each concrete problem of an easy type. The processes employed in the solution of a concrete problem must be symbolically written out. For example, every student in Form IV can reduce Rs. 8-6 as. to annas. The number of annas is $(16\times 8)+6$; generalising this Rs. $a-b$ as. $=16a+b$ annas. Again if Rs. a, b as. be equal to c as. establish a relation between a, b and c; Evidently $c=16a+b$. Again this may be looked upon as a formula and there are three letters, so that if any two of them be given the third can be determined. Thus, it will be seen, a number of questions may be proposed and these alone will give a thorongh grasp of the subject or of the topic. We propose some problems on generalised arithmetic. The teacher is requested to propose more problems.

1. How many quarter annas are there in m rupees?
2. ,, ,, ,, ,, in Rs. m and n annas?
3. ,, ,, ,, ,, in Rs. $m-n-6p$ pies?
4. How many half-rupees are there in b as.?
5. If p half-rupees be equal in value to q quarter annas, establish a relation between p and q.
6. A stick is a yds. b in. long. If c in. were cut off, what will be the length of the remaining stick?
7. A train performs a journey of x miles in a hrs.
   (i) What is its rate in feet per second?
   (ii) How many miles will it run in p hrs.?

(iii) How long will it take to run a distance of m miles?

(iv) If the train takes r hrs. to run q miles, establish a relation between a, x, r and q.

8. If a mangoes be bought at x annas each and b mangoes at y annas each, what will be the total amount spent?

If the total amount spent in Rupees be R, establish a relation between a, b, x, y and R.

9. If a merchant gains Rs. a by selling an article for Rs. m, what is the cost price of the article?

10. If a merchant loses Rs. a by selling an article for Rs. m, what is the cost price of the article?

11. If a merchant buys m mangoes at 3 for 2 annas and n mangoes at 3 for 4 annas and sells them at an anna each, find (a) his gain, (b) his loss.

The Simple Equation.

(1) A certain number added to 5 gives the result 7. This is a statement. This is represented symbolically.

$$x+5=7.$$

The equality $x+5=7$ is called an *equation*

For what value is $x+5=7$?

$x+5=7$ only when $x=2$.

Finding the value of the unknown quantity, for which the statement holds good is called solving the equation.

(2) *Laws of Equations.*

We know that $7+6=13$; Subtracting 6 from either side we have $7+6-6=13-6$ *i.e* $7=13-6$. The student will note that in the equality $7=13-6$, the 6 has changed side and when the side changes, the sign changes.

Hence if $a+5=12$; $a=12-5$

" $a-5=12$; $a=12+5$

" $a\times 5$ *i.e.* $5a=15$, $a=15\div 5$ or $\frac{15}{5}=3$.

" $a\div 5$ *i.e.* $\frac{a}{5}=3$; $a=3\times 5$ *i.e.* 15.

Examples:—(1) Think of a certain number; Add 5 to it. Multiply the result by 3; Divide the product by 7 and subtract 6 from the quotient. The result is 3. Find the number.

Suppose the number is x.
Adding 5 to it we have $x+5$
Multiplying $x+5$ by 3 „ „ $3(x+5)$
Dividing the product by 7 „ „ $\frac{3(x+5)}{7}$
Subtracting 6 from the quotient „ $\frac{3(x+5)}{7}-6$

This result is 3 (given)

$\therefore$ The equation is $\frac{3(x+5)}{7}-6=3$

$$\therefore \frac{3(x+5)}{7}=9$$

7 is in the denominator $\therefore 3(x+5) = 7\times 9$
i.e. $3x+15 = 63$
15 is added to $3x$ $\therefore 3x = 63-15$ *i.e.* 48
$x = 16$.

(2) Propose a problem of the type given above represented by the equation

$$\frac{2x-3}{2}+7=19.$$

Problem.—Think of a number; Double it; Subtract 3 from the result. Divide the remainder by 2. Add 7 to the quotient. The result is 19. Find the number.

On the solution of equations.—If a problem or a statement in words be expressed in equational form, the answer is obtained by the solution of the equation. The following rules may be followed.

i. Get rid of fractions—Multiply either side of the equality by the L. C. M. of the denominators.

ii. Remove brackets.

iii. Transpose and have unknown quantity on one side and the number on the other.

iv. +Simplify either side and determine the value of the unknown quantity.

11. *Algebraic multiplication and division.*

(i) The area of a rectangle 5 in. long and 4 in. broad is 20 sq. in.

Similarly the area of a square 5 in. each way is 5 × 5 sq. in. or 5^2 sq. in. or 25 sq. in.

The area of a rectangle a in. long and b in. broad is ab sq. in.

The area of a square a in. each way is $a \times a$ sq. in. or a^2 sq. in.

(ii) Hence represented symbolically we have :—

$$\left.\begin{array}{l} a \times b = ab \\ p \times q \times r = pqr \\ 3p \times 2q \times 5r = 30pqr \end{array}\right\} \text{when letters are different.}$$

$$\left.\begin{array}{l} a \times a = a^2 \\ a \times a \times b = a^2b \\ a \times a \times b \times b = a^2b^2 \\ 2a \times 3b \times a \times 2b = 12a^2b^2 \end{array}\right\} \text{Like letters and index law.}$$

(iii) To multiply a binomial by another binomial $(x+a)(x+b)$

$AB = x$, $BC = a$ and $AC = x + a$

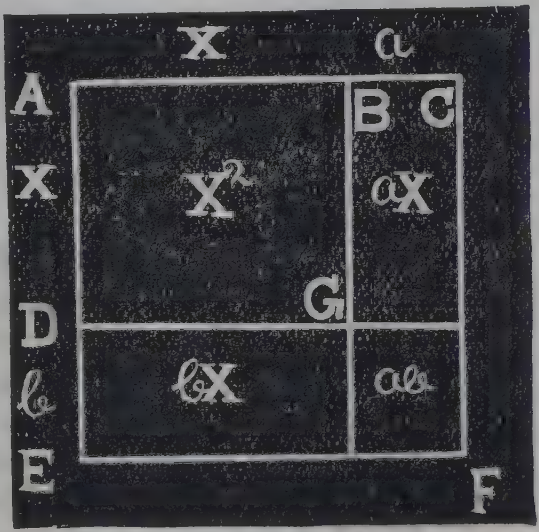
$AD = AB = x$, $DE = b$.

area $ACFE = (x+a)(x+b)$

$ACFE = AG + CG + GF + GE$

$= x^2 + ax + ab + bx =$

$= x^2 + ax + bx + ab$



Rule.—Multiply each term of the multiplicand by each term of the multiplier. Add the partial products.

$$\begin{array}{l} x + a \\ x + b \\ \hline x^2 + ax \\ \quad + bx + ab \\ \hline x^2 + ax + bx + ab. \end{array}$$

To the Teachers :—Diagrams of the kind in the previous article may be drawn to illustrate the following products :

$$x(a+b+c) = ax+bx+cx$$
$$(x+a)(p+q+r) = x(p+q+r)+a(p+q+r)$$
$$(x+a)(x-a) = x^2-a^2$$
$$(x+a)^2 = x^2+2ax+a^2$$
$$(x-a)^2 = x^2-2ax+a^2$$
$$(a+b)(c+d) = ac+ad+bc+bd$$

12. The student knows that $\frac{8}{4}=2$ is $\frac{8}{4}=\frac{2\times2\times2}{2\times2}=2$

Similarly $\frac{5\times3}{5}=3$

and $\frac{18}{3}=\frac{3\times6}{3}=6$

Likewise $\frac{ab}{a}=\frac{a\times b}{a}=b$

and $\frac{a^2b}{a}=\frac{a\times a\times b}{a}=ab$

Again $\frac{12a^2b}{3a}=\frac{3\times4\times a\times a\times b}{3\times a}=4ab$

13. Working Rule :

(a) *Algebraic Multiplication*—If like letters are multiplied add the powers.

Examples : $a^2\times a^3=a^{2+3}=a^5$
$a^3\times a^7=a^{3+7}=a^{10}$
$3a^2\times 2a^3b=6a^{2+3}b=6a^5b$

(b) *Algebraic Division*—If a monomial is divided by another, subtract from the power of each letter in the dividend, that power of a *like* letter in the divisor.

Examples : $a^5\div a^2=a^{5-2}=x^3$
$a^7b^3\div a^2b=a^{7-2}b^{3-1}=a^5b^2$
$12a^2b^3c\div 4ab^2=3ab.$

(c) **Note to Teachers :**—The students must be taught the division of a polynomial by another. The polynomials may preferably be functions of *one* letter only.

14. Fill in the following table :—

Length in feet.	Breadth in feet.	Area in square feet.	Perimeter in feet.
$3a$	$4b$	...	...
$2a$	...	...	$8a$
$3a+b$	...	...	$6a+4b$
$5x+2y$	...	...	$14x-2y$
$a+b$	$b+c$	...	...
$2m+n$	...	...	$12m+6n$
$m-4n$	$m+4n$	...	...
$3a$	...	$12ab$	...
$4x$	...	$28x^2$	...
$3p$	$2p-q$	...	...
$a+b$	...	a^2-b^2	...
$a+b$	...	$a^2+2ab+b^2$	...

EXAMPLES I.—(A)

EXERCISE 1.

Simplify :—(*Qs.* 1 – 13)

1. $3a+2+(a-2)$

2. $2a+3b-(a+2b)$

3. $3x+y-(2y-3x)$

4. $(2m-n)-(3m-n)$

5. $(a-2b+c)-(b-2c+a)$

6. $(2x-4)+(3x-2)-(2x-1)$

7. $2ab+3a^2+2b^2-(a^2+b^2-ab)$

8. $(3a+2b-5c)-(2a-3b-2c)$

9. $(x^2+xy+y^2)+(2y^2-2xy-x^2)-(3x^2-2xy+y^2)$

10. $7a-[3a-\{2a+(2a-b)\}]$

11. $(7a-3b)+\{(2a-3b)-3a-2b\}$

12. $\{3a-(2a+3b)-(3a+2a-3b)\}$

13. $(2x^2+y^2-z^2)-(x^2+y^2+z^2)-(3x^2-2y^2+4z^2)$

14. If $a=2$, $b=3$, $c=4$ find the value of

$$\frac{a-b+c}{a+b-c}+\frac{b-c+a}{b+c-a}+\frac{c-a+b}{c+a-b}$$

15. If $a=1$, $b=2$, $c=5$ find the value of

$$\{a-(b-c)\}^2+\{b-(c-a)\}^2+\{c-(a-b)\}^2$$

Also find the value of

$$\{a^2-(b-c)\}^2+\{b^2-(c-a)\}^2+(c^2-(a-b)\}^2$$

16. If $x=6$, $y=3$, $z=5$ find the value of

$$\frac{x-y}{4}\times\frac{z-4}{x}+\frac{x}{y}-\frac{y}{z}.$$

17. If $a=1$, $b=2$, $c=3$, $d=0$ find the value of

$$\frac{a^2b+b^2c+c^2d+d^2a}{(a+b)(c+d)-\{(a-d)+(c-b)\}}.$$

18. Simplify:

(i) $a-[2b+\{3c-3a-\underline{(a+b)}\}+2a-(b-3c)]$

(ii) $x-[a-\{2a-(3a-4a-x)\}]$

(iii) If $v=5a+4b-6c$, $x=-3a-9b+7c$, $y=20a+7b-5c$, $z=13a-5b+9c$ find the value of $v-(x+y)+z$.

EXERCISE 2.

1. Subtract $2a-5b$ from $3a+2b$.

2. From the sum of x^2+xy+y^2 and $2xy-y^2$ subtract x^2+2xy.

3. Find the remainder when $3a^2+2ab+b^2$ is taken away from the difference between $6a^2+3ab+5b^2$ and $2a^2-ab-b^2$.

4. If $P=a^2+ab+b^2$, $Q=2ab$, $R=2a^2+2b^2$ find in terms of a, b & c, the value of $3P-(Q+R)$.

5. (i) What must be added to $a-b$ to produce 0?

(ii) What must be subtracted from $c-d$ to produce (i) 0, (ii) c, (iii) d, (iv) $c-d$, (v) f?

6. Simplify $x(y-x)-(xy+x^2)$.

7. The divisor is a, the quotient is b and the remainder is r. Find the dividend.

8. The product is P, the multiplier is R, find the multiplicand.

9. Find a formula for

(i) all numbers which when divided by 3 give a remainder 1.

(ii) the product P of two consecutive numbers.
(iii) ,, ,, P ,, ,, odd numbers
(iv) ,, ,, P ,, ,, even numbers
(v) ,, ,, P ,, ,, multiples of 10
(vi) ,, ,, P ,, ,, ,, ,, 5.

10. (i) Multiply $x+3$ by $x-3$
(ii) ,, $2a+b$ by $a+b$
(iii) ,, x^2+x+1 by $x-1$
(iv) ,, x^2-x+1 by $x+1$
(v) ,, $a+b$ by $a-b$
(vi) ,, a^2+ab+b^2 by $a-b$
(vii) ,, $a+b$ by $a-b$
(viii) ,, a^2-ab+b^2 by $a+b$
(ix) ,, x^2-2x+1 by $x-1$
(x) ,, x^2+2x+1 by $x+1$.

11. Divide (i) x^2-x by x
(ii) $a^2-2ab+b^2$ by $a-b$
(iii) x^2-1 by $x+1$
(iv) x^2+5x+6 by $x+2$
(v) $x^3+6x^2+11x+6$ by x^2+3x+2.

12. Solve:

(i) $10x-12=3x+9$
(ii) $7x+8=2x-7$
(iii) $3(x+5)=x+25$
(iv) $9x+3(x+5)=5(2x+5)$
(v) $3x-(2-3x)=5x+6$
(vi) $2(3x-5)-(x-3)=7(x-6)$
(vii) $3(2x-5)-4(x-3)=3x+12$
(viii) $2(4x-9)-(3x+10)=4(x-3)-(x+2)$
(ix) $\dfrac{7x-5}{6}=x$
(x) $\dfrac{7x-5}{6}+5=x$
(xi) $\dfrac{7x-5}{6}+\frac{2}{3}=20$

(xii) $\dfrac{7x-5}{6}+\frac{1}{3}=2x+7$

(xiii) $\dfrac{4x-2}{5}+\dfrac{2x-1}{10}=\dfrac{x-5}{2}$

(xiv) $\dfrac{0{\cdot}25x+1{\cdot}5}{0{\cdot}125}=0{\cdot}5x$

(xv) $\frac{2}{3}(x-3)=\frac{1}{2}(x-2)$

(xvi) $\frac{1}{5}(6x+1)-\frac{1}{7}(5x-6)=\frac{1}{3}(2x+1)$

(xvii) $3-\dfrac{3x-1}{7}=1$

(xviii) $\dfrac{7x}{2}-\frac{1}{3}(x-8)-\dfrac{4(4x+2)}{5}=\dfrac{5x-3}{7}$

13. Divide Rs. 90 between two boys, so that one of them may have Rs. 15 more than twice the share of the other.

14. The sum of the ages of a father and his son is 46. The father is six years older than twice the age of his son. Find their ages.

15. The strength of a school is 640. The number of Brahmins is 20 less than 10 times the number of non-Brahmins. Find the non-Brahmin strength.

16. A straight line is 7·6 cms. long. It is divided into two parts such that one part is 0·8 cms. less than twice the other. Find the length of the shorter.

17. There are 660 voters in a voting list; on an occasion 100 were absent and the remainder voted. The elected candidate won by a majority of 36 over his rival. Find the number of votes he secured.

18. The perimeter of a rectangular field is 540 feet. The length is 10 feet less than two and a half times the breadth. Find the breadth.

19. The length of a rectangle exceeds the breadth by 5 cms. and is also 0·5 cm. more than one and a half imes the breadth. Find the length.

20. The sum of 5 numbers is 135. Each succeed-ng number is 7 more than the previous one. Find the umbers.

21. The number of boys in a primary school is 24 more than half the number enrolled and the number of girls is six more than one-fourth the number enrolled. Find the strength of the school.

22. The return ticket from T to M is 10 as. less than twice the cost of a single journey ticket. If 5 single tickets and 4 return tickets are purchased for Rs. 46-4 as. find the cost of a return ticket.

23. Some men agree to pay equally for the use of a boat and each man pays 15 as. If there had been two more in the party each would have paid 10 as. How many men are there?

24. The formula $A = Ks^2$ gives the area of the polygon; when $s = 12$, $A = 72$. Find s when $A = 144$.

25. A father is twice as old as the older of his two sons. The senior son is 6 years older than his brother; six years hence the father will be 12 years older than the united ages of his two sons. Find their present ages.

26. The father is 56 years of age and the son is 28 years; when will the sum of their ages be 100?

27. A man bikes a certain distance at 9 miles per hour and then walks an equal distance at 4 miles an hour and then is overtaken by a motor which takes him to his destination in $7\frac{1}{2}$ hrs., running at 18 miles an hour. If the destination is 54 miles from his starting-place, find the distance travelled by motor.

28. A man walks a certain distance going $\frac{2}{3}$ of the journey at 4 miles an hour and the remainder at 3 miles an hour. The whole journey occupies 9 hrs. 10 min. Find the length of the journey.

29. Divide 64 into two parts such that the first part divided by 7 is equal to the second part divided by 9.

30. Two boys measured the same length. One of them gave his answer in feet which could be represented by $2x + \frac{1}{2}$; the other gave it in inches represented by $16x + 7$; what was the length of the line in inches?

EXAMPLES I.—(B)

The Framing of Equations.

ORAL.

Find the No. x in the following cases (*Qs.* 1 - 11):—

1. (a) A number x increased by 5 is equal to 12.
(b) ,, x ,, a ,, b.

2. (a) Twice a number x increased by 5 is equal to 11.
(b) ,, ,, ,, a ,, p.

3. (a) A number x diminished by 5 is equal to 7.
(b) ,, x ,, a ,, m.

4. (a) Twice a number x diminished by 5 is equal to 7.
(b) ,, ,, ,, a ,, m.

5. (a) A number x increased by 5 and then doubled is equal to 18.
(b) ,, x ,, a ,, equal to q.

6. (a) A number x diminished by 5 and then doubled is equal to 18.
(b) ,, x ,, a ,, is equal to b.

7. (a) One half of a number x increased by 3 is equal to 5.
(b) ,, x ,, a ,, b.

8. (a) ,, x diminished by 3 is equal to 5.
(b) ,, x ,, ,, a ,, r.

9. (a) ,, x diminished by one-fourth of 12 is equal to 5.
(b) ,, x ,, ,, a. is equal to b.

10. (a) One half of three times a number x increased by twice the number 2 is equal to 4.
(b) ,, x ,, twice the number a is equal to b.

11. (a) From three times a number x, 7 is subtracted. The result is multiplied by 2 and the product is divided by 5. The resulting quotient is 8.

(b) From three times a number x, a is subtracted. The result is multiplied by 2 and the product is divided by 5. The resulting quotient is b.

12. (a) A man is now b years of age. What should have been his age 7 years ago? If that age be 13 what is his age now?

(b) A man was b years old, a years ago. His son was then half as aged as his father. Find the son's age now in terms of a and b.

13. The population of a town is 18 thousands. There are x persons who can read and write. The number of the illiterate exceeds that of the literate by 8 thousands. Find how many cannot read or write.

14. A stick a yards long is divided into two parts such that one part which is x in. long, is b inches longer than the other. Find the lengths of the parts.

15. If the number p be divided by another number x the quotient is a and the remainder is b. Find the number x.

16. A booking clerk sold a chair-tickets at Rs. m each and b bench-tickets at Rs. n each and c floor-tickets at p annas each. Find x the total collection in rupees.

17. In one scale-pan of a balance is placed a viss of sugar and in the other x viss. If b palams be taken from the former and put into the latter, the beam will be horizontal. How many viss in terms of a and b was there in the latter?

18. One of the base angles of an isosceles triangle contains 10 more degrees than the vertical angle. Find the angles of the triangle.

19. A rectangular plot of ground is a yds. long and b yds. broad. If I go x times round the field I would have walked m miles. Find x.

20. A man bought an article for Rs. x and by selling it for Rs. b gained Rs. p. Find x his cost price.

21. By selling an article for Rs. 11, a merchant lost half of what he paid for it. If he had bought it for Rs. x find his cost price x.

22. A man runs p miles in q hrs. How many miles can he run in an hour?

23. (a) A and B have together Rs. 50; but A has twice as much as B. How much has each?

(b) A and B have together Rs. a. B has twice as much as A. How many Rs. has A?

24. (a) A and B together own 26 acres of land. They divide it among themselves such that A has 3 acres more than half of what B has. How many acres does each get?

(b) An estate of a acres is to be divided among two reversioners such that one may have b more acres than twice the share of the other. What is the share of each?

Express the statements in Qs. 25-28, symbolically in the form of equations:—

25. The sum of two numbers a and b is less than the difference between two other nos. c and d by 5.

26. (a) The difference between x and y is 5 more than twice the difference between p and q.

(b) The difference between x and y is a more than b times the difference between p and q.

27. A person walks at p miles an hour for a hours; while another walks at q miles an hour for b hours. The latter has walked m miles more than the former.

28. The sum of p and q is divided by the difference between m and n and the quotient is t more than the difference between x times l and c times d.

29. The three angles of a triangle are together equal to 180°; angle A is a times angle B, and angle B is b times angle C; and angle C contains x degrees. Find the magnitude of each angle.

30. (i) $1/a$ is called the reciprocal of a; what is the reciprocal of—(i) $7\frac{1}{2}$, (ii) q, (iii) $q + 7\frac{1}{2}$ (iv) the sum of x and y.

(iv) The sum of a number and its reciprocal is twice the difference between another number and its reciprocal. Express the relation between the two numbers.

31. (a) One side of a rectangle is a inches. If this length be diminished by 3 inches, what will the new length be?

(b) If the breadth of the rectangle be b in. and it is diminished by 2 in., what will the new breadth be?

(c) If the length be diminished by x inches and the breadth by y inches, by how much will the area be diminished?

32. The length of a rectangle is x inches and its breadth y inches. The length is increased by 2 inches and the breadth is diminished by 3 inches. The area is not affected. Find the relation between x and y.

EXAMPLES II.

The Framing of Equations.

1. A person has Rs. x with him. He gives Rs. a to his friend and has Rs. b left. Find x in terms of a and b.
If $a = 230$ and $b = 474$, find x.

2. A person has Rs. x with him :—

i. He gives Rs. a to Rama and Rs. b to Krishna. He has still half of what he originally had.

ii. He gives Rs. a to Rama, but Krishna gives him Rs. b. On the whole, he has a third of what he had.

iii. He gives Rs. a to each of his three friends and wants Rs. b to give an equal sum to a fourth.

iv. He gives half of what he has to A and Rs. a to B and half the balance to C and has Rs. b left.

v. He gets an equal sum from each of his three friends and buys b things at a annas each and Rs. c remain over.

In each of the above cases express x in terms of the other letters involved.

3. Two boys A and B began to play. Each had x marbles. In the first round A lost a marbles. In the second round B lost b marbles and in the third round A lost c marbles, so that at the end of the third round B had half as much again as A. Find x in terms of a, b and c.

4. A person owned x cawnies of land; a of these were Nunja lands which could be leased at Rs. m per cawnie and the remainder consisted of Punja lands which could be let only at half the rate. If the total amount the property was let for, be Rs. p, what is the relation between x, a, m, and p?

5. I earn Rs. x a month and am able to save a fourth of my earnings. If the savings accumulate to Rs. y annually, write down the relation between x and y.

6. A gentleman bought a horse and a carriage for Rs. a. He sold the horse for 1½ times the price he paid for it and got Rs. x; he again sold the carriage for Rs. b which was only ¾ of the price he paid for it. On the whole, however, he gained Rs. c. Find the relations between x, a, b and c.

7. A merchant bought a cycle for Rs. x. He sold it to X for 1½ times the price he paid for it. X sold it to Y for Rs. a and gained $\frac{1}{5}$ of what it had cost him. Express x in terms of a.

8. Two different varieties of paddy were once bought by a merchant. The superior sort cost him Rs. a per kalam and the inferior Rs. b per kalam. He purchased 20 kalams of the former and 15 kalams of the latter and mixed them. By selling each kalam of the mixture for Rs. m, he gained Rs. x. Find x.

9. (a) The numerator of a fraction is x. The denominator exceeds the numerator by 2. Write down the fraction.

(b) If 5 be added to the denominator, the fraction becomes ½. Find the fraction.

10. An empty vessel weighs p seers; when full of water it weighs w seers. What will be the weight—

i. when the vessel is half full of water?

ii. when the vessel is full of milk which weighs a times as much as water?

11. A deal-wood box weighs d ℔. Half-pound packets of biscuits are packed in it. If h packets are contained in it and the packing materials such as paper, deal-wood shavings etc., weigh s ℔., find the whole weight of the box.

12. A box containing a dozen gross of match-boxes weighs p lbs. The box itself weighs b lb. If the wrapper paper weighs w lbs., find the weight of each match-box in ounces.

13. There were x anna coins in a bag. The owner gave change for Rs. a and there were still b anna coins left. Find x in terms of a and b.

14. A piece of cloth is x yds. long. The cloth is sold at a annas per yard. If Rs. b were realised by selling a portion and c yds. remain, find x.

15. There were p five-rupee currency notes with me. I bought a reams of paper at Rs. m per ream and two fountain pens at Rs. b each. With the balance I bought x yds. of cloth at Rs. n-q as. per yard. Find x, the no. of yds. I bought, in terms of the other letters involved.

16. In a distribution of cloths to the poor, a pieces, each 40 yards long, were purchased; 9 yds. were given to each woman, 3½ yds. to each adult, and 2 yds. to each boy or child. p women, q adults and r children were thus clothed and there still remained t yds. Express the relation between a, p, q, r and t.

17. The price of a gross of Exercise note-books is Rs. b. If bought in retail each note-book costs a pies more. A person buys in retail x note-books for c annas. Find x in terms of a, b and c.

18. a bulls were sold at Rs. p each and b cows at Rs. q each. With the proceeds x goats were bought at Rs. r each. Determine x.

19. A bag contains x two-anna pieces, twice as many four-anna pieces, thrice as many eight-anna pieces and y five-rupee currency notes. The value of the five-rupee notes is equal to the value of the other coins in the bag and the value of all the coins and the notes together is Rs. a. Find x and y separately in terms of a.

20. A man has Rs. a in a bank A and Rs. b in a bank B. He has Rs. b to invest. He invests Rs. x in bank A and the balance in bank B. The amount that after this investment stands to his credit in A is—

$$\left.\begin{array}{rll}\text{(i)} & \text{Twice that in Bank } B & \\ \text{(ii)} & \text{Half of that} & ,, \\ \text{(iii)} & \text{Equal to that} & ,, \\ \text{(iv)} & p \text{ times that} & ,, \end{array}\right\}\text{Find } x \text{ in each case.}$$

Harder Problems.

21. A dog which runs at a miles an hour, sets in pursuit of a hare, h yards ahead. The hare runs 3 miles an hour less than the dog. In how many hours will the dog overtake the hare?

22. A piece of cloth is x yards y feet z inches long. If two pieces each a yards long be torn off and the remainder be cut into strips each s inches long, how many strips will there be?

23. A book-shelf is b feet long. If v volumes each a inches thick, and t volumes each c inches thick are placed in it, how many more volumes each x inches thick will the shelf accommodate?

24. a viss of sugar cost Rs. x and b viss of coffee Rs. y. Find the cost of p viss of the former and v viss of the latter.

25. There are x telegraph posts to the mile at equal distances apart.

(a) Find in feet the distance between two adjacent posts.

(b) If a train clears n of these in m seconds find its rate in miles per hour.

26. The price of a house is Rs. x and that of the furniture in the house is Rs. y. If the house and the furniture together cost Rs. a, express the relation between x, y and a.

Again if the price of the house alone is b times the price of the furniture find x and y separately in terms of a and b.

27. (a) A father is now x years old. What was his age a years ago?

(b) His son is now y years old. What was the son's age a years ago?

(c) If a years ago the father was twice as old as his son, write down the relation between x, y and a.

28. A husband is a years old and the wife b years. When they were married x years ago, the husband was half as old again as his wife. Find x.

29. The difference between the ages of two brothers is a. One of them is x years of age. b years hence the elder will be one-fourth as old again as the other. Find x in terms of a and b.

30. i. A tank is rectangular and is a yds long and b yds. broad. To walk a distance of c miles, a person goes round the tank x times. Find x.

ii. If he goes p times round the tank, how many miles would he have walked?

iii. If the man takes 15 minutes to go round the tank once, find (a) his rate in miles per hour, (b) the time he took to take a walk of (1) c miles, (2) p times round the tank.

31. The number of coins I once paid to clear a V. P. P. was a. Some were two-anna pieces, and the rest one-rupee coins. The total amount of the V. P. P. was Rs. 4-12 as. If I had x one-rupee coins, find how many two-anna pieces there were in the amount I had paid.

32. A mason agreed to work for a days. It was stipulated that he should receive m annas for each day he worked and pay a fine of n annas for each day he was absent. If he was absent for x days, find how much he was entitled to.

33. A landowner finds the length and breadth of a rectangular piece of land by stepping along. If each step is on an average a inches and he takes x steps along the length and y steps along the breadth, find in terms of a, x and y:

(a) P, the perimeter of the field.

(b) A, the area of the field.

(*c*) D, the diagonal of the field.

Evaluate these when $a=2{\cdot}75$, $x=30$, $y=22$.

34. Two persons stood for a certain election. The voting list had x voters; a did not vote, and of the recorded votes b were rejected as illegal. The successful candidate won by a majority of c votes. If v denote the number of votes he secured, establish a relation between x, a, b, c and v. If $x=3390$, $a=50$, $b=25$ and $v=1720$, find c.

35. Two persons A and B, x miles apart, start at the same time and travel towards each other, A at a miles an hour and B at b miles an hour. Where will they meet? If P be the distance in miles, of the meeting place, from the terminus wherefrom A started, express x in terms of P, a and b. Find x when $a=3$, $b=5$, and $P=13{\cdot}875$; and P when $a=3$, $b=5$ and $x=45$.

36. A contractor supplied p planks of average length q ft. a of these were found useless and rejected. The rejected planks measured on an average b ft. r more were supplied for the rejected ones, each of average length c ft. Find the total number of planks accepted and the average length of each plank.

37. A workman is engaged for p days on condition he should receive q as. for every day he worked and pay a fine of r annas for every day he was absent. At the end of the period he received t annas. If he worked for x days and was absent for the remaining days, establish a relation between x, p, q, r and t.

If $p=35$, $q=20$, $r=6$ and $t=518$, find x.

38. There were a certain number of beggars, say x, and a gentleman had some rupees with him. If he gave each beggar a annas, p annas remained, and if he gave each b annas, q more annas were needed. Find x in terms of a, b, p and q.

If $a=7$, $b=7\frac{1}{2}$, $p=6$, $q=25$, find x.

39. To reach a shrine a pilgrim has to walk a ft. up an incline, b ft. along the level, and c ft., down an incline. His rates of walking uphill, on level ground and downhill are respectively p, q and r ft. per minute. If T

denote the hours he takes to reach the shrine, find the relation between T, a, b, c, p, q and r.

40. A man rides one-third of the distanoe from A to B at the rate of x miles an hour and the remainder at the rate of $2y$ miles per hour. If he had travelled at a uniform rate of $3z$ miles per hour he could have ridden A to B and back again in the same time. Establish a relation between x, y and z.

41. The average age of x boys in a school is a years. p boys left school and the average was reduced by b years. Find the average age of the boys that left the school.

If T denote the average, evaluate T correct to one decimal place when $x=230$, $a=14$, $b=1$, $p=29$.

42. If a man buys x dozen apples at a annas per dozen, and y dozen apples at b Rs. a dozen and sells $\frac{3}{8}$ of the whole lot at $2a$ annas per dozen and the rest at Rs. $1\frac{1}{2}$ per dozen, find his profit in annas.

Find his profit when $x=5$, $a=12$, $y=3$ and $b=2$.

43. A, B, C and D are four brothers a, b, c and d years of age respectively. A's age is equal to the sum of the ages of B, C and D. If x years hence, A's will be half as much as the combined age of B, C and D, find the relation between x, a, b, c and d.

44. A train has f first-class compartments, s second-class compartments and t third-class compartments. The charge per mile for each passenger is 1 anna, 6 ps., and $2\frac{1}{2}$ ps. by each class respectively. If each first-class passenger travels on an average x miles, each second class passenger y miles and each third class passenger z miles, what will be the income to the company for the train, if each first-class compartment can hold 4 passengers, each second class 3, and each third class 64 passengers, supposing that the train carried the full complement of passengers ?

45. m viss of ghee at Rs. 3-4-0 a viss are mixed with n viss at Rs. 3-8-0 a viss, and the mixture is sold at Rs. 3-5-0 a viss and the seller neither gains nor loses. Establish a relation between m and n.

46. In an election, out of x voters, a were absent, b remained neutral and the remainder voted. The rival candidates were P, Q and R; P got c more votes than R and Q was elected as he got as many votes as P and R together. Find, in terms of x, a, b, c, how many votes each secured.

47. From a purse containing Rs. a, b as., a boy stole away a as. b ps. on each of m different occasions and the purse then contained x as. Write down the relation that must exist between a, b, m and x.

If $a=13$, $b=6$ and $x=92{\cdot}5$, find m.

48. To ascertain the cubical contents of round timber the following instructions are given to the S. I. Ry. employees :—

"Take the girth of the log at the two ends and at the centre and add them together. Divide the total by 3 to get the mean girth of the log. Square the mean girth thus found, multiply it by the length of the log and divide the result by 12½ and the quotient will be the cubical contents of the log."

If a log measures a ft. at one end, b ft. at the other and tapers uniformly, the total length being c ft., find x the cubical contents in feet. If $a=6$, $b=10$, $c=16$, find x.

49. A man must be at his destination in an hour. He runs a distance of p miles at a miles an hour, then walks a distance of q miles at b miles an hour and again runs at x miles an hour, and thus covers the total distance s miles in the hour. Find in terms of the other letters the value of x.

If the total distance be 4½ miles, p, q, a, and b be respectively 1½, 1, 4 and 3, find x.

50. A piece of elastic rubber string of natural length p inches is attached by one end to a beam. To the free end a weight is attached and the string is stretched to a length of q inches. How far lower is (i) the mid-point of the original piece, (ii) a point originally a inches from the free end to which the weight is attached?

51. On a steamer there is stowage of s cub. ft. of luggage. When there are p passengers, their luggage occupies three-quarters of the storage. Assume that each passenger owns the maximum volume of luggage allowed by regulation and express this volume in cub. ft. If there were q additional passengers, prove that $(3q-p)\times s/(4p)$ additional cubic feet of stowage room would be required to avoid change in regulations.

Explain the result when $p=100$, $q=30$, $s=3600$.

EXAMPLES III.

Substitution in Formulæ.

Changing the Subject of the Formula.

1. $A=lb$, gives the area A of a room in sq. ft., the length of the room being l ft. and its breadth b ft.

(a) Find A if $l=18{\cdot}5$ and $b=8{\cdot}75$.

(b) Express l in terms of A and b, and find l if $b=8$ and $A=320$.

(c) Express b in terms of A and l and find b if $l=32$ and $A=288$.

2. $W=2h(l+b)$, gives the area of the four walls of a room in sq. ft.—l, b, h denoting the length, breadth and height respectively of the room in feet.

(a) Find l if $W = 600$, $b = 10$, $h = 12$
(b) ,, h if $W = 575$, $l = 15$, $b = 12$
(c) ,, b if $W = 500$, $l = 14$, $h = 10$
(d) ,, W if $l = 20$, $b = 15$, $h = 12$.

3. The length of a room is l ft., its breadth is b ft and its height is h ft. Find in terms of l, b and h, the value of T, which denotes the total area of the fou walls, floor and the ceiling of the room.

4. If A denote the area of a triangle in sq. in., b the length of its base in inches and h the length of the perpendicular on the base in inches, then $A = \frac{1}{2} bh$.

Find b if $A = 24$, $h = 6$
„ h if $A = 20$, $b = 8$
„ A if $h = 7$, $b = 8$.

5. If s denote the semi-perimeter of a triangle in inches and a, b, c, the lengths in inches of the three sides and $\triangle$ the area of the triangle in sq. in.

$$\triangle = \sqrt{s(s-a)(s-b)(s-c)}.$$

Also if h_a, h_b, h_c denote in inches the altitudes on sides a, b, c, respectively,

$$\triangle = \tfrac{1}{2}ah_a = \tfrac{1}{2}bh_b = \tfrac{1}{2}ch_c$$

Hence find the lengths of the three altitudes of a triangle whose sides are 12 inches, 13 inches and 5 inches.

6. If a and b denote the lengths in inches of the parallel sides of a trapezium, and h the distance in inches between the parallels,

$$\frac{h(a+b)}{2} = A$$

where A denotes in sq. in. the area of the trapezium.

Express h in terms of A, a and b, and a in terms of A, h and b. Also find b given that $A = 135$, $h = 10$, $a = 12$.

7. $(ab)/2 = \triangle$, gives the area $\triangle$ of a rt. angled triangle in sq. units, the lengths of the sides being a and b units.

$$\left.\begin{array}{l}\text{Express } a \text{ in terms of } b \text{ and } \triangle \\ \text{,,} \quad b \text{ in terms of } a \text{ and } \triangle\end{array}\right\} \begin{array}{l}\text{Find } a \text{ if } b = 6 \text{ and } \triangle = 15, \\ \text{,,} \quad b \text{ if } a = 8 \text{ ,, } \triangle = 36.\end{array}$$

8. If a side of an equilateral triangle is $2a$ inches, its area A in sq. in. is $a^2\sqrt{3}$. Hence find a if $A = 43{\cdot}03$ and find the length of a side of an equilateral triangle whose area is 43·03 sq. in.

9. The area S of a regular hexagon, one of whose sides is a is

$$S=\frac{3a^2}{2}\sqrt{3}$$

Express a in terms of S, and find S if the length of a side is (i) 2 in. (ii) 6 cms.

10. The area S, of a regular octagon, one of whose equal sides is a is

$$S=\frac{a^2}{4}(1+\sqrt{2}).$$

Find S if the side be 5 cms., and find a if $S=2{\cdot}14$ sq. cms.

11. The circumference of a circle of radius r is $c=\pi r$; and the area $S=\pi r^2$. Find r if $S=9\pi$, $c=6\pi$, and S if $r=4$.

12. The area of a circle is $S=\pi r^2$; if d be the diameter $d=2r$; hence express S in terms of π and d. Find S if $d=4$.

13. The simple interest I, on a sum of money P, for n years at r per cent per annum is $(Pnr)/100$.

Express P in terms of I, n and r.
Find n if $P=125$, $I=10$, $r=4$
" r if $P=150$, $I=15$, $n=2$
" P if $I=60$, $n=6$, $r=3\frac{1}{3}$.

14. If a sum of Rs. P be lent at compound interest at r per cent per annum, then the amount A in rupees at the end of n years is,

$$\left(1+\frac{r}{100}\right)^n \times P.$$

Find A if $n=3$, $P=100$, $r=5$.
" P if $A=1681$, $r=2{\cdot}5$, $n=2$.

15. If S denote in sq. in. the curved surface of a hollow cone the radius of whose base is r in., the height in inches being denoted by h, then $S=\pi r\sqrt{r^2+h^2}$.

Find S if $r=3$, $h=4$
" h if $S=16\pi$, $r=2$
" h if $S=30\pi$, $r=3$.

16. The volume v of a cylinder is $v=\pi r^2 h$, where r is the radius of the base and h is the height.

Find v if $r=3$, $h=4$
,, r if $v=12\pi$, $h=3$
,, h if $v=80\pi$, $r=4$.

17. The entire surface of a solid cylinder (the curved surface and the two flat ends), is $C=2\pi r(r+h)$.

Find C if $r=2$, $h=3$
,, r if $C=42\pi$, $h=4$
,, h if $C=80\pi$, $r=4$.

18. The curved surface of a cone S is given by the formula $S=2\pi r\sqrt{r^2+h^2}$ where r denotes the radius of the base, and h the height.

Find S if $r=2$, $h=4$
,, h if $S=130\pi$, $r=5$.

19. The volume V, of a cone whose height is h in. and the radius of whose base is r in., is $V=\pi r^2 h$.

Find V, if $r=6$, $h=8$
,, r if $V=12\pi$, $h=4$
,, h if $V=26\frac{2}{3}\pi$, $r=4$.

20. If a body weighs W grams in air and w grams in water, its specific gravity S is $W\,(W-w)$. Find w in terms of W and S. If $S=19$, $W=34$, find w.

21. The distance s feet fallen in time t seconds by a bomb dropped from an aeroplane vertically downwards is given by the formula, $s=ut+\frac{1}{2}gt^2$, where u denotes the initial velocity of the bomb.

(i) If $g=32$, $u=40$ and $t=18$, find s the height of the aeroplane.
(ii) Also express u in terms of s, g and t.
(iii) ,, ,, g ,, s, u and t.

22. If a certain temperature is F degrees Farenheit and the same temperature is C degrees Centigrade

$$F=32+\frac{9C}{5}.$$

Express C in terms of F and find C when $F=86$.

EXAMPLES IV.

Simultaneous Equations.

1. (a) The cost of 1 orange and 1 apple is 8 as. The cost of an apple is 5 annas. What is the cost of an orange?

(b) $x+y=8$; $x=5$. Find the value of y.

2. (a) The cost of 1 orange and 2 apples is 13 as. The cost of 1 orange is 3 as. What is the cost of an apple?

(b) $x+2y=13$; $x=3$. Find the value of y.

3. (a) The cost of 1 orange and 2 apples is 13 as. The cost of an apple is 5 as. What is the cost of an orange?

(b) $x+2y=13$; $y=5$. Find the value of x.

4. (a) The cost of 3 oranges and 4 apples is Re. 1–13 as. The cost of an orange is 3 annas. Find the cost of an apple.

(b) $3x+4y=29$; $x=3$. Find the value of y.

5. (a) The cost of 1 orange and 3 apples is Re. 1-2 as. The cost of 1 orange and 2 apples is 13 as. Find the cost of an apple; also find the cost of an orange.

(b) $x+3y=18$; $x+2y=13$. Find x and y separately.

6. (a) The cost of 3 oranges and 4 apples is Re. 1-13 as. The cost of 2 oranges and 2 apples is Re. 1. Find separately the cost of an orange and the cost of an apple.

(b) $3x+4y=29$; $2x+2y=16$. Find x and y.

N.B.—At this stage the teacher will show how, by multiplying one of the equalities by a certain number and the other equality by another number, the coefficients of one of the unknowns are the same in both the equations. The meaning of the word 'Simultaneous' and its applicability to this system of equations, should be explained. The student, of course, must have known by this time that finding the value of the unknown quantities is called 'Solving the equations'.

Solve the equations in examples **7** below :—

7. (i) $3x+2y=13$; $4x-y=10$.

(ii) $5x+y=17$; $y-3x=1$.

(iii) $11x-12y=8$; $7x+6y=46$.

(iv) $x+2y=8$. $6y-3x=3$.

(v) $2x-3y-\frac{3}{4}=0$; $5x+7y=20$.

(vi) $2x+y-2=0$; $x-\frac{1}{2}y=4$.

(vii) $4x+3y=0$; $x+y=\frac{1}{2}$.

(viii) $5x-4y=1$; $x+y+\frac{2}{7}=1$.

(ix) $3x+2y=6{\cdot}4$; $5x-3y=1{\cdot}8$.

(x) $x+3y=0$; $7x+11y=7$.

(xi) $3x+2y-11{\cdot}3 = x-y+1{\cdot}4=0$.

(xii) $5x+6y=2{\cdot}4$; $7x+11y=13{\cdot}5$.

(xiii) $5x-4y=x+2y-3=7x-6y-1$.

(xiv) $11x-7y=y-x=3x-2y+2$.

(xv) $4x+3y=3x+2y-4=11x+8y+2$.

(xvi) $\frac{6}{x}-\frac{4}{y}=4$; $\frac{3}{x}-\frac{1}{y}=3{\cdot}1$.

[*Hint*: Let $\frac{1}{x}=X$ and $\frac{1}{y}=Y$. Solve for X and Y.]

(xvii): $\frac{1}{x}+\frac{1}{y}=2\frac{3}{4}$; $\frac{3}{x}+\frac{2}{y}=7$. (xviii) $\frac{1}{x}+\frac{1}{y}=0$; $\frac{5}{x}+\frac{3}{y}+3=0$.

(xix) $\frac{5}{x}-\frac{4}{y}=\frac{1}{x}+\frac{1}{y}+\frac{2}{7}=1$. (xx) $\frac{2}{3x}-\frac{1}{y}=-\frac{1}{9}$; $\frac{3}{x}+\frac{3}{y}=8\frac{2}{3}$.

(xxi) $\frac{1}{x}+\frac{3}{y}=0$; $\frac{7}{x}+\frac{11}{y}=7$.

8. If $y=ax+b$ and if $y=0{\cdot}5$ when $x=2{\cdot}5$
and $y=2$ „ $x=3$
determine a and b.

9. The expression

$ax^2 + by^2$ has the value 15 when $x=3$, $y=2$
and has the value 16 when $x=2$, $y=1$.
Find its value when $x=5$, $y=3$.

10. If $P(x^2+y^2+z^2)+Q(xy+xz+yz)$ is equal to
20 when $x=3$, $y=2$, $z=1$
and 27 „ $x=0$, $y=3$, $z=2$
determine P and Q.

EXAMPLES V.

Equations—Problems.

N.B.—Problems marked with an asterisk may profitably be done in form IV.

*1. Two boys A and B begin to play a game of chance and skill. At the end of the game A found he had lost Rs. 8-8 as. and B then had half as much again as A. Originally B had Re. 1-8 as. more than A. What was the amount each had at the beginning?

*2. If 5 boys be seated on each bench 3 boys remain over, and if 6 were seated, there is room for 4 more boys. Find the number of benches and the number of boys.

*3. The denominator of a fraction is 6 more than the numerator. If 5 be subtracted from both the numerator and the denominator, the fraction becomes ½. Find the fraction.

*4. The denominator of a fraction exceeds the numerator by 5. If 2 be added to the numerator, the fraction becomes ¾. Find the fraction.

*5. From a certain number 5 is subtracted. The remainder is multiplied by three and the product is divided by 7. If 5 be added to the quotient the result is 17. Find the number.

*6. A father's age is thrice that of his son; in 18 years the father will be twice as old as his son; how old are they?

*7. A father is older than his son by 20 years. In 10 years the father will be twice as old as his son. How old are they now?

*8. In an examination the number of passes was ⅓ of the number of failures. The boys were re-examined and 15 more passed, so that the number of passes was ⅔ of the number of failures. How many sat for the examination?

*9. The number of spectators at a theatrical performance was 432. Some of them had paid Re. 1-8 as. and the rest 12 annas and the total collection was Rs. 468. How many took the costlier ticket?

*10. A certain number is 14 more than another. If the greater be divided by the smaller, the quotient is 2 and the remainder is 1. Find the numbers.

*11. A carpenter demands as a day's wages 8 as. more than a mason. I engage 6 masons and 5 carpenters and pay them Rs. 10-12 as. What is the daily wages of a carpenter?

*12. A milk-seller buys a certain quantity of milk at $2\frac{1}{4}$ measures per rupee and adds to it 23 ollocks of water, and by selling the mixture at $2\frac{1}{2}$ measures per rupee, gains 4 annas on the whole. How much did she buy? (1 measure = 8 ollocks.)

*13. A milk-seller buys milk at 3 measures for a rupee. How much water must she add to it, to reduce the price of an ollock to 6 ps., supposing she wants no profit.

*14. A person walks from A to B at the rate of 3 miles an hour, and after transacting some business which occupies him an hour, returns to A by the tramway, at the rate of 5 miles an hour. He then finds he has been absent 2 hrs. 20 min. Find the distance from A to B.

*15. The total fare for a journey of 504 miles, partly by main line and partly by branch line, was Rs. 17-11-6 ps. The rates per mile being 6 pies per mile on the main line and 8 pies on the branch line, what distance was travelled on the branch line?

*16. How far shall I drive at 10 miles an hour, with a friend who leaves the station at 9 A.M. that I may, by walking back at the rate of 4 miles an hour, reach home in $3\frac{1}{2}$ hours, from the time of starting?

*17. Forty-three friends started on an excursion. All of them journeyed on cycles. They had not the required number of cycles. Two rode on each of a few cycles and the number of cycles carrying two, was 13 less than the number of cycles carrying one. How many cycles were there?

*18. A father is now 35 years old. The elder son is 7 years and the younger is 5 years old. When will the father's age be equal to the sum of the ages of the two sons?

19. A company at a restaurant had Rs. 30-15 as. to pay. Two left without paying their share and each of the rest had 10 annas more to pay. How many were there at first?

20. A boatman rows 10½ miles up a river and back again in 5 hours; he finds that he can row 7 miles with the stream in the same time as 3 miles against it. Find the rate at which the river flows.

*__21.__ A certain sum of money can be divided equally among a certain number of people at 7 s. 6 d. each. If there had been 2 more, each would have received only 6 s. 8 d. How many people were there?

22. A walks half a mile per hour faster than B and three-quarters of a mile per hour faster than C. To walk 45 miles C takes ¾ of an hour more than B and two hours more than A. Find their respective rates of walking.

23. A cyclist had to go to a place 24 miles distant. He rode for a time at 12 miles an hour and then being tired, had to reduce his speed by 2 miles an hour and on the whole took 2 hrs. 7 min. to reach his destination. Find when and where he changed his speed.

*__24.__ If the breadth of a rectangular court be increased by 5 ft. and its length diminished by 7 ft. the area remains unaltered. If the breadth be 70 ft., find the length.

25. The length of a rectangle is 1¼ times the breadth. If the former be increased by 3 ft. and the latter diminished by 2 ft., the area is unaltered. Find the length and breadth of the rectangle.

*__26.__ A sum of Rs. 520 is lent partly at 5 per cent annual interest and partly at 7½ per cent. and the total annual income is Rs. 36. Find the sum lent at the higher rate.

*__27.__ A roll of cloth 50 yds. long is cut into pieces some 3 yds. long, and the rest 4 yds. long, and the total number of pieces is 14. Find how many there are of the longer piece.

28. A and B begin to take a walk of $3\frac{1}{8}$ miles. A gives B a start of 10 furlongs and overtaking him in 30 minutes completes the whole journey in 20 minutes more. Find the speeds of A and B.

29. A person had debts to the extent of Rs. 1800. On a certain portion of his debts he was able to pay at 12 as. in the rupee and on the rest at 8 as. in the rupee. The amount thus paid was Rs. 1150. On how many rupees did he pay the higher dividend?

30. An insolvent had two creditors, his debt to the former being Rs. 800 more than his debt to the latter. He paid the former 12 as. in the rupee and the latter 14 as. in the rupee. If he had divided his assets proportionately, he would have been able to pay $12\frac{1}{2}$ as. in the rupee. Find the amount of his debts.

31. One day a country postman went half his round at the rate of 3 miles per hour and the other half at the rate of 4 miles per hour and was 10 min. late. On the next day he went half his round at the rate of $3\frac{1}{3}$ miles per hour and the other half at the rate of $4\frac{1}{2}$ miles per hour and took 12 min. less than the allotted time. How far had he to go, and what time was he allowed?

32. A district which is in the shape of a square of 10 miles side, has been partly mapped to a scale of 4 in. to the mile and partly to a scale of 6 in. to the mile. The area of these two portions together on the map is 2500 sq. in. What area has been mapped to each scale?

33. A batsman has a certain average of runs for 16 innings. In the 17th innings, he makes a score of 85 runs, thereby increasing his average by 3. What is his average after the 17th innings?

34. I bought a horse and a carriage for Rs. 770. I sold the horse at a gain of 10 per cent. and the carriage at a loss of 15 per cent. and on the whole lost Rs. 28. Find the price I paid for the horse.

35. At a cricket match in the first innings, the runs scored by two batsmen A and B were in the ratio of 3 : 2. In the 2nd innings the former scored 12 runs, while the latter 58, and the ratio of the total runs in the two innings was reversed. How many did each score on the whole?

36. A boy playing marbles lost in the first hour all he had and got into debt for half a dozen more; in the second hour he gained double of all he had lost in the first and 40 per cent. besides of the number he started with, which, after he had paid his debt, increased his store of marbles at the end of the play by 48. How many marbles had he to start with?

37. A length of 60 centimetres is divided into equal parts. What is the number of these parts, if, when this number is increased by unity, the length of each part is decreased by 1 millimetre?

38. At an election where only seven-eighths of the whole number of voters voted, one of two candidates received a fourth as many votes again as the other and beat him by 896 votes. What was the total number of voters?

39. A caterer undertook to supply luncheon at 14 as. a head, but stipulated that the cost of those that might be absent should be borne by the rest. Five were absent and the rest had therefore to pay 2 as. each extra. For how many was the luncheon ordered and what was the total amount the caterer got?

40. How can 45 be divided into four such parts, that if to the first part you add two, from the second part you subtract two, the third part you multiply by 2 and the fourth part you divide by two, the sum of the addition, the remainder of the subtraction, the product of the multiplication, and the quotient of the division be all equal?

41. A man has 40 acres of land. The more fertile lands, he let on lease at Rs. 60 per acre and the less fertile at Rs. 45 per acre. Next year the same extent of fertile lands was let at Rs. 75 per acre and the rest at Rs. 35 per acre; but his income was unaltered. Find the extent of the fertile lands by framing two equations connecting the extent of the fertile lands and the corresponding total income.

42. Almost all we know of Diophantus, the only algebraist Greece produced, is contained in the following epitaph. From it find how long he lived: "Diophantus passed $\frac{1}{6}$ of his life in childhood, $\frac{1}{12}$ in youth, and $\frac{1}{7}$ more as a bachelor; five years after his marriage was born a son who died four years before his father at half his father's age at death."

43. Eleven persons contributed a certain sum. Nine of them gave Rs. 5 apiece, and the other two persons gave Rs. 5 and Rs. 10 more respectively, than the average subscription of the eleven subscribers. What sum did the two persons give?

44. From A to B is 52 miles by train and 11 miles by motor. If the train costs Rs. 2-4 as. per mile, no matter how many passengers there are, and the motor-car costs 10·8 pies per person per mile, how many passengers must there be in order to clear expenses and get a profit of Re. 0-4-0 per mile, if an anna a mile is charged per passenger for the whole journey?

45. I wanted 20 tons of casuarina logs for firewood. At a place A, 18 miles distant, it was sold at Rs. 7-8 as. per ton, and the cartage was at 3 as. per mile per ton. At another place B it was sold at Rs. 8 per ton, and the cartage was at 3 as. 10 ps. per ton per mile. I found it cost me the same amount if I ordered it from either A or B. How far is place B from my house?

46. Three motor-cars leave Madras at 9, 9-30, and 10 a. m., travelling along the same road at uniform rates. The second travels three miles an hour faster than the first, and the third five miles an hour faster than the second. If all the three meet one another t hours after the starting of the first car and if the rate of the first car be x miles an hour, frame equations and find the rates and the distance of the meeting point from Madras.

***47.** A man bought 15 railway tickets for Rs. 165-6-0; some of them being second class, and the rest third class ones. The price of each second class ticket was Rs. 15-12-0, and that of each of the third class Rs. 7-14-0. How many of each class did he buy?

***48.** The distance by road from a village to a railway station is x miles. A short cut by fields is 3 miles shorter. A person drives by the road at 8 miles an hour, and the next day he leaves his house at the same time and runs by the footpath at 5 miles per hour, and reaches the station just at that time as he did on the previous day. Find the length of each route.

49. A person wanted to engage the services of a female cook. Three cooks offered their services. One demanded Rs. 10 a month, free boarding and two cloths every year; the second demanded Rs. 15 a month and free boarding, while the third wanted Rs. 25 per month and two cloths every year. He found that it would cost him the same if he engaged any of them. At how much did he value the cloth and the boarding per month separately? What would it cost him per annum if he engaged one?

50. A caravan set out, each riding a horse and leading another. On the way 17 of the party and thirty horses died and 80 horses more were sold. If on their return each one had only the horse he rode on, find how many men were originally in the troop.

51. Two rival candidates A and B stood for an election and A was elected by a majority of 90. If a fifth of those that voted for A had, as feared, voted for B, the latter would have been elected by a majority of 12. How many votes did each secure, and how many did vote?

52. A cocoanut-seller bought 2000 cocoanuts at Rs. 6 per hundred, and assorted them into 3 varieties. The first variety he sold at 1 a. 3 ps. each, the second at 1 a. each and the third at 9 ps. each. He remembered that there were 400 in the last variety. How many should there have been in each of the other two varieties if he had gained 12½ % on the whole?

***53.** A person bought the materials of an old house for Rs. 1800, on condition that they should be removed within 12 days under a penalty of Rs. 15 a day for every day beyond the 12 days. He paid Rs. 52-8 as. a day for labour, and having sold the materials for Rs. 3217-8 as. found that he had made a profit of Rs. 450. How many days did he take to remove the materials?

***54.** A merchant mixes 80 maunds of one kind of rice at Rs. 4 per maund with a certain quantity of another kind at Rs. 3-8-0 per maund, and finds that the mixture costs him Rs. 3-13-0 a maund. Find how many maunds of the second kind there are in the mixture.

55. A man's total income is made up partly of his salary and partly of income derived from investments, his salary being 4 times as great as the rest of his income. He has to pay income-tax at the rate of 9 d. in the pound on his salary and at the rate of 1 s. 2 d. in the pound on his other part of his income. He is allowed an abatement of £ 120 from his salary and the total amount of income-tax that he pays is £ 18-8-4 d. Find his total income.

56. The price of a lady's ticket for a concert is 12 as. and a gentleman's Re. 1; the refreshments are contracted for at 10 as. a head and the miscellaneous fixed expenditure is Rs. 75. Find the smallest number that must be present on the supposition that there are as many gentlemen as ladies, in order that the expenses may be covered.

57. I lived in a village a few miles distant from the town. I was summoned as a witness to appear before the Judge in the town. On the day posted for hearing, I left my village and travelled by a bullock-cart at 4 miles an hour, and was half-an-hour late. Fortunately for me, I learned that the hearing was adjourned. Next time I biked the distance at 8 miles an hour. I was delayed on the road for one full-hour as the bike gave some trouble. I however reached the court 45 minutes before time. How far is the town from the village?

58. Two stout elastic cords are attached to a beam. One is $2\frac{1}{2}$ feet and the other 3·6 feet long. The former stretches to an additional length of 0·3 inch for every seer weight attached to the free end, while the latter for every seer stretches by 0·2″. Equal weights are attached to each and the weights hang at the same height from the floor. Find the weights attached.

59. In a cricket match between two schools A and B, A scored 85 runs more than B in the first innings; but in the second innings B scored ·64 per cent. more runs than A and won the match by 43 runs. What were he scores in the second innings?

60. The people of a certain village petitioned for a telegraph office in the village. The Superintendent opened one, tentatively for a year, on condition that the income of the telegraph department in the year must average Rs. 20 a month, failing which the office will be abolished. In the first eleven months it was found that 246 messages had been despatched from the office each of the average income of as. 14. To make up the deficiency 24 messages were sent in the last month by the villagers, some at Re. 1-2 as. each and the rest at 12 as. each. Find how many of each kind were despatched from the office in that month.

61. The owner of a motor-car organised a raffle to dispose of his car. He printed a number of tickets and found that if each ticket be priced at Rs. 5, he would get Rs. 100 more than what he expected for his car, and that if each ticket be valued at Rs. 4, he would get Rs. 350 less than what he expected. In each case however he deducted 10% of the collections towards commission to ticket-sellers and incidental charges. How many tickets did he print and how much did he want for his car?

62. (i) A cyclist goes at the rate of p miles an hour. How far can he ride so that by returning at q miles an hour, he may be back in r hours from the time of starting?

(ii) A man had to wait at a railway station for 3 hours. He had a cycle with him and wind was steadily blowing all the time from the station along a road. He can bike at 12 miles an hour with the wind and 4 miles an hour against the wind. How far can he take a ride on his cycle to be in the station in time to catch the train?

63. (i) To build a wall 12 masons and 22 coolies were engaged on the first day, and were paid Rs. 15-5 as. that evening. Next day only 10 masons and 16 coolies turned up and these were paid Rs. 12- 4 as., the rates being the same as before. What is the daily rate of wages for a mason and for a cooly separately?

(ii) If on the third day 11 masons and 19 coolies worked at it and finished the work, how much would they have received?

64. A retail dealer bought 35 gallons of wine at Rs. 6-3 as. a gallon and 28 gallons at Rs. 7-13 as. a gallon. He mixed the two and mixed some water also. The whole mixture was put into quart-bottles which cost him Rs. 8-11 as. He sold each bottle at Re. 1-12 as., and gained Rs. 60 on the whole. How much water did he mix?

65. In the second of two particular years the annual income of a gentleman was Rs. 60 more than in the first. While in the first year, the gentleman spent one anna in each rupee of income, on charities, in the second year he spent 1 a. 3 ps. in the rupee: but the balance in the two years was the same. Find his income in each of the two years.

66. The gross income of a certain person was Rs. 4 more in the second of two particular years than in the first: but as he paid income-tax at the rate of 4 pies in the rupee in the first year and at the rate of 5 pies in the rupee in the second year, his net income in the second year was Rs. 6½ less than his net income in the first. What was his gross income in each year?

67. When a viss of coffee seeds sold at Rs. 3-8-0 and a viss of sugar at 10 as., a consumer spent on those Rs. 6-10-6 per month. The price of coffee fell by 25% and the price of sugar rose by 20% and the cost of consumption was only Rs. 5-10 as. How much of coffee seeds and how much of sugar did he buy each month?

*__68.__ How much fresh water to the nearest gallon must be added to 50 gals. of sulphuric acid that the liquid may weigh as much as an equal quantity of salt water, the specific gravity of fresh water being 1, that of salt water 1·0263 and of sulphuric acid 1·85?

69. At an election 10 per cent. of the people on the voting list did not vote and 60 votes recorded were rejected as illegal. The majority of the successful candidate was 308 and it was found that he had been supported by 47 per cent. on the voting list. What was the number of legal votes recorded for each candidate?

70. A candle P was lighted at 6-6 p.m. and a candle Q at 6-30 p.m. They were of the same length at 8-30 p.m.; also *P* was 1 inch long at 10-15 p.m. and *Q* 1 inch long at 11 p.m. Assuming that they burned away uniformly, find their original lengths which differed by 2 in.

71. A farther's age is equal to the united ages of his five children. In 15 years, his age will be only one-half of their united ages. How old is the father?

*72. I wished to give a certain number of old men 1 anna 8 pies each and found that I had not money enough in my purse by 11 annas; so I gave them 1 anna 5 pies each and then I had money enough and 3 annas 3 pies to spare. Find the number of old men.

*73. *A* has Rs. 6 more than *B*, but if *A* give to *B* half his money, and then *B* give back to *A* one-quarter of his increased sum, they will have each the same sum. Find what amount each has.

74. A tradesman sells two articles together for Rs. 46, making 10 per cent. profit on one and 20 per cent. profit on the other. If he had sold each article at 15 per cent. profit, the result would have been the same. At what price did he sell each article?

75. A farmer bought equal numbers of two kinds of sheep, one kind at Rs. 6 each, the other at Rs. 8 each. If he had expended his money equally in the two kinds, he would have had three sheep more than he did. How many of each kind did he buy?

*76. A person bought a certain number of cocoanuts: half at 3 for 2 annas, and the rest at 5 for 4 annas. He sold them at 9 for 8 annas and gained Rs. 7-4-8. How many did he buy?

77. A sum of money was divided equally among a certain number of persons. Had there been 8 more each would have received 10 pies less and if there had been 10 fewer, each would have received 1 a. 8 ps. more. Find the number of persons.

78. The gross income of a person was £ 40 more in the second of two particular years than in the first, but in consequence of the income-tax rising from 4 d. in the pound to 6 d. in the pound in the second year, his net income after paying the tax was unaltered. Find his income in each year.

79. A market woman sells 1000 oranges, some at a gain of 25 per cent. and the rest at a gain of 15 per cent., and thereby gains 18 per cent. on the whole. How many of each sort did she sell?

80. Two passengers have together 7 maunds of luggage, and for the excess above the weight allowed free, one of them is charged Rs. 3 and the other Rs. 5. If all the luggage had belonged to one passenger he would have been charged Rs. 11. What amount of luggage is each passenger allowed free of charge?

81. A composition of copper and tin containing 140 cub. in. weighs 42 lbs. 3 oz. How many ounces of each are there if a cubic inch of copper weighs 5¼ oz. and a cub. inch of tin 4¼ oz.?

82. A person has a certain amount of money to divide among a certain number of people. If he gives Rs. 2 to each, he will have Rs. 20 left over; but there are 10 people too many for him to give Rs. 2-8 as. to each. Find the amount he has to give away, and the number of people.

83. A train performs a journey of 64 miles in 2 hrs. 5 min. A certain portion of the railway is uphill and the remainder 22·2 miles downhill. The rate of the train uphill is 5 per cent. less and downhill 20 per cent. more, than its rate on the level. Find its speed on the level.

84. In a certain municipal election two candidates A and B competed. A got 10 more votes than B; but 5 per cent. of A's votes and 4 per cent. of B's votes were declared illegal and therefore B won by the narrow majority of 1. Find the number who voted.

85. (1) A merchant mixed 80 bags of superior paddy worth Rs. 6 per bag with a certain quantity of inferior paddy worth Rs. 4-4 as. per bag and by selling at 7 measures for a rupee gained Rs. 184. If a bag contain 50 measures, find the quantity, in bags, of the inferior paddy in the mixture.

(2) At how many rupees per bag must he have sold so as to neither gain nor lose?

86. If 1 be subtracted from the numerator of a fraction, the fraction becomes $\frac{1}{2}$; but if 9 be added to the denominator, the fraction $= \frac{1}{3}$. Find the fraction.

87. If 2 be subtracted from the numerator and the denominator increased by 1, the fraction becomes $\frac{1}{2}$, but if 7 be added to the numerator and the denominator diminished by 1, the fraction becomes equal to 1. Find the fraction.

88. If the length of a rectangle be diminished by 10 yds., and the breadth increased by 10 yds., the area is increased by 30 sq. yards; but if the length be increased by 10 yards and the breadth diminished by 13 yards, the area is diminished by 500 sq. yards. Find the length and breadth.

89. If the length of a rectangular field be diminished by 30 ft. and the breadth increased by 20 ft. the area remains unaltered. If the length be increased by 5 ft. and the breadth diminished by 20 ft., the area is diminished by 3200 sq. ft. Find the length and breadth.

90. A farmer took to the market 50 cocoanuts and 15 pumpkins and realised Rs. 14-11 as. On another occasion he took 40 cocoanuts and 18 pumpkins and selling them at the same rate as before realised Rs. 16-1 a. What was the price of each pumpkin and each cocoanut ?

91. I bought a house and its furniture. I sold the house at a profit of 20 per cent. and the furniture at a loss of 10 per cent. and gained Rs. 540 on the whole. If as advised, I had sold them at a uniform profit of 12 per cent. I would have gained Rs. 792. How much did I pay for each ?

92. 4 viss of coffee seeds and $3\frac{1}{2}$ viss of sugar cost Rs. 15; 5 viss of coffee seeds and 4 viss of sugar cost Rs. 18-6 as. Find the price of a viss of sugar.

93. A person has two horses and a saddle worth Rs. 75. If the saddle be put on the first horse, his value becomes double that of the second; but if the saddle be put on the second horse, his value will be less than that of the first horse, by Rs. 350. What is the value of each ?

94. The dimensions of a rectangular court are such that if the length were increased by 3 yards and the

breadth diminished by the same, its area would be diminished by 18 sq. yds., and if its length were increased by 3 yards, and its breadth increased by the same, its area would be increased by 60 sq. yds. Find its dimensions.

95. I bought a horse and a carriage for Rs. 1350. I sold the horse at a gain of 12 per cent. and the carriage at a loss of 4 per cent. and gained on the whole 6 per cent. Find the prime cost of the carriage.

96. The opposite angles of a quadrilateral inscribed in a circle are together equal to two right angles. If ABCD be such a quadrilateral, and it is given that the number of degrees in B is half the sum of that in A & C increased by 10 degrees, and that in D is half their difference increased by 40; find separately the magnitudes of the four angles in degrees.

97. When a certain number is divided by 17, the remainder is 3, and when the same number is divided by 51, the remainder is 20; but the quotient in the latter case is seven less than that in the former. Find the number.

98. A mathematician was asked how many quires of paper he bought on a certain day. He gave the following reply :—"If I had 2 as. 6 ps. more than what I took with me and the price of a quire were 6 ps. less I would have bought 3 quires more than what I have bought. If I had 6 as. 6 ps. more and the price of a quire were 6 ps. less I would have bought $4\frac{1}{3}$ quires more." What was the amount he took with him, and what was the price paid by him for a quire?

99. The strength of the Sixth form of a certain school is 102, of the Fifth 88, and of the Fourth is 110. The rate of school fees in a certain instalment was Rs. 6-8, Rs. 5-8 and Rs. 4-8 respectively per student. The total collections came up to Rs. 1560 and the writer remembered that 16 students had not paid their fees and that all the students of the Fifth form had paid their fees. How many of the Sixth and how many of the Fourth had been defaulters.

100. At an election there were two rival candidates and their supporters were conveyed to the polling booths in carriages capable of accommodating 8 and 12 voters respectively. If the voters, 740 in all, just filled 75 carriages find by what majority the election was won.

101. In order to reach a certain place, a railway transports passengers x miles by train, and y miles by a service of motor cars. The train costs the company 34 d. per mile and the provision of the necessary motor cars 26 d. for each mile of the remaining distance. If 32 tickets for the journey are taken at a through rate of 1 penny a mile, the receipts are £ 10-13-4 d., and the railway just clears expenses, find the distance of service by train.

102. ABC is an isosceles triangle; AB = AC. The angle A contains $2x+3y$ degrees; B, $3x-y$ degrees; and C, $4x-2y$ degrees. Find in degrees the angles of the triangle.

103. If incomes are taxed at the rate of x pence in the pound, but y pounds out of the total are exempt from the tax, how much does a man pay whose income is m pounds? To what rate in the pound would this be equivalent, if his whole income were taxed?

Find x and y if the tax is equivalent to 1 s. 2 d. in the pound on the whole of an income of £ 3000, and to 1 s. 1 d. in the pound on £ 2625; and find to the nearest pound, the incomes on which the proposed tax will then be equivalent to 1 s., to 11 d., and to 10 d., respectively in the pound on the whole income.

104. (*a*) If a quadrilateral be inscribed in a circle the opposite angles are together equal to two right angles. In such a quadrilateral ABCD, angle A is $(6x-2y)^\circ$; B, $(2x+y)^\circ$; C, $(x+y)^\circ$; and D, $(5y-8x)^\circ$. Find the magnitude of each angle of the quadrilateral.

(*b*) Given that the diameter of the circle is 3·4 inches and that the side AB is equal to 1·2 inches, describe the circle with the quadrilateral and find the lengths of the other sides both by calculation and measurement.

105. A certain resolution was carried in a council by a majority which was equal to one-third of the number of votes given on the losing side: but if with the same number of votes, 10 more votes had been given to the losing side, the resolution would only have been carried by a majority of one. Find the number of votes given on each side.

106. I distribute a certain sum of money equally among a certain number of persons. If there were 24 less, each will receive 1 a. 6 ps. more, and if there were 40 more each will receive 1 a. 6 ps. less. How many are there ?

107. Twenty-two mansons and 30 coolies were one day employed on a building and were given for wages Rs. 33-3 as., while at the same rates 18 masons and 28 coolies were given Rs. 28-2 as. the next day.

Find—i. the wages of a mason and a cooly ;
ii. The total cost for the three days, if 40 masons and 73 coolies were set on the third day.

108. The distance from my house to the Railway station is t miles. Another foot-path is a miles shorter. I walk at x miles an hour, and reach the station b minutes after a friend, who took the shorter route and walked at m miles an hour. Find my rate in terms of t, a, b and m.

109. The marks secured by examinees in a certain examination had to be revised. The individual mark x was revised to y, so that $y=ax-b$; where a and b are arithmetical numbers. The original marks 32 and 68 obtained by two students were in the revised list 29 and 56 respectively. Find a and b and the revised marks of boys who originally secured 42 and 36.

110. If F and C denote the same temperature in the Farenheit and Centigrade thermometers they are connected by a relation $\mathrm{F}=p\mathrm{C}-q$ where p and q are arithmetical Nos. The student knows that 0 of Centigrade corresponds to 32 of Farenheit and 100 of Centigrade to 212 of Farenheit. Find p and q.

111. In a certain hostel attached to a college the establishment expenditure which does not depend on the number of members is Rs. a. If there be x students the total expenditure $y = px + a$. It was found that the total expenses was Rs. 241 when there were 26 members, and Rs. 343 when there were 38 members. Find p and a. Find the total cost when there are 40 members.

112. If a st. line passes through the point (x, y) its equation is of the form $y = mx + c$. You know that two points determine a st. line. If the line passes through the points $(0, -5)$ and $(-3, 4)$; find the equation of the line.

113. A man aged 30, has Rs. 4000, which he invests in a bank paying 6% simple interest. Out of his earnings he is able to save Rs. 100 per month which he goes on hoarding without lending for interest. He wants to resign his appointment, and lead a peaceful life at a time, when the money he has laid by, together with the amount he would then draw from the bank, would be sufficient to enable him to spend Rs. 100 a month till he attains his seventy-fifth year. Find, to the nearest integer, when he may resign his office.

114. The resistance R, in pounds per ton, to the motion of a railway train is roughly given by the formula $R = a + bv^2$, when v is the speed in miles per hour, and a and b are numbers which are the same for all values of R and v. When $v = 20$, $R = 9{\cdot}6$ and when $v = 40$, $R = 20{\cdot}4$; find the values of a and b and find R when $v = 35$.

115. In a sugar-refining factory the cost of production is partly constant and partly dependent upon the quantity of sugar produced. When 2,600 tons of sugar are produced the total cost is £ 7,870 and when 3,000 tons are produced the total cost is £ 8,350. What will be the expense when 10,000 tons are produced?

SECTION II.

Elementary Geometry.

Exercises are based on the following Geometrical truths.

1. *If a st. line stand on another st. line the adjacent angles are together equal to two right angles.*

2. *If two st. lines cut, the vertically opposite angles are equal.*

3. *If a st. line cuts two parallel st. lines,*
   - i. *the alternate angles are equal.*
   - ii. *the interior angles on the same side of the cutting line are together equal to two rt. angles.*
   - iii. *the exterior angle = the interior opposite angle on the same side of the cutting line.*

4. (a) *The three angles of a triangle are together equal to two right angles.*
   - (b) *The exterior angle = the sum of the two interior opposite angles.*

5. i. *The base angles of an isosceles triangle are equal.*
   - ii. *The bisector of the vertical angle of an isosceles triangle is perpendicular to the base and bisects the base.*

6. (a) *The interior angles of a polygon of n sides are together equal to* $(2n-4)$ *rt. angles.*
   - (b) *Each interior angle of a polygon of n sides is equal to* $\frac{2n-4}{n}$ *rt. angles.*
   - (c) *If the sides of a polygon be produced in order, the exterior angles are together = 4 rt.* $\angle$*s.*

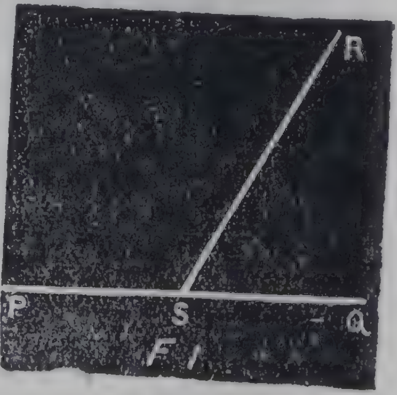
7. *Two triangles are identically equal if*
   - (a) *two sides and the included angle of the one are equal to two sides and the included angle of the other, each to each.*
   - (b) *the three sides of the one are equal to the three sides of the other, each to each.*
   - (c) *two angles of the one are equal to two angles of the other, each to each, and a side of the one = to the corresponding side of the other.*

8. *Any two sides of a triangle are together greater than the third.*

EXAMPLES VI.

N.B.—The geometrical principles involved in Problems 1—31 marked with an asterisk must be drilled in, in Form IV.

***1.** In figure 1, angle RSQ contains 73 degrees. How many degrees are there in angle RSP?

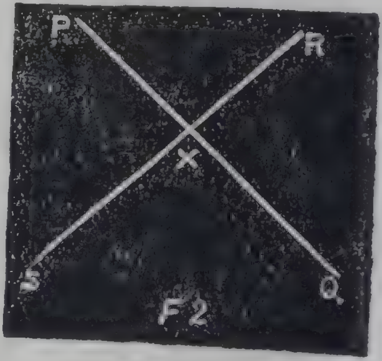


***2.** If angle RSP contains x°, how many degrees will there be in angle RSQ?

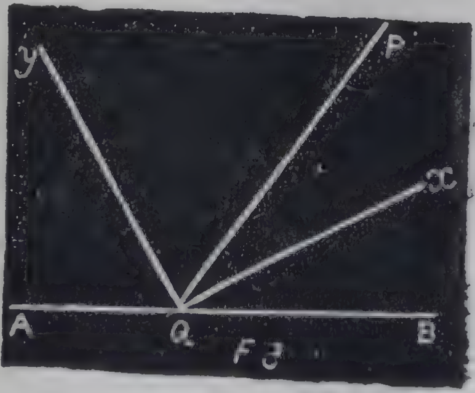
***3.** (*a*) The magnitude of angle RSQ is $(3x+7)$ degrees and that of RSP is $(2x+3)$. Find each angle in degrees.

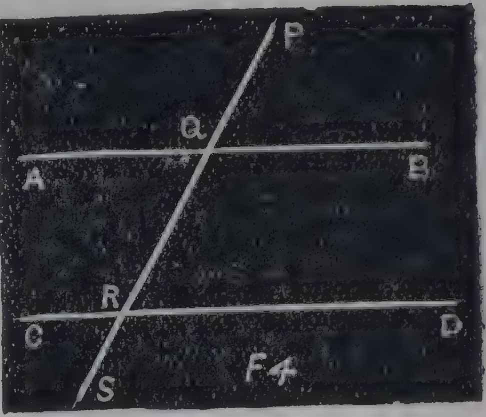
(*b*) From a point O, straight lines OA, OB, OC, OD, OE are drawn in order. Angle $AOB = 57^\circ$, angle $BOC = 91^\circ$, angle $COD = 32^\circ$, angle $DOE = 147^\circ$. Determine any pairs of lines which are (*a*) in the same st. line, (*b*) at right angles.

***4.** PQ and RS are straight lines cutting at X, Fig. 2. Angle PXR contains $3x - y$ degrees; angle RXQ contains $2x + 3y$ degrees and angle SXQ, $7x - 3y$ degrees. Find the magnitude of each angle in degrees.



***5.** In Fig. 3. Qx bisects angle PQB and Qy bisects angle PQA; angle PQB contains $3x - y$ degrees, angle PQA contains $7x - 3y$ degrees and angle xQy contains $2x + 10y$ degrees. Find the magnitude of each of the angles mentioned.





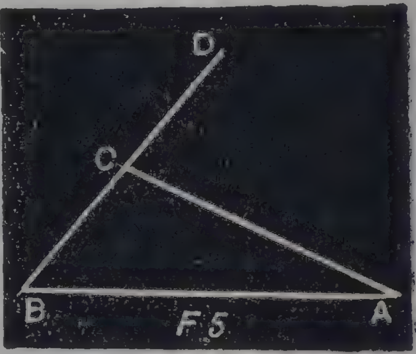
*6. AB and CD are parallel lines. PQRS cuts them.
$\angle$ PQB contains $(x+y)^\circ$
$\angle$ QRD „ $(4x-9y)^\circ$
$\angle$ BQR „ $(2x+y)^\circ$
Find the magnitude of each of the various angles in the figure. Write down (1) the two sets of equivalent angles, (2) the sets of supplementary angles.

*7. In the triangle ABC the side BC is produced to D. (Fig. 5).

The magnitude of $\angle$ A is $(2x+y)^\circ$
„ „ $\angle$ B is $(3x-3y)^\circ$

What is the magnitude of the angle ACB ?

If $\angle$ ACD is $(3x - y)^\circ$ what is the relation between x and y ?



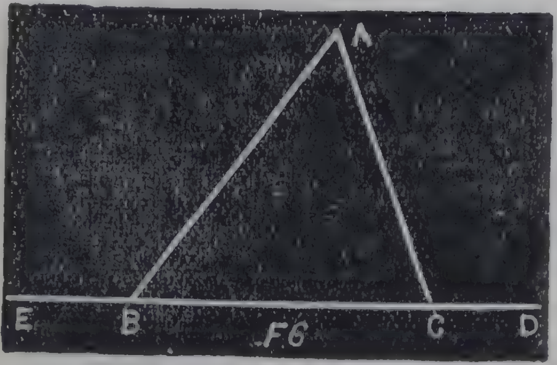
*8. In the same figure the magnitude of

$$\left.\begin{array}{ll}\angle A & =(2x+y)^\circ \\ \angle B & =(9x-3y)^\circ \\ \angle C & =(9x+2y)^\circ \\ \angle ACD & =(6x+3y \\ & \qquad +10)^\circ\end{array}\right\}\text{find the magnitudes of the angles of the triangle.}$$

*9. The number of degrees in an angle is denoted by $(x-1)$ and the number of degrees in its complement by $(6x+7)$. Find in degrees the magnitude of the angle.

*10. The number of degrees in an angle, its complement and its supplement are respectively denoted by $2x-y$, $3x-7y$ and $3x+2y$. Find the magnitude of the angle.

***11.** In Figure 6. ∠ ACD = 110 degrees. ∠ ABE = 120 degrees. Find the magnitude of each angle of the triangle.



***12.** (a) In the same figure if ∠ A = 60° and ∠ ACD = 132° find the angles of the triangle.

(b) If angle B = a degrees and exterior angle ACD = b degrees, find the magnitude of each angle of the triangle in terms of a and b.

(c) If $B + C = 88°$ and $B - C = 4°$ find the angles of the △ ABC.

***13.** (a) In fig. 2 angles PXR, RXQ and QXS are together = 258 degrees. Find the magnitude of each of the four angles.

(b) If in the same figure the three angles given above, be together equal to a degrees, find the magnitude of each of the other angles in terms of a.

***14.** If in Fig. 2 angle PXR be equal to $(2a - b)$ degrees find the magnitude of each of the four angles in terms of a and b.

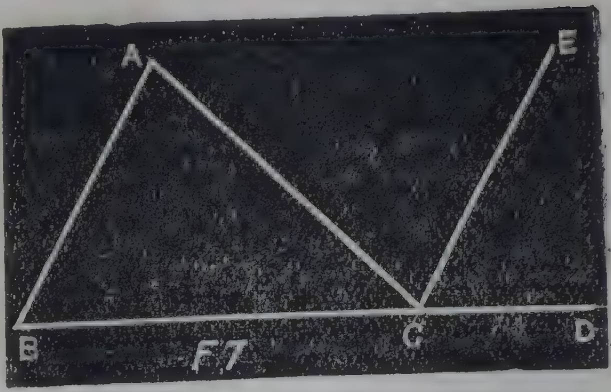
***15.** The angles A, B and C of a triangle are equal to x, y and z degrees respectively. If BO and CO bisect angles B and C, find the magnitude of the angle BOC in terms of

i. y and z; and hence in terms of
ii. x alone.

***16.** Similarly if BT and CT bisect the angles B and ACD, D being in BC produced, find the magnitude of the angle BTC.

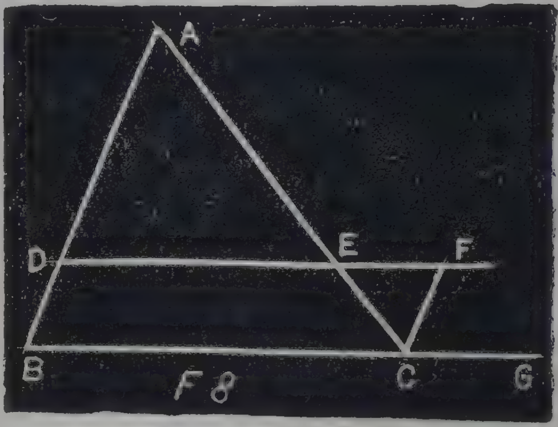
***17.** In figure 7. CE is parallel to BA, $\angle A = m^\circ$, $\angle C = n^\circ$ and $\angle ECD = p^\circ$; find the magnitudes of the angles ABC and ACD.

If $m = 65$, $n = 55$ find p.



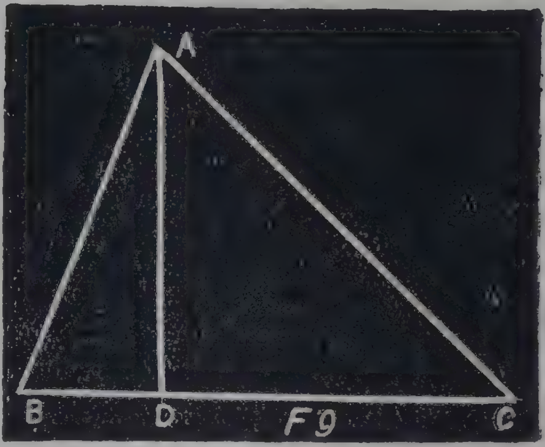
***18.** In Fig. 8. $\angle A = 60^\circ$, $AED = 50^\circ$, and $\angle FCG = 70^\circ$; CF is parallel to BA.

What do you know about the figure DFCB?

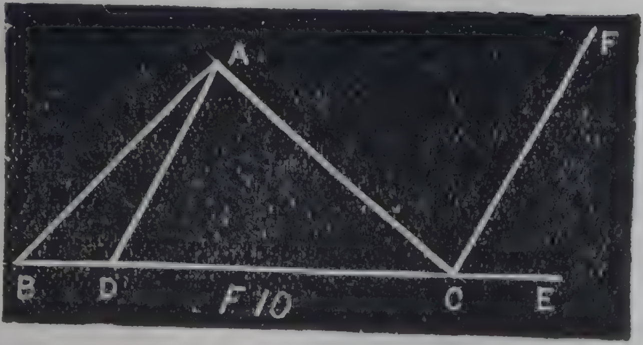


***19.** AD is perpendicular to BC in Fig. 9. $\angle BAD = x^\circ$;

$\angle ACB = y^\circ$; find the magnitude of each of the other angles in the figure.



*20. ABC is a triangle and BC is produced to E. D is any point in BC.



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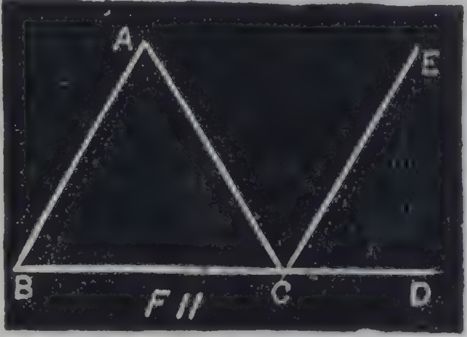
Given that $\angle ABD = x°$, $ADC = y°$ and that $DAC = z°$ and CF is drawn parallel to DA, find the magnitudes of all the angles separately, in terms of x, y and z.

*21. (a) If in a triangle ABC, the side AB = AC, what do you know about the angles ABC and ACB?

(b) If the angle $ABC = x°$, find the magnitude of each of the other angles.

(c) In Fig. 11, ABC is an isosceles △. AB = AC and BC is produced to D and CE is ∥ to BA.



F 11

If angle $B = x°$ and angle $ACE = 62°$; find the value of x and the magnitude of each angle in the figure.

*22. If one angle of a triangle is a rt. angle and the other two angles are equal to $x°$ and $y°$ respectively, what is the relation between x and y?

*23. ABC is a triangle rt.-angled at A : AD is drawn perpendicular to BC. Find the magnitude of each angle in the figure if one of the other two angles of the original triangle is $x°$.

*24. In Fig. 11, if AX be drawn perpendicular to BC, what do you infer about the angles BAX, CAX and the lengths BX and CX.

*25. In triangle ABC, side AC is greater than AB. From AC, AD is cut off = BA. If angle $A = a°$ and angle $B = b°$ find in terms of a and b the magnitude of each of the angles ABD, DBC, DCB.

***26.** The side BA of a triangle ABC is produced to D such that AD = AC. If angle A = 38° and C = 42° find the magnitude of the angles BDC, BCD.

***27.** D is any point in the base BC of any isosceles triangle ABC; DE and DF are perpendiculars from D on the sides AB, AC respectively and AG is the altitude from A on BC. Find by measurement the lengths DE, DF and AG. What relation exists between these three lengths?

***28.** From any point in an equilateral triangle drop perpendiculars on the sides. Shew by careful measurement that the sum of these three lengths = the length of an altitude of the equilateral triangle.

***29.** An angle contains $(5x-6y)$ degrees. Its complement contains $(6x+2y)$ degrees, and its supplement $(12x+8y)$ degrees. Find the magnitude of the angle itself.

***30.** (a) An angle contains x degrees and is $3\frac{1}{2}$ times its complement. What is the magnitude of the angle?

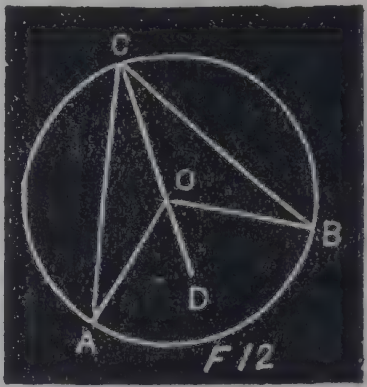
(b) An angle is a times its complement. Find its magnitude in terms of a.

***31.** (1) The complement of an angle is a times the supplement of the angle. Find its magnitude.

(2) If $a=\frac{1}{3}$ find the angle in degrees.

(3) If $a=4$ " "

32. In Fig. 12, O is the centre of the circle. ∠ ACO = 25° and ∠AOB = 110°; find the magnitude of the angles ACB, CBO, COB and AOD.



33. (a) Draw a number of lines OA, OB, OC.........OZ radiating from O. Find the sum of the angles AOB, BOC, COD...........ZOA.

(b) Draw a convex polygon of 8 sides. Take any point O within the figure. Join OA, OB, OC etc. A, B, C, etc. being the angular points of the polygon. How many triangles have been formed ? What is the sum of all the angles of all the triangles ? Hence what is the sum of all the angles of the polygon ?

34. If in 33 (b) the polygon be regular, what is the magnitude of each angle ?

35. (a) If a polygon of n sides be taken and a point O be taken within the polygon, how many triangles will you get by joining O to the angular points of the polygon ?

(b) What will be the magnitude of all the interior angles of the polygon ?

(c) If the polygon be regular, what is the magnitude of each angle of the polygon ?

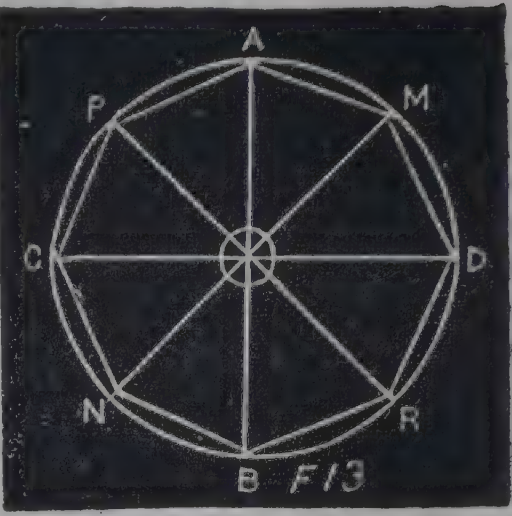
(d) If the sides of the regular polygon be produced in order, what will be the magnitude of each exterior angle ?

36. (a) If the exterior angle of a regular rectilineal figure be 90°, how many sides has the figure ? Any special name for the figure ?

(b) If the interior angle of a regular polygon be 120 degrees, how many sides has the figure ?

37. (a) Take a st. line AB = 2·2 cms. On AB describe an equilateral triangle ACB ; Produce AC and BC to D and E such that CD = CE = CA or CB ; Bisect angles DCB and ECA by FCG and make CE = CF = CA on one side and CD = CG = CB on the other. What kind of figure is ABGDEF ?

(b) With centre C and radius CA describe a circle. See that it passes through the other angular points. Hence devise a method for inscribing a regular hexagon in a circle.



38. AB and CD are two perpendicular diameters of the circle. MON and POR are st. lines bisecting the angles AOD and AOC respectively. What kind of figure is AMDRBNCP?

Hence what is the construction for inscribing a regular octagon in a circle?

39. Describe a circle with any convenient radius. Take a point P in the circumference of the circle. Set off PA, AB, BC, CD etc., along the circumference, each length PA, AB, BC, CD etc., being equal to the radius. If the points P, A, B, C, D etc., be joined in order, what kind of figure is PABCDE?

40. (a) Draw a st. line AB = 4 cms. Bisect the line at C. On AB as diameter describe a semi-circle. Let the perpendicular bisector of AB cut the circumference at D; What is the magnitude of angle ADB?

(b) With centre D and radius = DA or DB, cut off DE in CD produced. What is the magnitude of the angle AEB?

(c) Hence devise a method for describing a regular octagon on a given st. line.

EXAMPLES VII.

To be done in Form IV.

Construction of triangles from given data. The usual notation is adopted in this exercise.

I. (1) A = 60°, b = 2·4 in., c = 3·6″
(2) A = 30°, b = 2·3 in., c = 2·9″
(3) A = 45°, b = c = 3·4 in.
(4) A = 90°, b = 2·3 in., c = 3·4 in.
(5) B = 75°, c = 3·2″, a = 1·9 in.
(6) C = 72°, a = 2·3 cms., b = 2·8 cms.

Describe △s whose two sides and the contained angles are given.

II. The three sides a, b and c of different triangles are given below. Describe the triangles. Point out impossible cases and find out why the triangle could not be drawn in such cases.

(1) 7·3 cms., 2·8 cms., 6·4 cms.
(2) 6·4 cms., 2·3 cms , 5·9 cms.
(3) 4·6 in., 3·2 in., 2·9 in.
(4) 3·8 in., 7·4 in., 2·8 in.
(5) 3·3 cms., 2·7 cms., 4·6 cms.
(6) 5·3 cms., 8·7 cms., 2·9 cms.

Three sides with reason for impossible cases.

III. (1) $A = 45°$, $B = 39°$, $a = 3{\cdot}9$ cms.
(2) $A = 45°$, $B = 39°$, $b = 3{\cdot}9$ cms.
(3) $A = 45°$, $B = 39°$, $c = 3{\cdot}9$ cms.
(4) $B = 100°$, $C = 97°$, $a = 4{\cdot}3$ cms.
(5) $A = 78°$, $C = 47°$, $c = 3{\cdot}8$ cms.
(6) $A = 69°$, $C = 90°$, $a = 3{\cdot}7$ cms.
(7) $A = 110°$, $C = 77°$, $c = 4{\cdot}2$ in.

Do you meet with impossible cases in this set? If you do, what reason do you give for the impossibility?

IV. In a triangle $b - c = 1{\cdot}1''$, $b + c = 4{\cdot}5''$, $a = 1{\cdot}9''$. Find b and c geometrically and also algebraically. Describe the triangle.

V. If one side of a triangle be 3·7 in. and the other two sides be equal, describe the triangle, given that each base angle is half as much again as the vertical angle.

VI. In a triangle PQR, $P + Q = 98°$ and $Q + R = 114°$. Find the angles of the triangle. The side opposite to the angle $P = 3{\cdot}7$ cms.; describe the triangle.

VII. The altitude of an isosceles ∠ (i.e. the perpendicular from the vertex where the equal sides meet, to the base) is 2·9 in., and the vertical angle is 80°. Construct the triangle.

VIII. The altitude of an equilateral triangle is 3·4 cms. Construct the equilateral triangle and measure the length of a side.

IX. The perimeter of an isosceles triangle is 9 cms. and one of the equal sides is 3·4 cms. Construct the triangle. Find the magnitude of the vertical angle.

X. The perimeter of a triangle is 13·2 cms. The sides are in the ratio of 3 : 4 : 5. Construct the triangle. Do you observe any peculiarity in the triangle described?

XI. Construct a right-angled triangle having the hypotenuse = 2·6″ and one angle = 30°.

XII. Construct an isosceles triangle, base 3 in. and vertical angle 30°; On the same base, describe another isosceles triangle having the vertical angle, double that of the first.

XIII. Construct an isosceles triangle, altitude being 2¼″, the equal sides making angles of 25° with the base.

XIV. (a) On a base 4·2 cms. long, describe an isosceles triangle having the vertical angle = 40 degrees.

(b) Describe another triangle whose sides are to those of the triangle as 5 : 4. Find the length of one of the equal sides of this triangle.

XV. A street lamp-post is 10 feet above the ground and I stand at a distance of 5 feet from the post. My height is 5 ft. 4 in. The end of my shadow just reaches the foot of a tree. How far is the tree from the lamp-post?

XVI. (i) A is a water-tap and B is another and AB, 120 yds. long is the water-pipe. A third water-tap C is to be erected at a distance of

(a) 170 yds. from A and 130 yds. from B.

(b) 120 yds. from either A or B.

Locate the tap C in the figure drawn to scale.

(ii) If to conduct water to the tap, the main AB is to be bored at a place as near to C as possible, where should it be done in each case? What length of piping will be necessary in each case?

XVII. One side of a triangle is 2 in. long. The lengths in inches of the other sides are the values of x and y given by $2+y+x=7$, $2y+x=8$. Describe the triangle. What kind of triangle do you get?

XVIII. On a base of 2 in. construct a triangle with sides 2·3 in. and 1·7 in. Shew all possible positions and shew that the angles of one of the triangles are respectively equal to those of the other.

XIX. A triangle has two of its sides of lengths 2·7 in. and 2·1 in. respectively. If the angle opposite the shorter of these be 30°, how many distinct triangles are there satisfying the given conditions? Construct them.

If the length opposite the angle of 30° were 3 in. would your result be altered? If so, how?

XX. Describe, with your instruments, triangles having parts as given below.

Note under each figure as many particulars about each triangle, as you can find.

(1) $a=2$ in., $b=4$ in., $c=5$ in.
(2) $a=3$ in., $b=3$ in., $c=5$ in.
(3) $a=3$ in., $b=4$ in., $c=5$ in.
(4) $a=3$ in., $b=3$ in., $c=3$ in.
(5) $a=3$ in., $b=3$ in., $C=60^\circ$
(6) $A=45^\circ$, $B=45^\circ$, $c=5$ in.
(7) $A=60^\circ$, $a=3$ cms., $c=6$ cms.

XXI. Draw a straight line AB, 2·5″ long and through A, draw a line AX so that angle $BAX=55^\circ$. Show how to find points Q and S in AX such that the angle $BQA=65^\circ$ and the angle $BSA=103^\circ$. Explain your construction and justify it by geometrical reasoning.

Also find the distances of Q and S from A.

XXII. A car going one night along a straight road saw, when passing a milestone, a light in a cottage off the road in the direction making an angle of 30 degrees with the road. At the next milestone the light was seen in a direction making an angle of 45° with the road. Find from a diagram the least distance of the cottage from the road.

EXAMPLES VIII.

To be done in Form IV and to be revised in Forms V & VI.

Introduction.—

Exercises are based on the following constructions and geometrical truths.

i. To bisect a given finite straight line.

ii. To bisect a given arc of a circle.

iii. To bisect a given angle.

iv. To draw a perpendicular to a given straight line, from a given point in the line:—(Special method when the given point is near the end of the line).

v. To draw a perpendicular to a given straight line from a given point outside the line:—(Special method when the given point is above one extremity of the line).

vi. Through a given point to draw a straight line parallel to a given straight line.

vii. To make an angle equal to a given angle.

Note.—Incidentally the teacher will elicit the following facts:—(No formal proof need however be attempted).

i. The perpendicular bisector of a straight line is the locus of points equidistant from the ends of the line.

Exercises (a) A is a well and B is a tank. A straight row of houses is to be built so that each house in the row may be as far away from the well as it is from the tank. Where should the row be built?

(b) S is the surgeon's bungalow and P the nurse's house. A ward is to be built such that each patient's room may be equidistant from the surgeon's or from the nurse's quarters. Where should the row of rooms for patients be built?

ii. ***The bisector of an angle is the locus of points equidistant from the arms of the angle.***

iii. ***The locus of points at a given distance from a given st. line is, the st. line parallel to the given line at the given distance.***

iv. ***The distance of a point from a given line is the perpendicular distance and this is the shortest.***

v. ***What is the locus of points at a constant distance from a given point?***

The Simple Constructions—Loci.

Bisection of angles and st. lines, Drawing of Perpendiculars, Parallels Making angles equal to a given angle.

1. Draw a st. line AB; take any point C in it. At C erect a perpendicular CD. By repeated bisection construct an angle of 22½°.

2. In the figure of the previous question mark the angle = 67½°.

3. How will you geometrically (without the use of the protractor) construct angles of

135°, 112½°, 101¼°, 56¼°, 33¾°?

4. Draw any st. line AB. How will you erect a perpendicular at A, without producing the line? If AD be the perpendicular, cut off from AD a length AC = 3·4 cms. On AC describe an equilateral triangle AEC. Bisect the angle CAE by the st. line AF. Find the magnitude of each of the angles BAE, EAF, FAC.

5. Construct geometrically angles of 97½°, 105°, 18¾°, 52½°.

6. The base angles of a triangle are 105° and 37½°, and the base itself is 4·3 cms. long. Using the foot-rule and compasses only, describe the triangle and find the lengths of the other two sides by measurement.

7. Describe a △ with the following data, $a = 3{\cdot}7$ cms., $b = 2{\cdot}9$ cms., $c = 5{\cdot}2$ cms. Produce BC both-ways, BC to D and CB to E. Make BE = BA and CD = CA. Join AE and AD.

(i) What kind of triangle is ABE? Is there any other triangle of the kind, in the figure?

(ii) By measurement find the length of ED. Find also the perimeter of the △ ABC. What do you observe?

(iii) There are two angles in the figure, each being half of ABC. Name them. Find also the two angles, each of which is half of ACB.

8. The perimeter of a triangle is 13·7 cms., and the base angles are 75 and 45 degrees. Using the foot-rule and compasses alone, construct the triangle.

9. S.S.L.C. Q. 2—1921.

10. Two roads meet at an angle of 75°. There is a well equidistant from the two roads and 100 feet from the crossing. Mark the 100 feet length to scale and locate the well.

11. A river is 4 furlongs wide. A bridge crosses the river at an angle of 60°. A boat in the river is equidistant from either bank and also from the bridge. Locate the boat.

12. Two roads cut one another at an angle of 100°. A cow is tethered to a peg and the peg is equidistant from the roads. Where should the peg be?

(*a*) Another road cuts one of the roads at an angle of 60° and at a distance of 50 yards from the corner. Shew this road in the diagram. The peg is equidistant from these two roads also. Where should it be?

(*b*) Find the distance of the peg from any one of the roads.

(*c*) If the rope is exactly of the length obtained in (*b*) above, find the area, the cow can graze through.

13. A field is triangular, the sides being 100, 170, 200 yds. There is a treasure equidistant from the three corners. Where should it be?

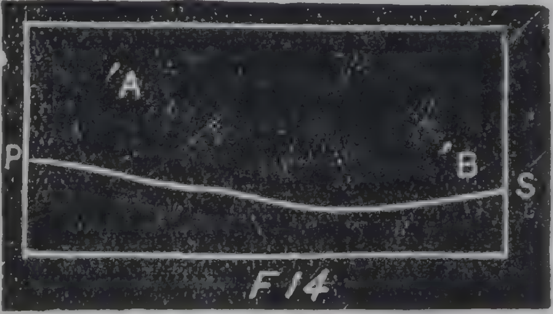
14. Two lines cut at an angle of 120°. Find a point which is

i. 4 cms. from each line.
ii. 3 cms. from the one and 4 cms. from the other.

15. Draw carefully a triangle with AB, 3 inches long, BC, 6 inches, and CA, 5 inches. Find points P and Q in the side BC such that (i) P is equidistant from the sides CA, AB, (ii) Q is one inch nearer to the side CA than to the side AB.

16. Draw two st. lines intersecting at an angle of 60°. Find a point which is at a distance of 5 cms. from one of the lines and 7 cms. from the other.

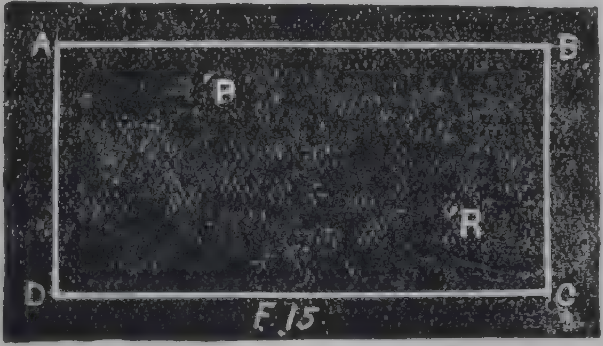
17. A and B are two towns, PS is a railway track. A station X is to be opened, so that it may be as far away from A as from B. Where should it be? If the distance between the towns be 1½ miles find the distance of the station from (i) either town, (ii) from a village which is on the straight road AB nearest to the station?



F 14

18. P, R, S, are three villages on a straight road. P to R is 1 mile 2 fur. and R to S is 6 fur. A railway station is equidistant from P and R, and is 7 furlongs from either village. The nearest point of the same railway line from S is an exact mile. Draw the line to represent the railway track.

19. ABCD is the plan of a billiard-table. When the billiard-ball is hit with the cue, to strike the edge of the table, the angle of incidence = the angle of reflection. P and R are two balls on the table. If R be hit with the cue (i) to strike the edge CD and the ball P on reflection, in which direction was the ball aimed? (ii) If it first strikes the edge CB and then the ball P which point in CB did it come in contact with?



F 15

20. AB is the straight bank of a river. T is a tree on the opposite bank. It is found that angle BAT = 75° and the angle ABT = 60°. The distance AB is 100 ft. Find by measurement, from a figure, the breadth of the river.

21. Three steamers P, Q, R lie at anchor in a harbour. PQ = 120 yards, QR = 80 yards, PR = 70 yds. A third steamer enters the harbour and is anchored at a place equidistant from the three steamers. Locate it in a figure drawn to scale.

22. A and B are two villages 10 furlongs apart. A school is to be built, such that it may not be more than 6 furlongs from either village. Shade the area within which the school must be built. If the school is to be exactly 6 furlongs from either village, where should it be built? Find from the diagram the distance of the school from the straight road running from A to B, if the school is 6 fur. from each village.

23. The sides of a triangular plot of ground are 60, 50 and 40 yds. A treasure is said to be at a distance of 15 yds. from the longest side and equidistant from the other two sides. Where within the field must it be?

24. It is required to find the length of a tunnel AB. Two points C and D, 800 yds. apart, are chosen on a straight road. CA = DA = 850 yds. CB = 450 yds. and DB = 900 yds. Find, from a drawing, the length of the tunnel.

25. An officer wants to build a bungalow equidistant from his office, the recreation grounds and the railway station.

The distance between the office and the recreation
grounds is 4½ furlongs.
" " railway station 6 "
" " the railway station and the
recreation ground is 3 fur.

Draw, to suitable scale, a figure and locate the bungalow.

Find its distance from any one of the three other buildings.

EXAMPLES IX.

To be begun in Form IV and continued in Forms V and VI.

The Ambiguous case, The Rt.-angled triangle, Heights and Distances.

N.B.—The pupils of the 3rd and 4th forms shall do the exercises on the Rt.-angled triangles by simple drawing and measurement. Questions marked with an asterisk may be done in the 4th. Pupils of the 5th form should be asked to verify their results obtained by drawing, by theoretical calculation.

1. Describe a triangle given that $a=3{\cdot}4$ cms., $c=2$ cms., and $C=30^\circ$. Shew that there are two triangles satisfying the conditions. Find the difference between the lengths of the side b in the two triangles. Also find the magnitudes of the angle A, in the two triangles. Note that angle A of one of the triangles is the supplement of the angle A of the other triangle. Hence deduce that when two sides and one angle are given, an obtuse-angled triangle and an acute-angled triangle can generally be drawn.

2. Describe triangles with the following data and note the peculiarities referred to in Q. 1 *Supra.*

(i) $a=3{\cdot}8$ cms., $c=2{\cdot}9$ cms., $C=45^\circ$
(ii) $a=4{\cdot}9$ cms., $b=3{\cdot}4$ cms., $B=20^\circ$
(iii) $c=2{\cdot}2$ in., $a=2{\cdot}9$ in., $C=40^\circ$
(iv) $b=1{\cdot}2$ in., $c=1{\cdot}9$ in., $B=35^\circ$
(v) $A=25^\circ$, $b=2{\cdot}4$ in., $a=1{\cdot}3$ in.

3. Describe an acute-angled triangle with the following data :—$a=1{\cdot}9$ in., $b=1{\cdot}5$ in., $\angle B=47^\circ$. Find the magnitude of the angle A and the magnitude of the angle C. Also find the length of c.

4. Describe an obtuse-angled triangle given that $b=4{\cdot}8$ cms., $A=48^\circ$ and $a=3{\cdot}8$ cms. Find the magnitude of the angle B and the length of c.

5. The lengths AB. AC of two adjacent sides of a triangle are given and the perpendicular distance of A from the base BC. Shew that two triangles can be constructed satisfying these conditions. Find by measurement, the difference between the lengths of their respective bases when AB = 8·1 cms., AC = 6 cms., and the distance from A to BC = 3·6 cms.

6. Given that $a = 2$ in., $c = 1·4$ in. and one of the angles of the triangle is 28 degrees, describe all the triangles possible with the given data. In each case find the length of the third side.

7. Given that AB = 3·4 cms. and BC = 4·6 cms. and the perpendicular from A to BC is 2·8 cms., how many triangles can be described? Find the length of AC in each case.

8. (a) $c = 3$ cms., $b = 4$ cms. and B = 55°. How many triangles could be described with these data? See that all the conditions are satisfied by the triangle.

(b) If c and a be of the same length and B = 90°, how many triangles will you get? What peculiarity do you observe in this case?

(c) Find b in the latter case.

9. Describe rt.-angled $\angle$s with the following data :—

(i) A = 90°, B = 40°, $a = 3·8$ cms.
(ii) B = 90°, C = 30°, $b = 2·9$ cms.
(iii) C = 90°, B = 46°, $a = 7·8$ cms.
(iv) A = 36°, B = 90°, $a = 4·7$ cms.
(v) $c = 3·8$ cms., $b = 4·7$ cms., A = 90°
(vi) $b = 2·7$ in., $c = 3·8$ in., C = 90°
(vii) $a = 3·4$ in., $c = 1·9$ in., B = 90°

10. A boy was flying a kite and the length of the thread was 100 yds. The kite was caught on the top of a tower and the boy himself was standing at a distance of 80 yds. from the foot of the tower. What must be the height of the tower?

11. A ladder is 10 feet long and reaches a cupboard in the wall 8 feet above the floor. How far from the foot of the wall is the other foot of the ladder?

12. Two roads cross each other at right angles. From the crossing two men start at the same time and walk, one along each road. The rate of the one is 3 miles an hour and that of the other 4 miles an hour. What will be the shortest distance between them at the end of (*a*) 1 hr. 20 mts., (*b*) 2 hrs. 45 mts.?

13. One side of a rectangular field is 40 yds. and an adjacent side is 30 yds. What is the length of the diagonal?

14. A tank is rectangular in shape. A boy swims from one corner to the opposite corner in 40 min. His rate of swimming is known to be 50 yds. in 25 min. One side of the tank is 48 yds. What is the length of the other side? What is the surface area of the tank?

15. One end of a rope 340 yds. long is fastened to the top of a tower while its other end reaches the ground 1 fur. from the foot. Find by measurement the height of the tower.

16. A man on one side of a brook finds that he can just rest a ladder 20 ft. long against a branch of a tree exactly opposite on the other bank; the branch is 12 ft. above the ground. How wide is the brook?

***17.** A ladder placed in a street reaches a window 32 feet high, making an angle A with the wall. On being turned round without removing the foot, it reaches a window on the other side of the street, making an angle (90°—A) with the wall. Find the breadth of the street if A = 24°. (*N. B.*—Congruence of Rt.-angled triangles.)

18. A rectangular plank 15 inches wide, is placed against the top of a wall 8 feet high, while the other end rests on the ground 6 feet from the wall; find the area of the plank.

***19.** On a base 4 cms. long describe an equilateral triangle ABC. From A drop AD perpendicular on side BC. Find the lengths of the sides of the triangle ABD. What is the magnitude of the angle BAD?

20. A ladder 55 ft. long is placed so as to reach a window 33 ft. high on one side of the street and on turning the ladder over to the other side of the street, its top rests at 44 feet from the ground. Find the breadth of the street.

21. The central post of a tent is 30 feet high. At a point 2 ft. below the top, a rope is attached and this is drawn tight and tied to a peg 21 feet from the foot of the post. What is the length of the entire rope, if 6 feet of rope be required for the knots at the two ends?

***22.** When a tree is cut, a rope is generally attached to the trunk and to a peg driven in the ground, that the fall may not be sudden. A tree 40 feet high is cut at a point 3 feet above the ground level. A rope connects a point 12 feet above the ground and a peg at a distance of 16 feet from the foot. What is the length of the rope? If after some time, owing to the weight of the branches etc., the part above the cut inclines on the side remote from the rope, making an angle of 10 degrees with the vertical, find from careful drawing the increase in the length of the rope due to elasticity.

***23.** A hall with a flat terrace is 40 feet by 18 feet by 12 feet. From the centre of the ceiling a lantern is hung by a rope passing through a pulley there and attached to a nail,

(*a*) in the centre of the breadthwise wall

(*b*) " " lengthwise wall

(*c*) at a point 1 foot vertically below the centre of a lengthwise wall

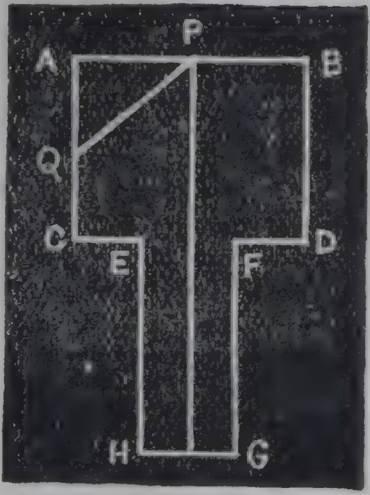
(*d*) at a point 1 ft. vertically above the centre of a breadthwise wall

(*e*) at a point 6 ft. vertically above a corner of the building

(*f*) at a point midway between the centre of a lengthwise wall and an adjacent bottom corner.

Find, from figures carefully drawn, the lengths of the rope required in each case, allowing 2 feet of rope extra for the knots etc.

*24. AC and BD are two vertical posts each 6 ft. high and EFGH the section of a well. The cross beam AB is 10 feet. P is the pulley at the centre. A rope 25 feet long, is attached to the middle point of AC. It goes round the pulley and the other end just touches the bottom of the well. Find by careful drawing the depth of the well.



25. A sanitorium is 600 feet high. What should be the least length of a telephone wire connecting the office on the hills with a station, 1 furlong from the foot of the hills?

*26. (*a*) Each leg of a pair of compasses is 8 in. long. If the legs be held in a position at rt. angles to each other and a circle described, what will be the radius of the circle?

(*b*) Find also the lengths of the radii, if the angles between the legs be 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80°, 120°, 150°, and tabulate the results.

*27. (*a*) Each step of a staircase is 6 in. high and 8 in. broad. If there be 13 such steps, what should be the length of a carpet spread above the steps, if it projects to a length of 1½ ft. beyond the foot of the staircase and along the topmost landing.

(*b*) What length of banister will be required to run the whole length of the staircase?

(*c*) What will be the height of the landing above the ground-floor?

*28. A table drawer is 6 in. deep, 18 in. broad and 2 ft. long. What is the longest ruler that can be placed in the drawer if (*a*) it should lie flat on the bottom of the drawer; (*b*) if it might be placed in any position?

*__29.__ A rectangular tank is 80 yds. long and 60 yds. broad. A pleasure boat is at a distance of 20 yds. from the longer side and 25 yds. from the shorter side. What is its distance from each of the four corners of the tank? Find also its distance

(i) from the middle pt. of an opposite shorter side
(ii) do. do. longer side
(iii) from the centre of the tank.

*__30.__ In a tank a lotus flower is seen one foot above the surface of water. Forced by the wind, its top is in a line with the surface of water at a distance of 2·5 ft. from the point in the surface where it was previously. Find the depth of water in the tank by careful drawing and measurement.

*__31.__ A flagstaff 36 ft. high breaks over without actually parting and the top touches the ground 12 ft. from the base. What is the length of the portion that fell?

*__32.__ At the bottom of a post is a serpent's hole. On the top of the post is perched a peacock. The post is 9 ft. high, and the peacock descries the serpent running to its hole when distant from it three times the height of the post; the peacock flies down obliquely and seizes the serpent when both have passed over an equal space. At what distance from the serpent's hole did they meet? Draw a figure to any convenient scale.

33. The front *Gopuram* of a temple is 80 feet high. A bird is sitting at the topmost point. A gun rests against the shoulder of a man standing at a distance of 44 ft. from the tower. If the shoulder be 4 ft. above the ground at what angle must he point the gun to shoot the bird?

34. The cannon in a fort is mounted at a height of 60 ft. above the sea-level. The cannon is inclined in a shooting position downwards at an angle of 15° to the horizontal and is fired. The bullet strikes the perioscope of a submarine in the sea. What is the distance of the submarine from the shore? The fort is on the shore.

35. A and B are the camps of two opposing armies. An aeroplane belonging to A, reaches a position vertically above B and signals its height as 1600 ft. If the angle of elevation of the aeroplane be 60 degrees, what must be the least range of the guns at A, that the bullet may fall on the camp of the opposing army?

*36. An aeroplane leaves a fort A and having gone a certain distance wirelesses to the fort that its height is 3000 ft. and that the angle of depression of an alien steamer is 30°. The angle of elevation of the aeroplane is found from the foot of the fort to be 60°. If the guns in the fort be at a height of 100 feet above sea-level, through what angle must the guns be lowered to strike the steamer? What is the distance of the steamer from the base of the fort?

37*. Two aeroplanes of party A are vertically above the enemy's camp B. One signals its height as 1500 feet above the other. The angle of elevation of the one is 60° and of the other 45°. How far is the enemy's camp from the camp of A?

*38. An aeroplane 1800 ft. high, finds the angle of depression of the top of a tower to be 30°, while from the foot of the tower the angle of elevation of the aeroplane is found to be 45°. Find the height of the tower.

39. Standing at a distance of 200 feet from the foot of a tower the angle of elevation of the tower is found to be 60°. Find the height of the tower.

40. An enemy cruiser was sighted and the guns in the fort were depressed by an angle of 20° and fired. The bullet fell at a distance of 30 yds. beyond the cruiser. The guns were then depressed by 15° more and the bullet fell 20 yds. before the cruiser. By how many degrees must the guns be depressed that the bullet may strike the cruiser straight? Also find the height of the guns above sea-level.

SECTION III. (A)

The square and square root of a number, and the Theorem of Pythagoras.

EXAMPLES X.

N.B.—Qs. 1—11 in Form IV and the rest in Forms V and VI.

1. a multiplied by itself is $a \times a = a^2$. If $a = 3$ what is the value of a^2?

2. $2p$ multiplied by $p = 2p^2$. If $p = 2{\cdot}5$, what is the value of $2p^2$?

3. If $a = 2{\cdot}5$ and $p = 3$ what is the value of $a^2 + 2p^2$, and the value of $2p^2 - a^2$?

4. Given that $a = 2$, $b = 3$ and $c = 1\frac{1}{2}$ find the value of (i) $3a + 2b - c^2$, (ii) $3a^2 + 2b^2c^2$, (iii) $a\,(2a + b^2 - c)$.

5. If a be multiplied by b, the result is put down as ab
If a be multiplied by a ,, ,, as a^2
If ab ,, ,, a ,, ,, a^2b
(i) What will be the result when bc is multiplied by b?
(ii) ,, ,, abc ,, ,, a?
(iii) ,, ,, abc ,, ,, bc?

6. There were a subscribers to a fund and each one subscribed as many annas as there were subscribers. If R denote the total amount in rupees, find a relation between R and a.

If $a = 32$, find R.

7. In a certain school there were a boys in the College classes and b boys in the High School classes. Each College class student subscribed a annas and each High School student 3 annas. Find R, the total collection in rupees.

If $a = 38$ and $b = 412$, find R.

8. Multiply the sum of 4 and 3 by 5. This may be done either by adding 4 and 3 and then multiplying the result by 5 or by multiplying 4 by 5 and then 3 by 5 and adding the results. Similarly find (i) $a(b + c)$, (ii) $2x(y + 2)$, (iii) $7y(2x + y)$, (iv) $3a(2a + b)$, (v) $2a(2b - a)$.

9. Extend the rule and multiply $(a+b)(c+d)$.
$(a+b)(c+d)=a(c+d)+b(c+d)=ac+ad+bc+bd$.

Similarly multiply (i) $(a+b)$ by $(a+b)$, (ii) $(c+d)$ by $(c+d)$, and (iii) $(x+y)$ by $(x-y)$; (iv) $(a-b)$ by $(a-b)$, (v) $(c+d)$ by $(c-d)$, (vi) $(a+b)$ by $(a-b)$; (vii) $(c-d)$ by $(c-d)$.

N.B.—The forms of these results must be drilled in by the teacher.

10. (i) If $a=2·5$ and $b=1·5$ find the value of $(a-b)^2$ and a^2+b^2-2ab.

(ii) With the same values for a and b find a^2-b^2 and $(a+b)(a-b)$

(iii) Also find the value of $(a+b)^2$ and a^2+b^2+2ab

(iv) Simplify 2001^2-1999^2 using the formula $a^2-b^2=(a+b)(a-b)$.

(v) Simplify 1002×998 " " $(a+b)(a-b)=a^2-b^2$.

(vi) Find 1998^2 using the formula $(a-b)^2=a^2+b^2-2ab$.

(vii) Multiply 2016 by itself using the formula $(a+b)^2=a^2+b^2+2ab$.

11. (*a*) What number multiplied by itself gives the result ?

(i) 4. (ii) 9. (iii) 36. (iv) 144. (v) 256.

(*b*) If $x^2=144$, what is the value of x? (the meaning of the negative value.)

(*c*) If $(a+b)^2=144$ find the value of $(a+b)$

(*d*) $(a+b)^2=49$. If $a-b=2$ find a and b.

(*e*) $(x+y)^2=121$, $(x-y)^2=9$ find x and y.

(*f*) $a=5$, $b=4$ and $c=3$. Shew that $b^2+c^2=a^2$.

(*g*) Describe a triangle whose sides are 3, 4 and 5 inches. Find the magnitude of the angle opposite to the longest side.

(*h*) Describe a rt.-angled triangle, the sides containing the rt.-angle being 5 and 12 cms. respectively. Find the length of the hypotenuse, and verify the truth of the theorem :—The square on the hypotenuse of a rt.-

angled triangle is equal to the sum of the squares on the other two sides. If the hypotenuse is a inches long the square is a^2 sq. in. If each of the other sides be b and c in. long, the sum of the areas of the squares on the sides is (b^2+c^2) sq. in. The proposition states that $a^2=b^2+c^2$.

12. Perform the following constructions and verify geometrically that the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

ABC is a △ rt.-angled at A.

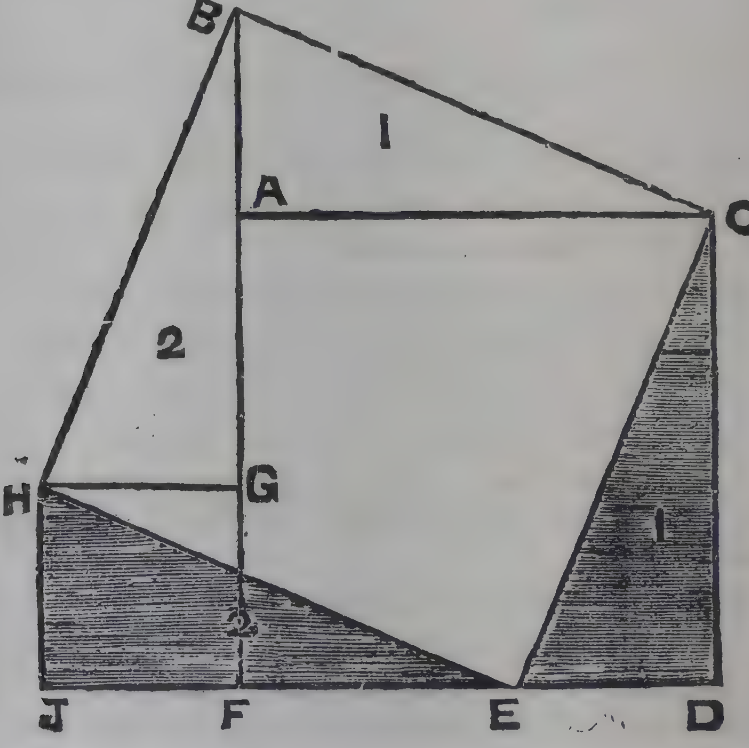
ACDF is the square on AC.

BCEH is square on BC or hypotenuse.

FGHJ is equal to square on AB, for FG is cut = AB, and JE = AC, ED = AB.

The sum of the squares on AC and AB is the figure ACDJHG.

From this, two △s, i.e., CED and HEJ are cut and placed in positions CBA and HBG and thus the figure ACDJHG is transformed into figure BCEH, which is a square.

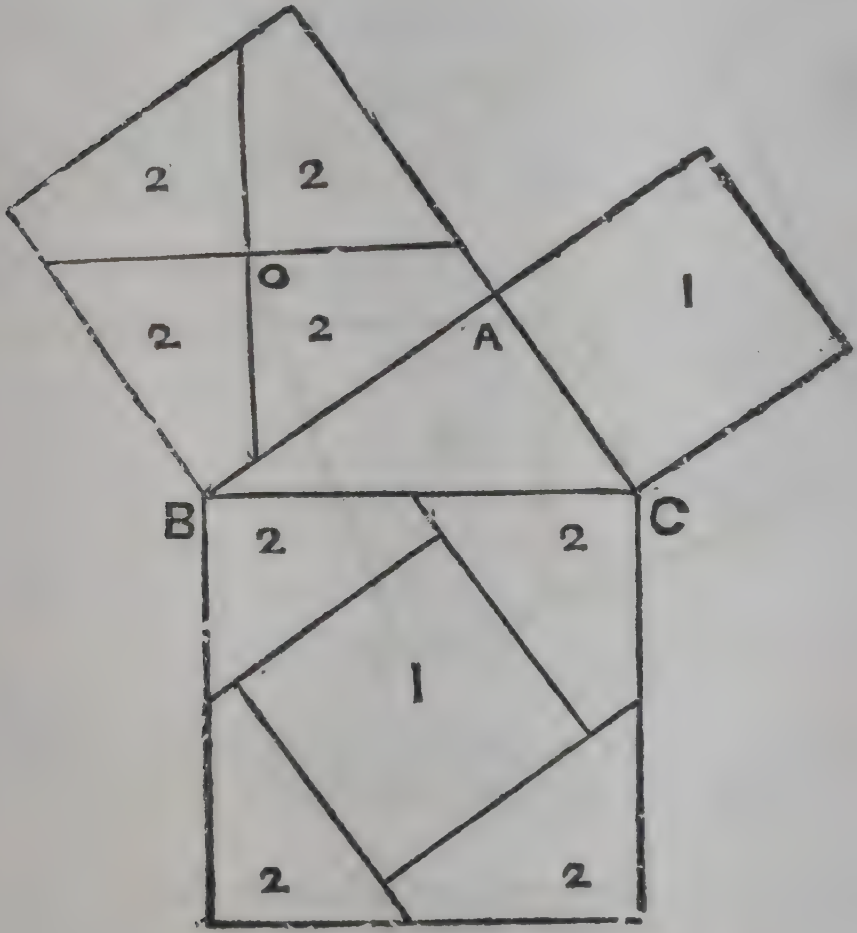


13. ABC is a △ rt.-angled at A and squares are drawn on the sides.

O is the mid-point of the square on AB.

Through O draw parallels and perpendiculars to the hypotenuse to meet the sides.

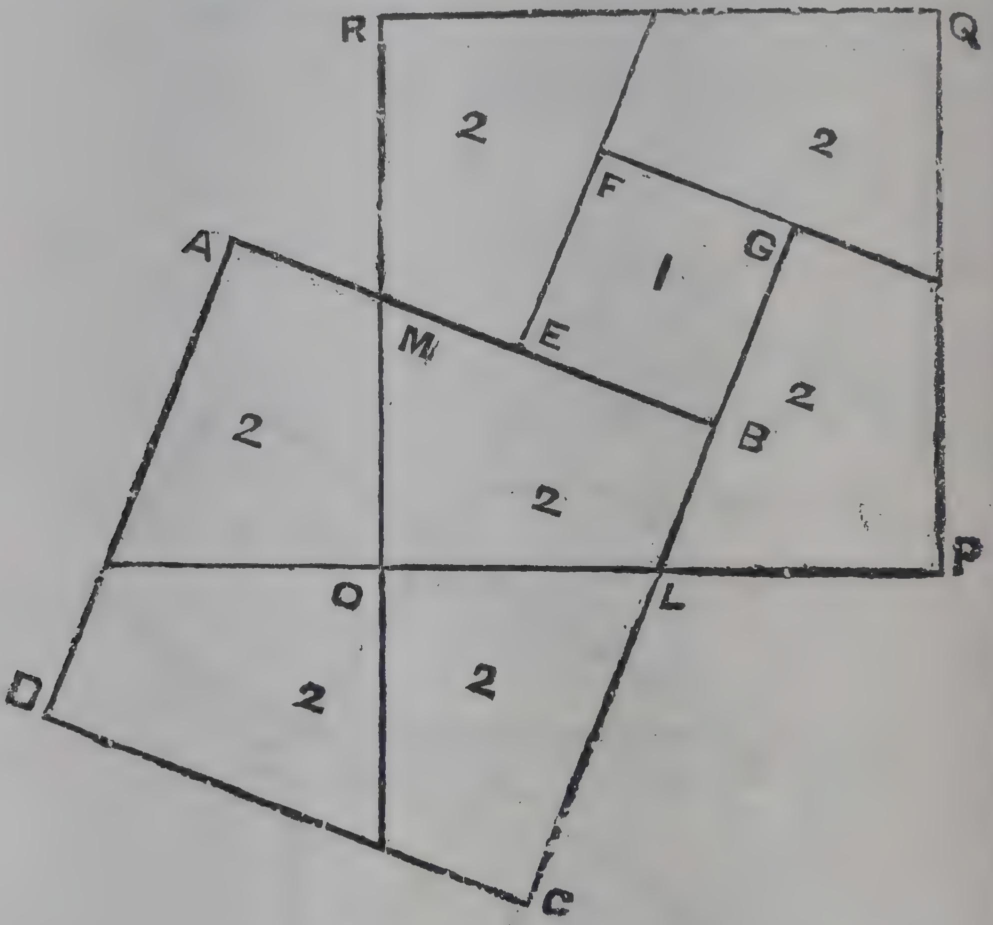
Again through the mid-points of the sides of the square on BC draw ∥s in order, to the sides of the △. The pieces marked 2 are identically equal and the piece marked 1 is equal to the square on AC.



14. Let AC, BF be two squares. Place them so that a side of one is in the same st. line with a side of the other.

L is the mid-point of CG.
O " " of the square.
M " " AE

RM = MO and OL = LP. Pieces marked 2 will be identically equal, and one piece is common.



N.B.—Examples 12 and 14 give methods for transforming two given squares into a single square of equal area.

15. (a) If c denote the hypotenuse and a and b the sides of a rt.-angled triangle, see that the relation $c^2 = a^2 + b^2$ is satisfied by the sides of each triangle in Q. 9, Examples IX.

(b) Also give theoretical solutions of all the problems given in Examples IX and check the answers obtained, by measurement.

16. (a) If one side of a rt.-angled triangle be 1·2 in. long and the other side ·5 in. long, what will be the length of the hypotenuse?

(b) If the length of the hypotenuse be 3·7 in. and the length of a side be 3·5 in. what will be the length of the third side?

Apply the formula $a^2-b^2=(a+b)(a-b)$.

17. (a) One side of a rt.-angled triangle is $2x$ inches long and the other side is (x^2-1) in. long. What is the length of the hypotenuse? [This is an exercise on the application of the formula $[a+b]^2=[a^2+b^2+2ab.]$

(b) Give values 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 to x and hence determine the different sets of integers for the lengths of the sides of a rt.-angled triangle.

18. (a) Shew that: $\left(\frac{n+1}{2}\right)^2-\left(\frac{n-1}{2}\right)^2=(\sqrt{n})^2$.

(b) Find by drawing and measurement the square root of 23.

This is done, from the above formula, by describing a rt.-angled triangle, whose hypotenuse is 12 in. and one of whose sides is 11 in.

19. Find geometrically the sq. root of :—

25, 47, 39, 27 and 26.

20. (a) Establish the relation:

$$\left(\frac{a+b}{2}\right)^2-\left(\frac{a-b}{2}\right)^2=(\sqrt{ab})^2.$$

(b) If $a=5$ and $b=3$, find geometrically the sq. root of 15.

(c) Find by this method the sq. root of

48, 39, 64, 37.

Solution.—To find the sq. root of 63 : two factors of 63 are 7 and 9 ; $b=7$, $a=9$, $\frac{9+7}{2}=8$ and $\frac{9-7}{2}=1$; Hence if the hypotenuse be 8 units and a side 1 unit, the other side will be $\sqrt{63}$ units.

21. Find geometrically the sq. root of 48 by employing all the methods you now know.

Sol. i. $48=49-1=7^2-1^2$

ii. $\left(\frac{49}{2}\right)^2-\left(\frac{47}{2}\right)^2=48$

iii. $\left(\frac{24+2}{2}\right)^2-\left(\frac{24-2}{2}\right)^2=48$

$\left(\frac{16+3}{2}\right)^2-\left(\frac{16-3}{2}\right)^2=48$

$\left(\frac{12+4}{2}\right)^2-\left(\frac{12-4}{2}\right)^2=48$

$\left\{\frac{8+6}{2}\right\}^2-\left\{\frac{8-6}{2}\right\}^2=48$

Which method of geometrically extracting the sq. root do you prefer in this case ?

22. (i) ADEF is a st. line of unlimited length. AC = 1 inch and is perpendicular to AF.

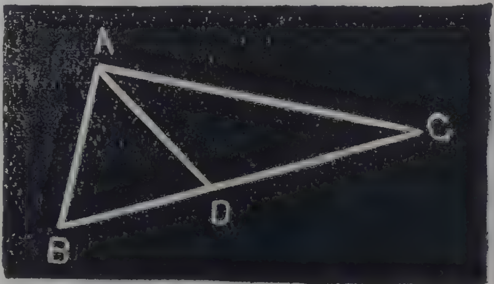
If AD be cut off = AC, what will be the length of CD ?

(ii) If again AE be cut off = CD, find the length of CE.

(iii) Further if AF = CE find the length of CF.

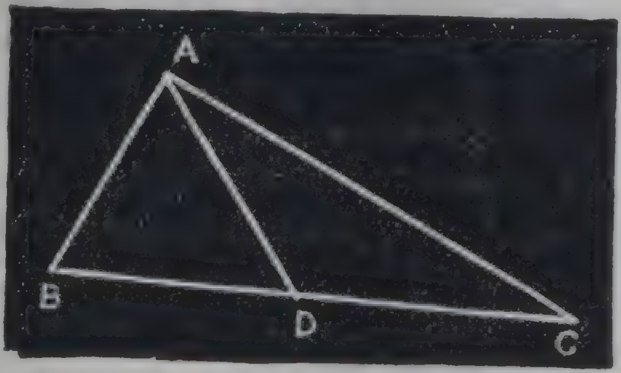
(iv) Find the square root of (*a*) 5, (*b*) 7.

23. (*a*) ABC is a △ rt.-angled at A. If AD be drawn such that ∠ BAD = ∠ ABD, prove that BD = DA = CD.



N.B.—A circle with centre D and radius DB will pass through B, A and C.

(*b*) If in any triangle ABC, the longest side being BC, the join DA of the mid-point to the opposite vertex be equal to half the longest side, then A is a rt. angle.



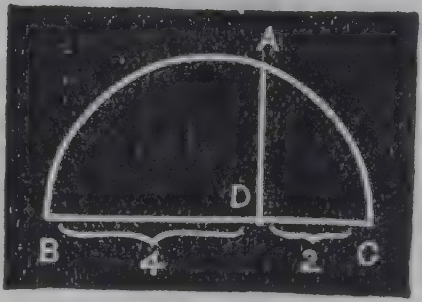
Proof :—DA = DB ; Hence $\angle$DAB = $\angle$DBA.
Also DA = DC ; „ $\angle$DAC = $\angle$DCA.

∴ $\angle$BAC = $\angle^s$ ABC and ACB : But the three angles are together equal to two rt. angles.

∴ $\angle$ BAC is a right angle.

N.B—***The property that the angle in a semi-circle is a rt. angle must be inferred.***

24. To find the square root of 8 : Split it into two factors 4 and 2 ; Take a line 6 in. long.



Take a point D in BC such that BD = 4 in. and CD = 2 in. ; on BC describe a semi-circle.

At D erect a perpendicular to meet the ⊙ce at A : The length of DA gives the sq. root of 8. How ?

Hint : $\angle$ BAC is a rt. angle ; $BC^2 = BD^2 + CD^2 + 2BD.DC$ for $(a+b)^2 = a^2 + b^2 + 2ab$. etc.

25. Describe a triangle given that,

(i) $a = 3$ cms., $b = 3$ cms. and $C = 90°$.
(ii) $c = 3\sqrt{2}$ cms., $A = 45°$, $B = 45°$.

Find the length of c in (i) and the magnitude of C and the lengths a and b in (ii).

Inference :—The sides of an isosceles rt.-angled triangle are in the ratio $1 : 1 : \sqrt{2}$.

26. (*a*) On a base BC 4 cms. long describe an equilateral triangle ABC. Bisect the angle A by the line AD to meet the base in D.

(i) What is the magnitude of $\angle$BAD?
(ii) ,, ,, of $\angle$ADC or ADB.
(iii) ,, length of the line BD or CD.

(*b*) If BC be a in. long, find in terms of a, the lengths of the three sides of the triangle ABD.

Inference :—The sides of a rt.-angled triangle, whose acute angles are 60° and 30°, are in the ratio $1 : 2 : \sqrt{3}$.

Note :—Different sizes of set squares may be taken and the lengths of the sides be measured by the boys.

27. (*a*) Find the lengths of the third sides in Q. 9 sub-divisions (v), (vi), and (vii): Examples IX.

(*b*) Find by calculation the height of the tower in Q. 10, Examples IX.

(*c*) Find by calculation the lengths required in Qs. 11, 12, 13, 14, 15, and 16: Examples IX.

(*d*) If $\angle$A in Q. 17, Examples IX be 60°, find by theoretical calculation (i) the height of the window, (ii) the breadth of the street, given that $\sqrt{3} = 1{\cdot}732$.

(*e*) Find by calculation the lengths or distances etc., required in Qs. 18, 19, 20, 21, 22, 23, 24, 25, 26 (a), 27, 28, 29, 30, 31, 32, 35, the distance required in 36, 38, 39.

28. A person standing at the crossing of two roads at right angles W to E and N to S observes two objects, one due West and the other due South on the road. He walks by a footpath 30°S of W, a distance of p yards, when, he sees one of the objects due North and the other due East. Find in terms of p the distances of the objects from the man and the distance between the two objects. If $p = 210$ find the distances to the nearest foot.

29. ABCD is a rectangular field; AB = 60 yards, BC = 40 yards. A point P is 15 yards from AB and 45 yards from BC. If this point be joined to the corners of the rectangle what fraction of the whole field is each of the four triangular portions thus divided?

Draw a neat figure to scale.

30. Standing on the bank of a river running direct West to East, a person observes that a tree on the other bank is 30° East of North. Having walked 150 yards along the bank with the river, he finds the same tree 30° West of North.

Find from a carefully drawn figure the distance of the tree from either of the positions and also the width of the stream.

All the constructions required should be done geometrically. Check the measurements by calculation.

31. A square field contains 10 acres. Two persons start from one corner, and while one goes along the sides, the other goes along the diagonal. What should be the rate of the man walking along the sides, if the rate of the one going diagonally is 10 miles an hour and both reach the opposite corner at the same time?

32. A duck enters water at an angle of 60° to the horizontal, and after it has gone in the direction a length 12 ft. rises up making an angle 60° with the vertical. Find from a carefully drawn figure and by calculation :—

(1) the greatest depth it reaches;

(2) the distance of the point where it emerges, from the point where it entered;

(3) the total distance it has gone through.

33. A railway cutting is 180 ft. broad at the top and the sides slope uniformly towards the bottom. The inclination of the side to the top is 30°. If the width of the bottom be 27 ft. find the depth of the cutting. Draw a plan of a vertical section of the cutting to scale, using your foot-rule and compasses only.

34. (i) How often must I run round a square of 10 acres to run a mile?

(ii) If ABCD be the square and if I walk at the rate of 3 miles an hr., how long will I take for the circuit A to C, C to B, B to D, and D to A?

35. Two roads meet at an angle of 60°. My house is on one of these roads at a distance of 150 yards from the crossing. Between the two roads, is the market, and it is equidistant from the two roads and also equidistant from my house and the crossing. Find by calculation and from a figure neatly drawn to scale (1) the distance of the market from either of the roads, (2) its distance from my house or the crossing, (3) the angle at which the two footpaths, the one from the crossing to the market and the other from my house to the market, cut.

36. (i) A ladder 65 ft. long leans against a wall and its foot is 33 ft. from the foot of the wall. To what height on the wall will the ladder reach?

(ii) If the ladder be raised so that the end against the wall reaches a point 7 ft. higher, how much must its foot be brought nearer to the wall? Draw diagrams to scale and verify your results by calculation.

37. A rectangular sheet of paper ABCD is taken. P and Q are points in AB such that $AP = BQ = a''$; and R and S are points in CD such that $CR = DS = b''$; If the paper be cut along PS and QR and the strips removed find the perimeter of the figure PQRS, given that the original sheet was p'' long and q'' broad.

38. A ladder 9·5 ft. long leans against a wall and touches a point 7 ft. from the floor. The ladder slips and the end against the wall now touches a point 1 ft. vertically below the original place. How many inches farther has the other end moved away? Draw a neat figure to scale.

39. A ladder 25 feet long leans against a wall and touches a point 20 feet above the floor. The ladder slips and the top against the wall touches a point, 5 feet vertically below the original position. How far would a man, who was standing half-way up the ladder have gone vertically down?

Draw a big diagram to scale and by measurement, find the length required. Check your result by calculation.

40. A picture is hung on a nail by a piece of twine 1½ yds. long attached to the ends of the top frame which in the position is horizontal. (*a*) If a piece of twine 6 in. longer be substituted how far lower will the bottom be? (*b*) If again a piece 8 in. shorter than the original be substituted how much higher will the picture be? The distance between the ends of the twine attached to the frame is 12 inches.

41. The vertical section of a hill is represented by a right-angled triangle with the right angle at the highest point. The height of the point above the foot along a slope is 300 feet and this side slopes at 60° to the horizontal. A straight tunnel is to be made from this foot so that its inclination is 30 degrees to the horizontal. The operation of boring is to be commenced on either side. At what height along the other slope should the operation be commenced? What is the vertical height of the end of the tunnel above the ground floor?

42. A and B are two points 60 yds. from each other and on the same side of a tower whose foot is O. At A and B the angles of elevation of the top of the tower are 30 degrees and 60 degrees respectively. Find the angle of elevation of the top of the tower at a point C on the opposite side of the tower, on the same road as A and B, whose distance from O is ½(OA + OB). Determine another point on the same side as A and B having the same elevation as at C and express its position relative to A and B.

Also find the height of the tower and its distance from A and B by calculation, to the nearest foot.

43. A person owned three square fields whose sides were 96, 128 and 384 feet respectively. He wanted to have a single *square* field equal in area. Another landlord had one and they transferred ownerships. What should be the length of a side of this field?

Draw a figure to scale and construct this length geometrically.

44. Two posts are situated on a straight road. At a point on the road between the posts, which is twice as far from one post as the other, the angular elevation of the nearer is 60°. If from the same point, the angular elevation of the farther is 30°, compare the heights of the post, given that the farther is 30 feet high. Also find the least distance between the tops of the posts.

45. A vessel sailing due E, is in distress and observes a lighthouse 30° N. of her course. It continues on its course a distance of 1 mile and then finds the same lighthouse 60° N. of her course. If from this point it makes for the lighthouse, how far must it go? If it had made for the lighthouse, from the former position, what distance might the ship have saved? Find these, both by calculation and measurement.

46. The longer side AB of a rectangle ABCD is a in., and the shorter side BC is b in.; P is a point equidistant from AB, AD, BC. Find its distances from the four corners in terms of a and b.

47. A cliff is 800 ft. high and the sides slope at angles 60° and 30° to the horizontal. Find the saving in distance that might be effected by boring a tunnel through it, instead of having to go through a track over it.

48. A triangle ABC is right-angled at C. The sides AB, BC, CA are respectively l, m and n in. long. A point P is in BC and $BP = a''$. Similarly another point Q is in AC and $AQ = b$ in. Express the lengths AP, BQ, PQ in terms of a, b, l, m and n and hence show that $AP^2 + BQ^2 = PQ^2 + AB^2$.

SECTION III. (*B*)

Square Root and Surds.

N.B.—What is meant by the sq. root must be told in Form IV. Finding sq. root by factorisation must be taught in Form IV. The rest may be reserved for Form V.

1. The student knows that a^2 is the square of a and that a is called the square root of a^2.

Symbolically $\sqrt{a^2}=a$. Note that $+a\times +a=a^2$; also $-a\times -a=a^2$; $\therefore$ Strictly speaking $\sqrt{a^2}=\pm a$.

Similarly $\sqrt{121}=\pm 11$; $\sqrt{144}=\pm 12$.

2. A knowledge of the multiplication table helps us to give at once the square roots of numbers of three digits which are *perfect* squares. 144 is a perfect square. Is 140 a perfect square? When the square root of a number cannot be expressed *exactly*, the square root is called a surd. For example, $\sqrt{5}$, $\sqrt{10}$, $\sqrt{12}$ are surds—We shall speak at some length on surds later on.

3. If the arithmetical number be rather big we cannot find out at once its square root, even though it is an exact square. Find the square root of 2401.

The student will easily find that $50^2=2500$ and that therefore the sq. root consists of two digits and is less than 50.

Rule.—Begin with the digit in the unit's place and put dots or dashes on alternate digits going to the left and right also (in case the given number has both the whole number and the decimal.)

The number of dots gives the number of digits in the square root—(The rationale may be explained to the boys.)

4. To find the sq. root of 2401.

If $N=2401$, $\sqrt{2401}=\sqrt{N}$. Suppose $\sqrt{2401}=a+b$.
$\therefore (a+b)^2=2401$; Hence $a^2+2ab+b^2=2401$.
$2401-a^2=b(2a+b)$.
$(2401-1600)=b(80+b)$.

$801=b(80+b)$; $\therefore \dfrac{801}{80+b}=b$; By *trial* $b=9$.

5. *The Arithmetical Rule.*

Find the square root of 19321.

$$
\begin{array}{r|l|l}
 & \dot{1}9\dot{3}2\dot{1} & 139 \\
 & 1 & \\ \cline{2-2}
23 & 93 & \\
 & 69 & \\ \cline{2-2}
269 & 2421 & \\
 & 2421 & \\ \cline{2-2}
\end{array}
$$

The first digit 1 is easily obtained. Take the next period and put down. Twice the number already found *i.e.*, 20 goes 4 times into 93, but 24 goes only 3 times into 93; hence take 23 and put 3 at the ten's place of the sq. root. Subtract from 93 three times 23 and to the remainder affix the next period; take twice the sq. root already found *i.e.*, 260 and it goes nine times into 2421; 269 × 9 gives 2421; so that the sq. root is 139.

In teaching the arithmetical rule, no amount of literature on paper will make the student grasp the working. Oral teaching is the only effective way here, as everywhere else. The following points must however be impressed:

i. The number of digits in the square root and the *pointing* deduced therefrom.

ii. The remainder at every stage is the result of subtracting the square of the part of the root already found, from the original number.

iii. How the arithmetical rule is derived from the algebraical method of extracting the square root.

6. Find the square root of 205209.
The square root consists of 3 digits.
It must be less than 500 and greater than 400.

$$\begin{array}{rl|l}
 & 2\dot{0}5\dot{2}0\dot{9} & 453 \\
 & 16 & \\ \cline{2-2}
 & 452 & \\
(2a+b)b=(80+5)\times 5=85\times 5 & 425 & \\ \cline{2-2}
 & 2709 & \\
(900+3)3=903\times 3 & 2709 & \\ \cline{2-2}
\end{array}$$

7. The following points should be remembered.

(*a*) Nos. ending in 2, 3, 7 and 8 or in an odd no. of zero's cannot have an exact square root.

(*b*) If the number of significant decimal places be odd, there can be no exact sq. root.

(*c*) If a perfect square number be multiplied or divided by another perfect square number, the result must be a perfect square number.

8. The tests of divisibility by 2, 3, 4, 5, 6, 8, 9, 10, 11, 12 will often help the extraction of the square root of a number which is a perfect square. If the number be not a perfect square, the number will in most cases be capable of being expressed in the form $A\sqrt{b}$ where b is a very small number. Example :—Extract the square root of 65367225.

$$\begin{array}{r|rl}
 & & \text{Evidently 5 is a factor} \\
5 & 65367225 & \\ \hline
5 & 13073445 & \text{Again we see 5 is a factor} \\ \hline
9 & 2614689 & \text{Sum of digits for factor 9} \\ \hline
11 & 290521 & \text{Try 11 (sums of alternate digits)} \\ \hline
11 & 26411 & \qquad\qquad ” \\ \hline
7 & 2401 & \text{Failing known tests try 7.} \\ \hline
7 & 343 & \\ \hline
7 & 49 & \\ \hline
 & 7 &
\end{array}$$

The number is equal to $5\times 5\times 3\times 3\times 11\times 11\times 49\times 49$.
Hence $5\times 3\times 11\times 49$ is the square root.

9. $\sqrt{5}$, $\sqrt{3}$, $\sqrt{13}$ cannot be exactly determined. These are called *irrational* numbers or 'surds'. They may however be expressed to any degree of accuracy so that the square of the result differs from 5, 3 or 13 by a very small fraction which may be neglected. This we explain later on.

10. The following properties of surds should be clearly understood.

(*a*) i. $7\sqrt{3} = \sqrt{7^2 \times 3} = \sqrt{49 \times 3} = \sqrt{147}$; or in general terms,

(ii) $a\sqrt{b} = \sqrt{a^2b}$; and conversely,

(*b*) (i) $\sqrt{605} = \sqrt{121 \times 5} = \sqrt{11^2 \times 5} = 11\sqrt{5}$

or in general terms

(ii) $\sqrt{x^2y} = x\sqrt{y}$

(*c*) $\sqrt{2} \times \sqrt{3} \times \sqrt{7} = \sqrt{42}$; or in general terms $\sqrt{a} \times \sqrt{b} \times \sqrt{c} = \sqrt{abc}$.

11. The square root of a fraction is the square root of the n$^{\text{r}}$ divided by the square root of the d$^{\text{r}}$.

(1) The sq. root of $\dfrac{25}{36} = \dfrac{\sqrt{25}}{\sqrt{36}} = \dfrac{5}{6}$; or

(2) The sq. root of $\dfrac{a^2}{b^2} = \dfrac{a}{b}$

(3) The sq. root of $\dfrac{45}{39} = \dfrac{\sqrt{45}}{\sqrt{36}} = \dfrac{\sqrt{5 \times 9}}{6} = \dfrac{3\sqrt{5}}{6} = \dfrac{\sqrt{5}}{2}$

(4) The sq. root of $\dfrac{36}{45} = \dfrac{\sqrt{36}}{\sqrt{45}} = \dfrac{6}{\sqrt{(9 \times 5)}} = \dfrac{6}{3\sqrt{5}}$

Again multiplying the n$^{\text{r}}$ and the d$^{\text{r}}$ by $\sqrt{5}$ we have

$$\frac{6}{3\sqrt{5}} = \frac{6\sqrt{5}}{3 \times (\sqrt{5})^2} = \frac{6\sqrt{5}}{3 \times 5} = \frac{2\sqrt{5}}{5}$$

12. When the denominator of a fraction is a surd, the approximation to the value of the fraction is conveniently found, by multiplying both the n$^{\text{r}}$ and the d$^{\text{r}}$ by the surd, thus making the d$^{\text{r}}$ an integer and not altering the value of the fraction. This is called *Rationalisation.*

Find the sq. root of $\frac{39}{48}=\frac{\sqrt{39}}{\sqrt{48}}=\frac{\sqrt{39}}{\sqrt{(16\times 3)}}=\frac{\sqrt{39}}{4\sqrt{3}}$

i.e. $\frac{\sqrt{39}\times\sqrt{3}}{12}$ i.e. $\frac{\sqrt{117}}{12}$; $\sqrt{117}=10{\cdot}1166538$...........

Hence the value of the fraction $=0{\cdot}8430544$.

13. Application of the identity $(a+b)(a-b)=a^2-b^2$.

Simplify $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$.

Multiplying both the n^{r} and the d^{r} by $\sqrt{5}-\sqrt{3}$,

We have $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\frac{(\sqrt{5}-\sqrt{3})^2}{5-3}=\frac{5+3-2\sqrt{15}}{2}$

$$=\frac{8-2\sqrt{15}}{2}=4-\sqrt{15}=4-3{\cdot}8729833.$$

$$={\cdot}1270167.$$

14. *The cube root*: $a\times a\times a=a^3$. a is called the cube root of a^3. Find the cube roots of : 2^3, 8^3, 8, 64, 125.

15. The following points should be remembered : The cube of a number ending with 1 must end with 1.

2	,,	8
3	,,	7
4	,,	4
5	,,	5
6	,,	6
7	,,	3
8	,,	2
9	,,	9
0	,,	0

To the teacher

Hence, known that a given number less than 100000 has an exact cube root, as the number of places in the cube root cannot consist of more than two digits, the cube root itself may by inspection be determined.

The curious point to be noted is that nos. ending with 1, 4, 5, 6, 9, 0 have their cubes ending with the same digit and nos. ending with 2, 3, have their cubes ending with 8, 7, and nos. ending with 8, 7, have their cube roots ending with 2, 3.

Note.—The student is advised to turn to the section on Similar Triangles and to the section on Contracted Methods, for further information.

EXAMPLES XI.

1. The numbers given below are exact squares. Determine their square roots :

196249, 16337764, 5396665444.

2. Given that $\sqrt{3}=1{\cdot}7321$ find correct to the nearest tenth, the values of $\sqrt{17787}$, $\sqrt{160083}$.

3. Given that $\sqrt{5}=2\,2361$ find correct to the first decimal place the values of

(*a*) $\sqrt{605}-\sqrt{45}+\sqrt{125}-3\sqrt{5}$

(*b*) $\sqrt{20}-\sqrt{845}+\sqrt{980}$.

4. Find theoretically the square roots of

$$\frac{51}{16};\ \frac{121}{27}.$$

5. Evaluate $\dfrac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}};\ \dfrac{\sqrt{7}+\sqrt{3}}{\sqrt{3}-\sqrt{2}};\ \dfrac{\sqrt{6}+\sqrt{12}}{\sqrt{6}+\sqrt{24}}.$

6. A rectangular field is 214·35 yards long and 123·26 yards wide. Express its area in acres, roods and poles ; and find to within an inch the side of a square field of equal area.

7. Find to five places of decimals the value of

$$\frac{\sqrt{3}\times\sqrt{5}}{\sqrt{5}+\sqrt{3}}$$

8. What length of wire will go round the edges of a cube, the surface of which contains 187 sq. yds. 54 sq. inches.

9. The length of a field containing 21 ac. 3 ro. 25 ps. $3\frac{7}{8}$ sq. yds. is twice its breadth. Find the length of the field.

10. A certain number of persons agree to subscribe as many pies each, as there are subscribers. If the total collection be Rs. 60-12 as. find the number of subscribers.

SECTION IV. (A)

N. B.—The whole of this section, IV (A) and (B) may be done in Forms V & VI.

EXAMPLES XII.

The Triangle and the Parallelogram.

1. (*a*) What are the necessary and sufficient conditions for two triangles to be identically equal?

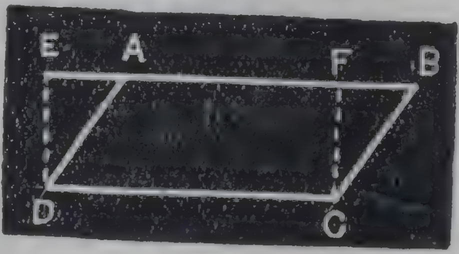
(*b*) ABCD is a parallelogram. Draw a diagonal and using the property of parallel lines prove that the two triangles (into which the parallelogram is divided) are congruent.

***Theorem*:—(The opposite sides and angles of a parallelogram are equal)**

2. AB and DC are two parallel lines; if AB = DC show that ABCD is a parallelogram, by proving that the triangles ABC, CDA are congruent.

***Theorem*:—Straight lines which join, towards the same parts, the extremities of two equal and parallel st. lines are themselves equal and parallel.)**

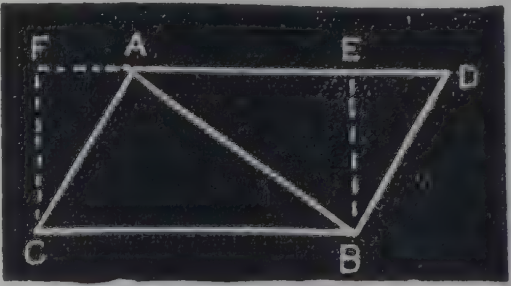
3. (*a*) ABCD is a parallelogram; Draw the perpendiculars DE and CF and shew that the area of the parallelogram is equal to the area of the rectangle DEFC. Establish the congruence of the riangles BCF and ADE and roceed.



(*Theorem*:—Parallelograms on the same base or on equal ases and between the same parallels are equal in area.)

(*b*) How will you proceed to find the area of a arallelogram?

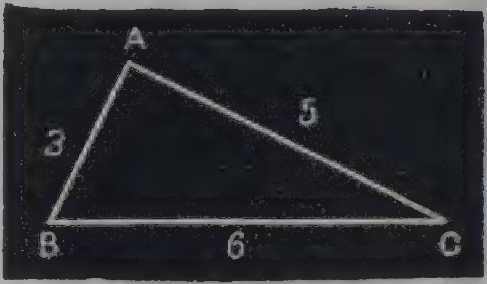
4. (*a*) ABC is a triangle. Through B draw BD parallel and equal to CA. Join AD. The area of the △ ABC is half that of the parallelogram ADBC. But the area of the parallelogram = that of the rectangle BEFC.



∴ The area of the △ ABC is ½ BC. FC i.e., ½ base × altitude.

Theorem:—If a triangle and a parallelogram be on the same base or on equal bases and between the same parallels the area of the triangle is half that of the parallelogram.

Rule:—The area of a triangle is ½ base × altitude.



5. (*a*) The sides of the triangle ABC are 3, 6, 5 in. long. Find its area.

Draw its altitude AD.

Let BD = x in., DC = $6 - x$ in.

$$\text{If AD} = h'', \text{ then } h^2 = 9 - x^2 \qquad \dots \text{(i)}$$
$$h^2 = 25 - (6 - x)^2 \dots \text{(ii)}$$

Subtracting (ii) from (i) $(6 - x)^2 - x^2 = 16$

$$\text{or } 6\,(6 - 2x) = 16 \text{ or } 6 - 2x = \frac{16}{6}$$

$$\text{or } 2x = \frac{20}{6} \text{ or } x = \frac{10}{6} \text{ or } \frac{5}{3}$$

$$\therefore h = \sqrt{(3 + \tfrac{5}{3})\,(3 - \tfrac{5}{3})} = \sqrt{\tfrac{14}{3} \times \tfrac{4}{3}} = \tfrac{2}{3}\sqrt{14} \text{ in}$$

The area of the triangle $= 3 \times \frac{2}{3}\sqrt{14} = 2\sqrt{14}$ sq. in
$= 2 \times 3{\cdot}742 = 7{\cdot}5$ sq. in. nearly

Rule.—The formula for the area of a △ in terms o the sides is $\sqrt{s(s - a)\,(s - b)\,(s - c)}$ where s denotes lengt of the semi-perimeter and a, b, c denote lengths of th sides.

(*b*) If the three sides of a triangle be given, the lengths of the three perpendiculars on the sides from the opposite angles, can be calculated, since $\sqrt{s(s-a)(s-b)(s-c)} = \frac{1}{2}$ altitude × base, and each side will be successively the base.

Example. Find the lengths of the three altitudes of a triangle whose sides are 10, 12, and 18 inches.

Here $s = 20$, $s - a = 10$, $s - b = 8$, $s - c = 2$. Taking the side of 10″ as the base, $\sqrt{20 \times 10 \times 8 \times 2} = 5 \times$ altitude.

$\therefore\ 40\sqrt{2} = 5 \times$ altitude $\therefore$ altitude $= 8\sqrt{2}''$

Similarly $40\sqrt{2} = 6 \times$ another altitude.

The second one $= \frac{20\sqrt{2}''}{3}$. The third $= \frac{40}{9}\sqrt{2}''$.

Remember that the altitudes are those, on the sides taken, from the opposite vertices.

6. Describe a triangle given that the three sides are respectively 2·5, 2·4 and 0·7 inches. Find the lengths of the three altitudes of the triangle and also the area of the triangle.

7 Half the product of the sides of a rt.-angled triangle gives the area of the triangle.

How is this rule deduced from the formula for the area of a triangle?

8. (*a*) On a base 3·4 cms. long describe an equilateral triangle. Find the lengths of the three altitudes of the triangle.

(*b*) Take any point within the triangle and draw perpendiculars to the sides. Find the sum of the lengths of the perpendiculars. What is the sum equal to?

9. (*a*) A side of an equilateral triangle is $2a$ inches. What is the length of an altitude?

(*b*) Describe a triangle ABC given that $a = 2·5$ cms., $b = 5$ cms. and $c = a\sqrt{3}$ cms., finding c geometrically. Find the magnitude of each angle of the triangle.

10. (*a*) On a base 4 cms. long, describe an isosceles rt.-angled triangle. Find theoretically the length of a side.

(*b*) If an isosceles rt.-angled triangle be described on a base x'' long, what will be the length of a side?

11. How will you calculate the area of a parallelogram, whose adjacent sides are 3 and 1½ cms. long and the contained angle is 60°. Describe the parallelogram.

12. The adjacent sides are 4 cms. and 2·8 cms. and the contained angle is 45°. Draw the figure and calculate its area.

13. The distance between the parallel sides of a parallelogram is an exact inch. One of the equal sides is 3·7 cms. and an angle of the parallelogram is 135°. Describe the parallelogram. Find its area.

14. The diagonal of a parallelogram is 3·7 cms. and the sides are 2·8 and 3·4 cms. Draw the parallelogram. Calculate the distance between the longer parallel sides.

15. The diagonals of a parallelogram are 6·3 and 5·8 cms. and the length of a side is 3·8 cms. Describe the figure and find (1) its area, (2) the distance between the longer parallel sides.

16. The diagonals of a parallelogram are 4·8 and 6·4 cms. and the angle between these is 60°. Describe the figure.

17. The diagonals of a parallelogram are 4·8 and 6·4 cms. and the angle between these is a rt. angle. Draw the figure. Do you note anything special with regard to the sides of the figure?

***Theorem*:—*The diagonals of a rhombus cut at rt. angles*.**

18. Each of the four sides of a parallelogram = 3·4 cms. and one of the four angles is 60°. Find, both by calculation and measurement, the length of each diagonal of the figure.

19. (i) The area of a triangle is 1710 sq. in. Two of the sides are 68 and 57 in. Find by calculation the length of the third side.

(ii) Similarly if two of the sides of a triangle be 77 and 40 in. and the area be 924 sq. in., find the third side.

(iii) In each of the triangles in (i) and (ii), find by calculation the lengths of the three altitudes.

20. Show how the formula $\sqrt{s(s-a)(s-b)(s-c)}$, for the area of a triangle, is altered if

i. the triangle is isosceles ;
ii. the triangle is equilateral ;
iii. the triangle is rt.-angled ;
iv. the triangle is rt.-angled and isosceles ;
v. the triangle is rt.-angled and one of the acute angles 60° ;
vi. the triangle is isosceles and the greatest angle is 120°.

21. ABC is a triangular plot of ground. AB = 65 chains, BC = 36 ch., and CA = 61 ch. D is a point in the side AB and from it DE is drawn at rt. angles to AB to meet AC in E. The portion ADE is sold. If AD = 40 ch., find by measurement from a diagram, the lengths AE and DE. Also find, by calculation, what fraction of the original extent is left.

22. The sides of a triangle are 17, 15 and 8 in. respectively. Find the length of the st. line, joining the middle point of 17, to the opposite angular point.

23. The sides of a triangle are 34, 42 and 20 in. long. Find the length of the median on the longest side.

SECTION IV. (B).

The Quadrilateral, The Trapezium, Reduction to equivalent triangle.

(To describe a quadrilateral or any other rectilineal figure from given data.)

Method I :—1. *Draw a rough free-hand figure.*

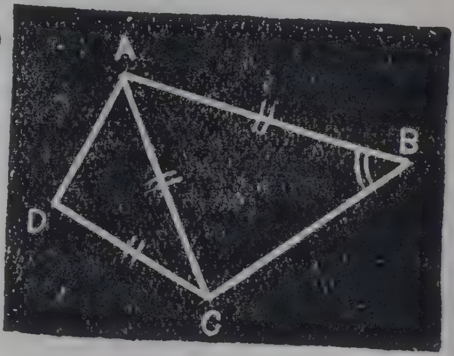
2. *Mark the given parts.*

3. *Find any triangle that can be described with the given parts.*

4. *Evolve the figure from the triangle described.*

Exercise I.—Describe a quadrilateral ABCD given that:—

AB = 3·3 cms., AC = 4·3 cms., ∠B = 90° and CD = 3·9 cms., and that D is equidistant from A and C.



i. We find that the triangle ABC can be described.

ii. D is in the perpendicular bisector of AC.

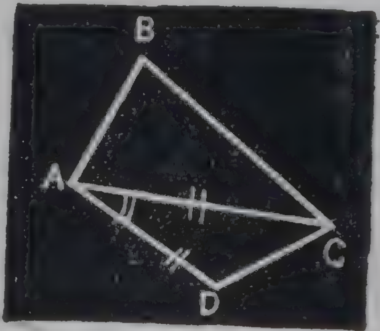
iii. D is then fixed, as CD is given.

Construction.—On AC describe a semi-circle and cut off AB = 3·3 cms. Join BC.

Bisect AC and draw the perpendicular bisector; In this find D, so that CD = 3·9 cms.

Ex. 2.—Draw the figure ABCD given that AD = 1 in., AC = 1½″, angle DAC = 60°, BC = ½ AC, AB = $\frac{3}{2}$ BC.

Analysis :—



Triangle DAC can be described; BC and AB can be geometrically constructed as AC is given.

N.B.—The student will do this exercise without the use of the protractor and without calculating the lengths of AB and BC.

3. ABCD is a quadrilateral field; BC = 20 yds., AD = 24 yds., the diagonal BD = 32 yds., the angles A and C are rt. angles. Draw a figure to scale (1″ = 10 yds.) and determine the area (S. S. L. C. 1914.)

The two triangles one on either side of the diagonal BD, can be drawn, and the two triangles together form the quadrilateral.

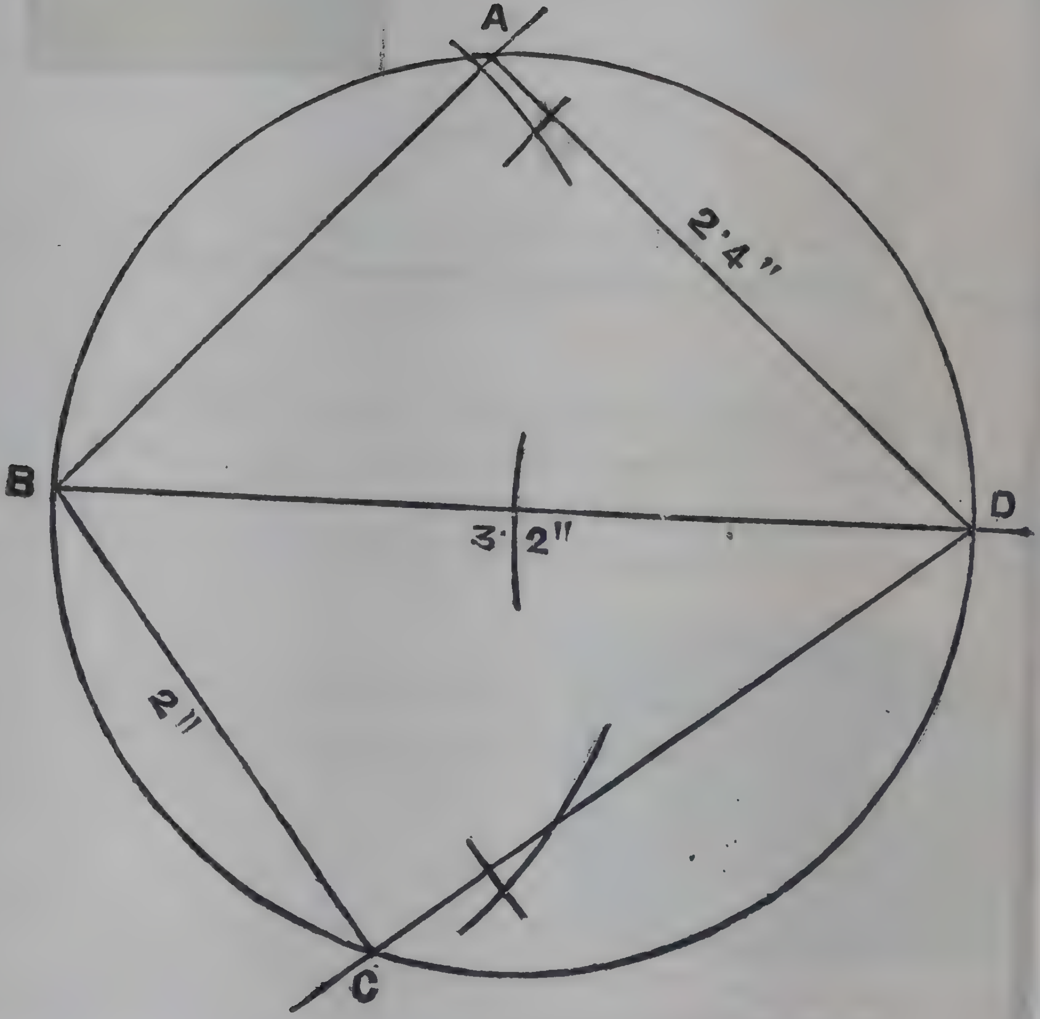
Draw a straight line BD = 3·2″; on this as diameter describe a circle. With B as centre and a length of 2″ radius, cut off BC an arc. Join BC and CD. Again with D as centre and radius = 2·4″ cut off an arc DA in the other semi-circle. Join DA and AB. Then ABCD will be the required quadrilateral.

$$\text{Area} = 10\sqrt{32^2 - 20^2} + 12\sqrt{32^2 - 24^2}$$

$$= 10\sqrt{52 \times 12} + 12\sqrt{56 \times 8}$$

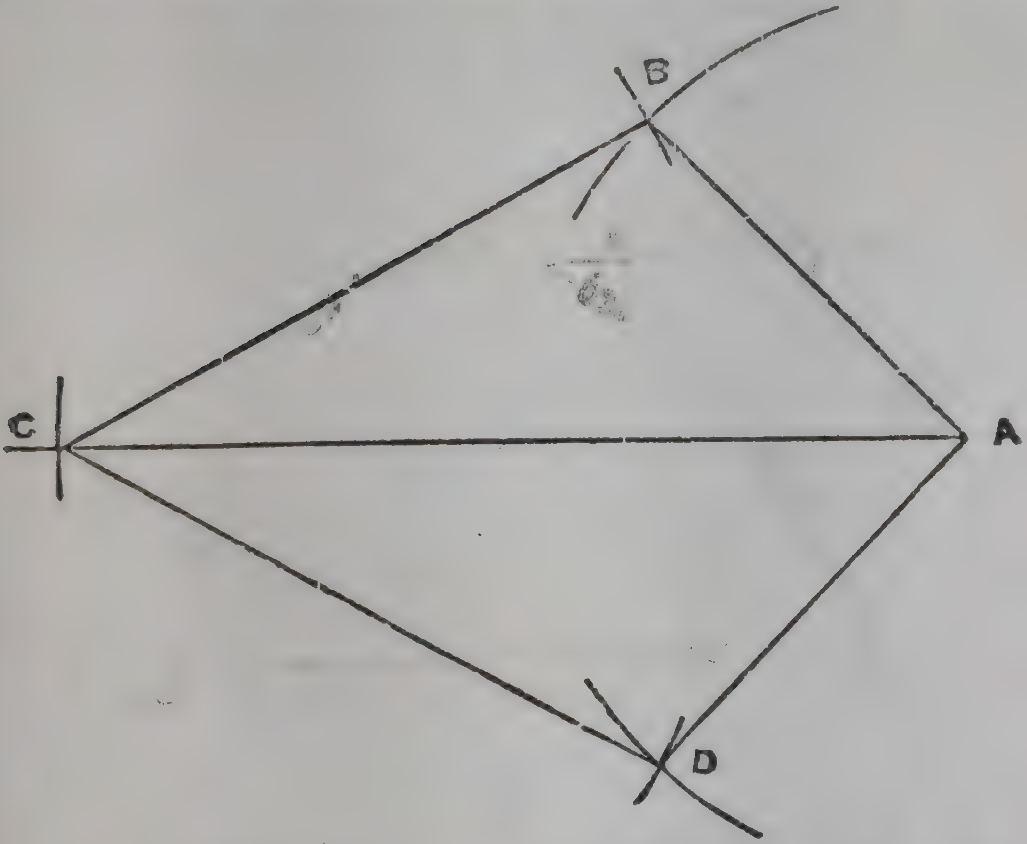
$$= 40\sqrt{39} + 96\sqrt{7}$$

$$= 40 \times 6{\cdot}24 + 96 \times 2{\cdot}65 = 249{\cdot}6 + 254{\cdot}4 = 504 \text{ sq. yds}$$



4. Construct a quadrilateral ABCD having AB = AD = 4 cms., AC = 8 cms., and CB = CD = 6 cms. Measure BD and the angles B and D.

Analysis. Describe two triangles one on either side of the diagonal AC. See Fig. below.



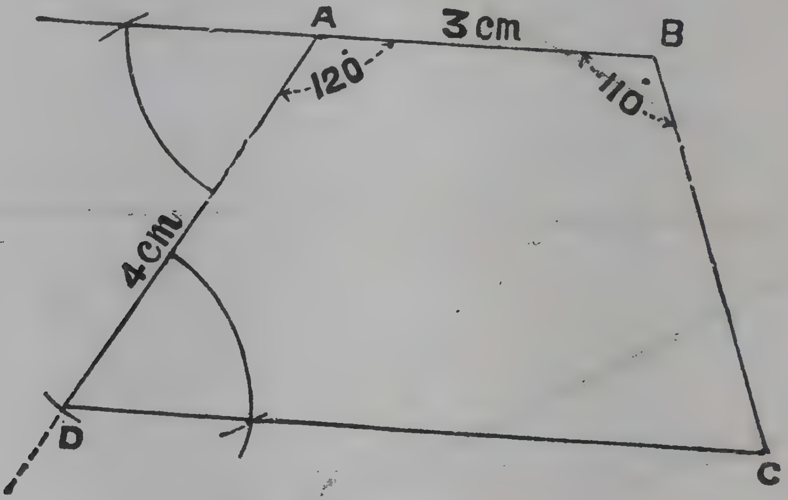
5. If from the data given, a triangle that could be described is not plainly seen, the application of some geometrical truths will get you a triangle that can be described.

Ex. Construct a quadrilateral ABCD given that AB = 3 cms., DA = 4 cms., A = 120°, C = 70°, and AB is parallel to CD. Measure BC. (S.S.L.C. 1918.)

$\angle A = 120^\circ$ $\therefore \angle D = 60^\circ$ as AB and CD are parallel. Similarly $\angle B = 110^\circ$.

Method II. Draw a line AB = 3 cms. At A and B draw angles equal to 120° and 110° respectively. Cut off AD = 4 cms. Through D draw a parallel to AB to meet BC at C.

In this case the figure is described by the gradual putting up of parts, which is another method we get for the drawing of rectilineal figures, if no triangle could be drawn with the given data.



6. ABCD is a quadrilateral field in which AB = 198 yards, BC = 264 yards, and AC = 330 yards. D is equidistant from A and C, and at a distance, of 214·5 yards from the diagonal AC.

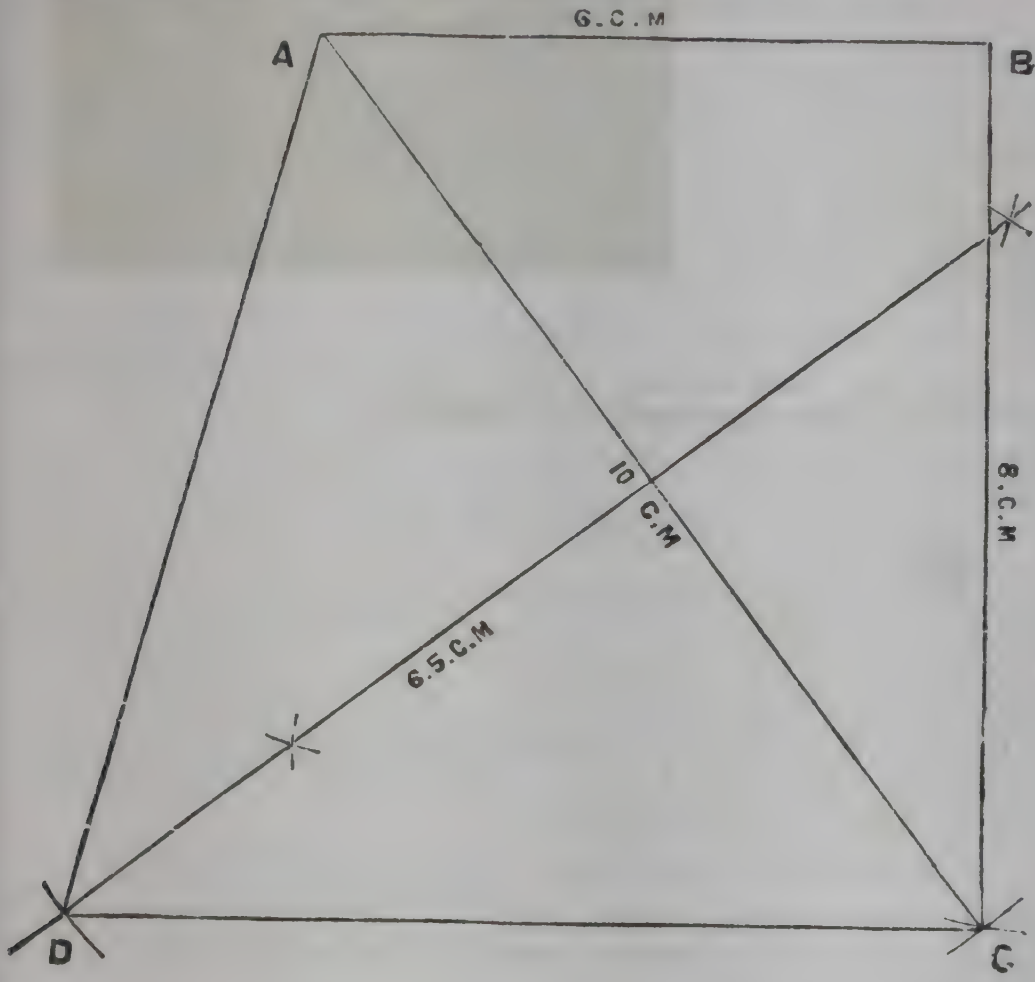
(1) Draw a plan of the field to a scale of 1 cm. = 33 yards.

(2) Find the area of the field in acres.

(1) See the figure.

Describe the △ ABC. Bisect AC. Along the perp^r bisector measure off a length = 6·5 cms.

(2) Triangle ABC is evidently a right-angled one and its area is 264 × 99 = 26136 sq. yds.



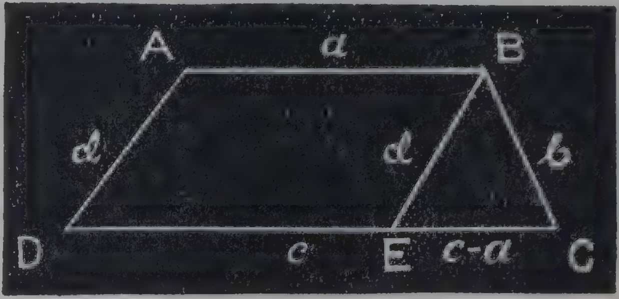
Area of the △ ADC is $\frac{330 \times 214{\cdot}5}{2}$ sq. yds.

$= \frac{6435 \times 11}{2}$ sq. yds. $= \frac{70785}{2}$ sq. yds. $=$ 35392·5 sq. yds.

7. The special name for the quadrilateral, which has one pair of opposite sides parallel is, 'Trapezium'. The figure in 5 *supra* is a trapezium.

If the lengths of the four sides of a trapezium be given, describe the trapezium.

Let a, b, c and d denote the lengths of the sides of a trapezium.



Through B draw BE a ||l to AD to meet CD in E.

The △ BEC can now be described for BE = AD = d; BC = b and EC = $c - a$. Proceed.

Half the sum of the parallel sides × distance between parallels, gives the area of the trapezium.

EXAMPLES XIII.

1. (a) Describe a triangle ABC with the following data:—$b = 7$ cms., $c = 6$ cms. and $a = 5$ cms. On AC describe another triangle ADC remote from B, given AD = 4 cms. and CD = 4 cms. What kind of figure is ABCD? How many measurements have you used to construct the new figure? How many measurements are necessary and sufficient?

(b) The four sides AB, BC, CD and DA of a quadl. are 4 cms., 3 cms., 2 cms., 3 cms. respectively and the diagonal AC = 2 cms. Describe the quadl. and measure the other diagonal.

2. The sides AB, BC, of a quadl. are 4·3 cms. and 3·8 cms. and the angle contained by the sides is 75°. If CD and DA be each equal to 5 cms., describe the quadl. Find by measurement, the diagonals.

3. Describe a quadl. ABCD, given AB = 1·2″, BC = 1·5″, AD = 1·8″, $\angle A = 120°$, $\angle B = 90°$. Draw perpendiculars from A and C on BD and measure them. Find the area of the triangles BCD and BAD and hence of the quadrilateral ABCD. Verify your answer by using the formula "Area of a quadl. = $\frac{1}{2}$ the product of the diagonal and the sum of the offsets (perpendiculars from the opposite vertices on the diagonals)."

4. PQRS is a quadrilateral with PQ = 4 cms., QR = 3·4 cms, $\angle P = 90^\circ$, $\angle Q = 60^\circ$, $\angle R = 100^\circ$. If this represents the site of a school building, draw to a scale of 1 cm. = 20 yds., a plan of the site, and find its area.

5. ABCD is a quadrilateral field whose sides AB, BC, and AD are 40 yards, 80 yards, and 35 yards respectively. If its diagonal AC = 100 yards, and $\angle A = 120^\circ$, draw a plan of the field and find its area.

6. The diagonals AC and BD of a quadl. ABCD are 180 yards and 200 yards respectively. If AB = 120 yards and angles BAC and ABD are 30° and 45°, draw the quadrilateral and find its area.

7. A quadrilateral ABCD is such that AB = 5·2 cms., BC = 3·9 cms., CD = 2·5 cms., DA = 6 cms., and angle ABC = 90°. Draw the figure and measure the length AC, and the angle CDA. Verify the measurements by calculation.

8. (a) Describe a quadrilateral with the following data:—AB = BC = CD = 6 in., AD = 8 in., the angle DAB is a rt. angle.

(b) If the figure represents a field drawn to a scale of 1 in. = 10 yards, find the area of the field in acres, correct to four significant figures.

9. (a) Draw the plan of a quadrilateral field ABCD with the following data: AB = 13 feet, BC = 14 feet, CD = 24 feet. The perpendicular from A on DC is 9 feet, and this perpendicular divides DC into two equal parts.

(b) Also find the area of the quadrilateral.

10. ABCD is a quadrilateral field. AD = 220 yards, BC = 260 yds. and the diagonal AC = 380 yds. The perpendiculars from D and B meet the diagonal at E and F so that AE = 100 yds. and CF = 70 yards. Draw a plan of the field and find its area.

11. Construct a quadrilateral ABCD with the following data:—

[*N.B.*—The constructions must all be done geometrically, *i.e.* foot-rule and compasses alone must be used.]

AB is parallel to CD; AB = 11·2 cms., CD = 5·3 cms., A = 75° and B = 60°.

Shew that there are two ways of constructing the quadrilateral. Describe one of the methods you employ, and outline the other.

12. The plan of a quadrilateral field ABCD is drawn to a scale of 1″ = 10 chains. The diagonals cut at right angles. AB = 2·4″, angle ABD = 72°, AD = 2·6″ and CD = 2·2″. Draw the plan and find the area of the field in acres.

13. KLMN is a plot of ground in the form of a quadrilateral with KM = 30 chains, NM = 24 chains, ∠N = 90°, ∠L = 120° and side KL = LM. Draw a plan of this plot of ground choosing a suitable scale and find its area in acres.

14. ABCD is a garden in which AB runs East and is 200 yards long. BC runs North and is 150 yards long. D is equidistant from A and C and at a distance of 162·5 yards from the diagonal AC. Draw a plan of the garden and find out the length of the road that runs through it cutting BC at 80 yds., from B and AD at 40 yds., from D.

15. In a quadl. ABCD angles B and D are right angles and the diagonal AC = 5 cms. If BC = 3 cms. and ∠CAD = 40° draw the quadrilateral and find the length of the diagonal BD.

16. A, B, C and D are the four stones that mark the corners of a certain field. The distance between A and B = 500 links, A and D = 450 links, B and C = 600 links. If the lines AC and BD cut at right angles and ∠BAD = 120°, mark the four stones in position and find the distance between C and D.

17. KLMN is a park with KL = 132 yards, diagonal KM = 198 yards, diagonal LN = 220 yards and ∠ MKL = 60°. If the diagonals intersect at right angles, draw a plan of the park to a scale of 1 cm. to 22 yards and find the length of fencing required to completely enclose the park.

18. PQRS is a lawn in the shape of a quadrilateral such that PQ = QR = PR. The distance of Q from PR = 34·6 ft., QS = 48 ft. and ∠S = 90°. Draw a plan of the lawn and find its area.

19. A plot of ground is in the form of a quadrilateral KLMN with the diagonal KM = 25 chains. The offsets LR and NS on KM are 15 ch. and 8 ch. long and are 16 ch. and 20 ch. respectively from M. Draw a plan of this plot and find its area in acres.

20. ABCD is a garden in which AB runs North and is 180 yds. long; BC runs East and is 90 yards long. AD runs North-east and is 120 yards long. Draw a plan of the field and mark in it, the position of a well which is 100 yds. from B and 50 yds. from CD. Find its distance from A.

21. The corners ABCD of a quadrilateral are respectively 2 cms., 3 cms., 4 cms., 3·4 cms., from a point O within it. If the angles AOD, BOC and COD are 60°, 80°, 140° respy., construct the quadrilateral and measure its sides.

22. A rectangular frame ABCD, 18″ by 10″ gets loosened and is to be held tight by strings tied to the nails K, L, M, N fixed to the sides AB, BC, CD and DA respectively. If K divides AB in the ratio of 9 : 11, L divides BC in the ratio of 3 : 5, M and N bisect CD and DA, draw a sketch to show the strings in position and find the length of the string required.

23. ABCD is a sheet of paper 7″ by 9″ wherein is drawn the plan of a quadrilateral field PQRS. P is 5″ from A and 2½″ from B. Q is 4″ from B and 5·5″ from C. S is 5·2″ from A and 4·6″ from D. R is 4·5″ from C and the angle at this vertex is a right angle. Show that there can be two such quadrilaterals and draw them.

24. Draw AB, 3″ long. Draw DC parallel to AB at a distance of 1·2″ from it, so that AD = 1·3″ and ∠BAD is an acute angle. What kind of figure is ABCD? Find its area by any method. Show that the result is equal to $\frac{1}{2}h(a+b)$, where a and b give the lengths of the parallel sides and h the distance between them.

25. Draw the plan of a quadrilateral field ABCD from the following measurements taken in links. Find the area of the field in sq. yds. and find the length of the shortest distance from A on CD.

BM the perpendicular from B on AC = 400.
DN ,, ,, D on AC = 300.
AM = 300, AN = 400, AC = 625.

26. Describe a quadrilateral ABCD with the following data :—

AB = BD = 3 in. ; BC = 3·25 in., AC = 4 in., AD = 3·4 in. Measure CD. Also find the area of the quadrilateral by any method.

27. (a) A field is in the form of a trapezium ABCD. The parallel sides are AB and CD, 180 and 200 yds. respectively. ∠A is a rt. angle ; ∠B is obtuse = 120 degrees. Draw the plan of the field. Find the length of BC.

(b) If this field is to be converted into a rectangular one without altering the area, shew how it could be done.

28. Construct a trapezium ABCD, (AB and CD being the parallel sides) with the following data and find its area. AB = 5 cms., CD = 3 cms., AD = 2·4 cms. and ∠A = 60°.

29. The four sides of a trapezium KLMN are as follows : KL = 7 cms., LM = 3 cms., MN = 5 cms., NK = 2·5 cms. and KL is parallel to MN. Construct the trapezium and find its area. If this figure represents a field drawn to a scale of 1 cm. to 22 yds. find its area in acres.

30. The parallel sides of an isosceles trapezium are 220 yds. and 160 yds. and the distance between them is 40 yds. Draw a neat diagram to scale and find the length of the oblique sides.

31. The side glass of a Municipal lantern is in the form of a trapezium with the parallel sides equal to 6″ and 8″ respectively. If the other two sides are both inclined to the base (shorter of the parallel sides), at an angle of 100°, draw a neat sketch of the lantern glass and find the area of the glass required for the lantern, if it has four such glass sides.

32. ABCD is a trapezium with AB = 2″, BC = 1·2″, ∠B = 105° and ∠D = 60°. If AB is given to be parallel to CD describe the trapezium and measure CD.

33. I own a plot of ground in the form of a trapezium PQRS. The parallel sides PQ and RS are 3000 links and 1800 links long. The third side PS is at right angles to them and is 1200 links in length. Find the length of the fourth side which is a narrow foot-path. If I desire to make the plot rectangular without changing the area, how can I effect the alteration? What will be the length of the fourth side after the change?

34. The diagonals AC and BD of a trapezium ABCD are 6 cms. and 3·5 cms. respectively. If the parallel sides AB and CD are 5 cms. and 2·5 cms. construct the trapezium and measure the oblique sides.

35. The two parallel sides AB and CD of a trapezium ABCD are respectively 3 inches and 2 inches and the diagonals AC and BD are 3·4 and 4·1 inches respectively. Describe the trapezium and measure the lengths of the sides BC and DA.

36. Describe a trapezium ABCD given the parallel side AB and DC are 2·3 and 3·5 inches long and that the angles BAC and DBA are 45 and 30 degrees respectively. Find the lengths of the other sides.

37. The sides MN and OP of a quadrilateral field are parallel and are 300 yds. and 180 yds. respectively. The diagonal MO is 320 yds. and makes an angle of 130° with the other diagonal. Draw a plan of the field and mark in it the position of a circular well 6 yds. in diameter having the point of intersection of the diagonals as its centre.

38. In a quadrilateral ABCD, the sides AB and CD are parallel and are in the ratio of 9 : 5. The diagonals AC and DB are 4·5 cms., 3·9 cms. and enclose between them an angle of 130°. Describe the quadrilateral and measure the parallel sides.

39. (a) Describe a triangle ABC given AB = 2·5 cms., BC = 1·3 cms. and CA = 2·4 cms. Find a point D, 1·5 cms. from C and 2·3 cms. from A. Find another point E, 1·3 cms. from A and 2·2 cms. from D. What kind of figure is ABCDE?

Note that for constructing the base triangle ABC, (*i.e.*, for fixing the first three vertices of a rectilineal figure) we want three measurements and that for fixing the 4th vertex and the 5th vertex we require two measurements each.

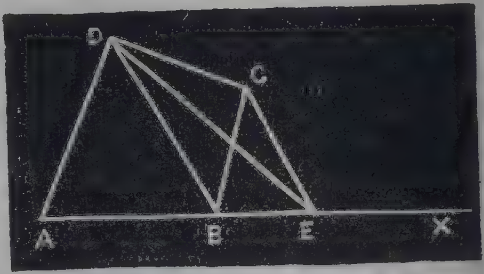
How many measurements have we used in constructing the 5-sided figure ABCDE? How many shall we require to construct a polygon of 6 sides, 8 sides and n sides?

(b) ABCDE is a field in the form of a pentagon whose sides AB, BC and CD are resp. 60 yards, 50 yds. and 30 yds. in length; $\angle A = 135^\circ$, $\angle B = 90^\circ$; $\angle C = 120^\circ$ and the distance of E from AD is 25 yds. Draw a plan of the pentagon choosing a suitable scale and find its area.

40. (a) Draw a st. line AB, 2 inches long. Draw another st. line XY parallel to it and at a distance of 1·5 inches from it. Take any two points C and D in XY and find the areas of the triangles ABC and ABD. What do you observe?

(b) On a base AB equal to 2 inches describe a quadrilateral ABCD with the following data:—BC = 1·5 inches, CD = 2·1 inches, DA = 2 inches, and BD = 2·4 ins. Produce AB to X. Draw CE parallel to DB to meet AX at E. Join DE. Show that the quadrilateral ABCD is equal in area to the triangle ADE.

(c) Hence devise a method for reducing a quadrilateral or any polygon to an equivalent triangle.



N.B.—The student will get considerable practice if he attempts the reduction to equivalent triangles, of quadrilaterals described as per conditions given in the questions above.

The area of the reduced triangle may be calculated from measurements taken. Preferably the student will retain one side of the quadrilateral and measure the altitude (of the triangle) on the side. The area of the triangle thus obtained and the area of the quadrilateral (calculated wherever possible) must afford a good verification.

SECTION V.

Graphs.

LESSON I.

To be done in Form IV.

Plotting of Points.

N.B.—We give below some teaching notes only. The author will only be too glad to receive instructions towards the improvement of the scheme laid down.

1. Ask a boy to mark a point anywhere on the blackboard.

2. Let the boy take a foot-rule and measure its distance from the right hand side edge and also its distance from the bottom edge. Now ask him to define the position of the point.

3. Let another boy mark another point on the blackboard and let him define the position of the point.

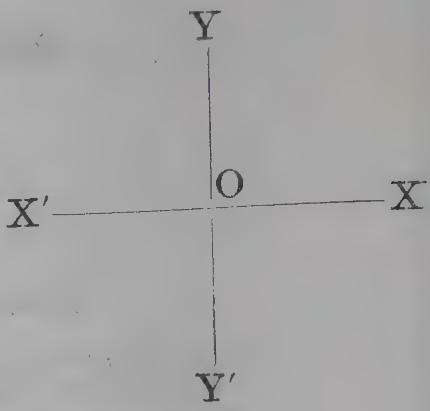
4. The teacher will ask a boy to mark a point 1 ft. above the bottom edge. If he marks a single point, the teacher will mark another point and ask him if that point also is not 1 ft. above the bottom edge and thus elicit the fact that all points in a parallel line 1 ft. above the bottom edge satisfy the condition. The idea of '*a locus*' must be introduced at this stage or revised if the pupils are already familiar with it.

5. Let the boy learn that one measurement cannot fix a point.

6. Can two measurements fix the point, one from the bottom edge and one from the top edge? The measurements are related and hence the points cannot be fixed.

7. Two independent measurements—distances from two lines at rt. angles—are necessary.

8. Let the boys be asked to fix positions of a few points on the blackboard always mentioning the rt. hand edge and the bottom edge.



9. Taking two lines at rt. angles, let the boys be asked to plot points thus :—

(*a*) 1 in. above the horizontal and 1½ in. to the right of the vertical.

(*b*) 1 in. above the horizontal and 1½ in. to the left of the vertical.

(*c*) 1 in. below the horizontal and 1½ in. to the right of the vertical.

(*d*) 1 in. below the horizontal and 1½ in. to the left of the vertical.

Let the boys be then told that, instead of repeating horizontal, vertical and left, right, below, above, we may use some abbreviation.

10. The terms 'co-ordinates' 'abcissa,' 'ordinate,' etc., may be brought in now.

EXAMPLES XIV.

[To be done in Form IV.]

Oral.

1. What are the co-ordinates of the origin?

2. „ „ of any point on the abcissa?

3. ... ... on the ordinate?

4 The ordinate of a point is – 8, and it is on a line parallel to the Y-axis at a distance of 1½ in. from the Y-axis. What are the co-ordinates of the point, the unit being $\frac{1}{5}''$?

5. If you plot the points (– 5, 3) (– 5, 8) (– 5, 12) (– 5, 18), (– 5, – 12) (– 5, 7), what peculiarity do you note about the line joining these points?

Written.

6. Plot the following points taking $\frac{1}{10}''$ as the unit: (3, 7), (2, −3), (−3, 8) (7, −8).

7. With $\frac{1}{5}''$ as the unit, plot the three points (−3, 8), (7, −4) and (−8, −5).

Find by counting squares the area of the triangle formed by joining the three points.

8. With $\frac{3}{10}''$ as unit, mark the points (4, 10), (4, 0), and (−8, 5). Show that the triangle formed by joining the three points is isosceles.

Hint.—Find the lengths of the sides by applying the theorem of Pythagoras.

9. Join the following points in order: (0, 5), (6, 0), (0, −5) and (−6, 0). What kind of figure do you get? How can you prove your statement?

10. Plot the following points with unit $\frac{1}{10}''$ and join them in order (10, 10), (10, 6), (5, 6), (5, −10), (−5, −10), (−5, 6), (−10, 6) and (−10, 10). The figure represents the ground-plan of a building to a scale of 1 in. = 30 feet. Find the area of the ground-floor in square feet.

11. (a) Plot the following points and join them in order. The unit is $\frac{1}{5}''$ along the X-axis and 1 in. along Y-axis:

(0, 5), (1, 4·8), (2, 4·6), (3, 4·5), (4, 3·9)

(b) What, with this scale, will be the co-ordinates of the points specified below.

(i) A point 1½ in. to the right of the Y-axis and 3½ in. above the X-axis.

(ii) A point whose co-ordinates are (−30, 85), if the unit be $\frac{1}{5}''$ along both axes.

12. You are given squared paper and are asked to find by measurement the length of a ladder whose foot rests at a point 5 feet from the foot of a wall, and whose top reaches a point 12 feet high on the wall. You are also told that the diagram drawn must be to the scale of $\frac{3}{10}''$ = 1 foot. If the foot of the ladder is to be at the origin, specify the co-ordinates of the points you must plot.

Is it allowable in this case to have different units, one for measurement along the X-axis and another for measurement along Y-axis ? Give reasons.

LESSON II.

To be done in Form V.

The graph as Locus and Simultaneous Equations.

1. Let the student plot the points (2, 4), (3, 6), (4, 8), (6, 12), (10, 20), (20, 40).

The student's attention will be drawn to the fact that the ordinate of each point is twice its abscissa.

2. These points all lie in a st. line! also points (5, 10), (7, 14), (9, 18) etc......i.e. points whose ordinates are twice their abscissa, all lie on this line.

3. Let the student be made to propose points whose ordinates are half their abscissa.

4. Instead of saying that all points, whose ordinates are twice their corresponding abca. lie on the st. line, we say that the line is the locus of points whose ordinates are twice their abca ; or because the ordinate i.e., the Y of each point is twice its abca., i.e. is 2 of its X, in shortened form we call the line, the graph of $Y = 2X$.

5. Let the boys propose co-ordinates of points whose ordinates are 3 less than twice their corresponding abca. Graph of $y = 2x - 3$.

6. In just the same way as $y = 2x - 3$ is a relation between y and x, the line is also a relation.

7. Draw graphs of the equations

i. $y = 5x - 2$, ii. $2y = 3x - 1$, iii. $4y = 6x + 7$, iv. $4y - 3x = 2$, v. $3x + 2y - 2 = 0$, vi. $3x + 1\frac{1}{2}y - 1 = 0$, vii. $3\frac{1}{2}x - 1\frac{1}{2}y + 1 = 0$.

8. Draw the graph of $y = 3x - 4$. In the same figure draw the graph of $3y - 5x = 8$.

What is the graph of $y = 3x - 4$, the locus of ?

What is the graph of $3y - 5x = 8$, the locus of ?

Derive graphical solution of simultaneous equation.

9. To draw the graph of $y=3x-4$.

Any value given to x, must give a corresponding value for y. However, the student should note that, though $y=143$ when $x=49$, and $y=6$ when $x=3\frac{1}{3}$, it is not advisable to give such big values or such fractional values, for the simple reason that in the former case there may be no room in the paper itself and that in the latter case the plotting of fractional values will not lead to accurate results.

The values given and obtained must be such that the points to be plotted lie, all of them, at the corners of squares already drawn in the paper. When fractional values have to be plotted, the unit must be changed. If $\frac{3}{10}''$ in the X-axis is to be the unit, $x=3\frac{1}{3}$ may be conveniently plotted.

EXAMPLES XV.

To be done in Form V.

Solve the following equations graphically.

(1) $7x+5y=17$
$3x-2y+1=0$

(2) $3x-y=1$
$4x-6y=1$

(3) $12x-3y=12$
$6x+y=11$

(4) $2x-4y=0$
$x+y=4{\cdot}5$

(5) $2x+y-3=0$
$4x-y=27$

(6) $3x-20y=15$
$x-3y=-0{\cdot}5$

N.B.—(1) The light which a theoretical solution throws on the choice of the scale, should afterwards be shown to the student.

(2) The equations in Examples IV will also be solved graphically.

LESSON III.

To be done in Form V.

Constants and Variables.

Conversion Graphs.

1. The distance between Madras and Trichy is 198 miles. How long will it take a train to cover the distance, if the train runs at an average speed of (i) 18 miles an hr., (ii) 20 miles an hr., (iii) 24 miles an hr., (iv) 30 miles an hr., (v) 36 miles an hr.?

The distance is constant, i.e., does note change; but the duration of the journey changes as the speed of the train changes. Therefore we call the *duration* and the *speed*—'variables.'

What is the relation between the time or duration of the journey, the speed and the distance?

If t denotes time, s the speed and d the distance, $d=ts$. The student knows that $d=ts$ is called an equation.

2. A dramatic troupe staged some plays during a season and the cost of each bench ticket was 8 as. For the season, the 8 as. is a constant. The receipts from bench tickets depend on the persons taking bench tickets. If y denote total receipts from the sale of bench tickets and x the number of spectators who purchased the bench tickets, we have the relation $y=8x$.........an equation.

The value of y depends on the value of x, i.e., the income depends on the number of spectators. x is therefore called the *independent variable* and y the *dependent variable.*

3. If you go in for a gramaphone and records, you have to pay a fixed amount towards the cost of the machine and an amount which varies according to the number of records you buy.

If the price of the gramaphone machine is Rs. 250 and the price of each record is Rs. 4, then the money y, you will have to pay will be, $y=4x+250$; x denoting the number of records you have purchased.

Whatever be the number of records you purchase, the price of the machine is fixed and the amount you have to pay, depends on the number of records you buy.

4. A boy who wants to learn cycling goes to a cycle dealer who lets cycles on hire. The dealer demands 8 as. for the use of the bicycle and four annas for every hour used. If at the end of 'x' hrs. the cycle is returned to the dealer, the amount (in annas) the boy will have to pay is, $y=4x+8$.

5. The student must have noted that the income of the company in Q 2, depends on the spectators alone, while the costs in Qs. 3 and 4 are partly constant and partly variable. The general forms of the equations are $y = mx$ and $y = mx + c$, respectively, m and c being constants.

6. Taking any linear equation in x and y, the student will note that, by transposition, it can be thrown into the form,

$y = mx + c$ where m and c are arithmetical numbers.

Ex. $7x - 5y = 17$; or $7x - 17 = 5y$, or $y = \frac{7}{5}x - \frac{17}{5}$; m is here equal to $\frac{7}{5}$ and $c = -\frac{17}{5}$.

7. As every equation is the locus of points satisfying the condition stated in the equation, the co-ordinates of any point in the line, substituted in the equation should satisfy the relation

i.e., $(5, -1)$ is a point on the graph of $y = 3x - 16$: for, $-1 = 15 - 16$.

8. The pupil must clearly understand that an equation of the form $y = mx + c$, represents symbolically a quantity y which is partly constant and is partly variable, that it is also the relation expressed between two variables y and x and that it is the locus of points each of whose ordinates is c more than m times its abscissa.

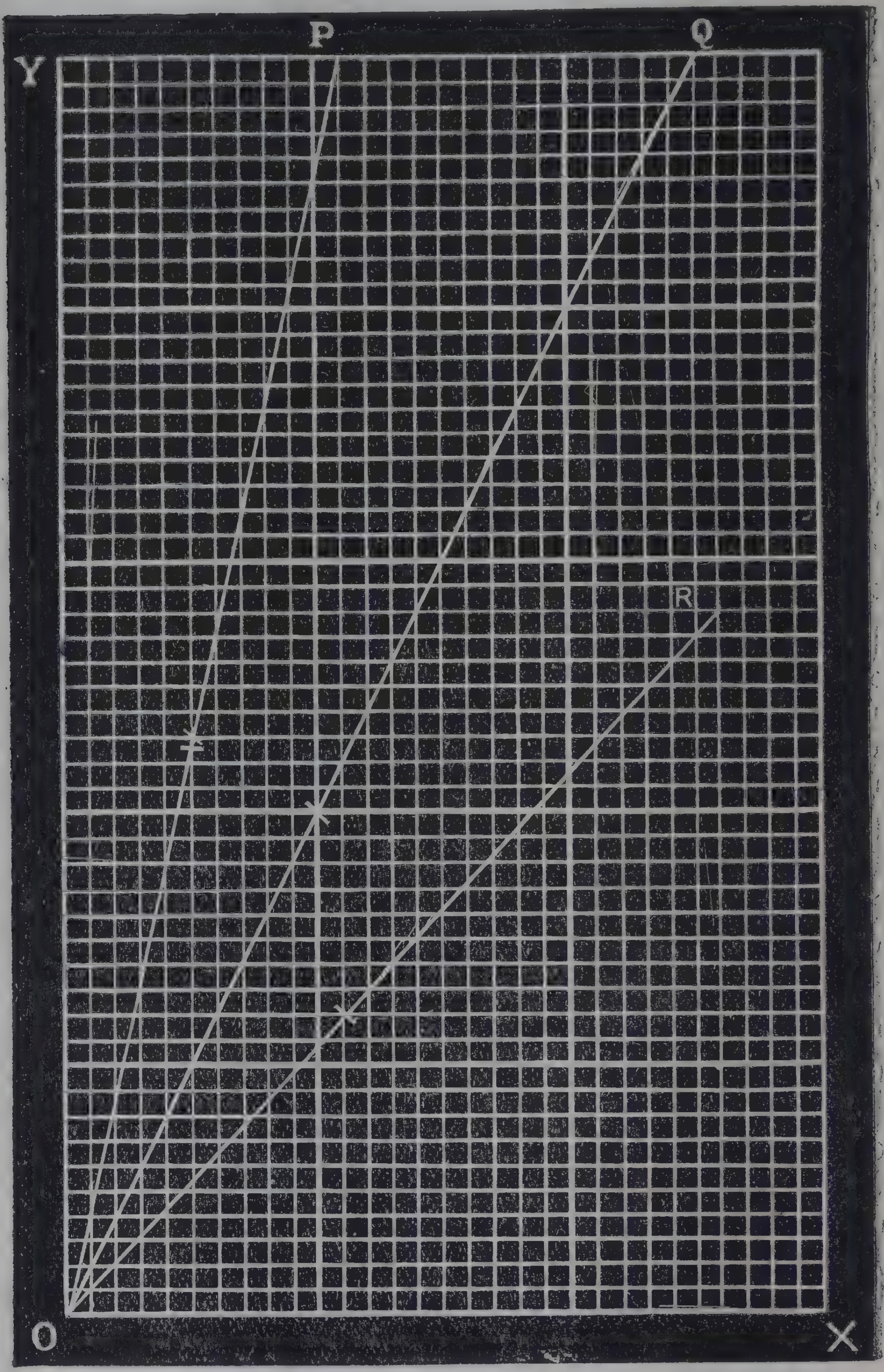
9. It follows therefore that if two quantities are related with one another, the relation can be expressed both algebraically and can also be exhibited graphically.

We know that Re. $1 = 16$ annas. Therefore if y denote the number of Rs. and x the number of annas equal in value to Rs. y then the algebraical relation is $16y = x$.

We may draw the graph of $16y = x$ and read off from the graph the value in annas of a given number of Rupees and *vice versa*; or plot points (1, 16), (2, 32) etc., to suitable scale and join them.

10. (i) 1 Kg. = 2·25 lb.
(ii) 1 Re. = 1 s.-4 d.
(iii) 1 metre = $1\frac{1}{11}$ yds.
} The graphs are overleaf.

(i) Scale along X-axis $\frac{1}{10}''$ or 1 dn. = 1 Kg.
" " Y-axis $\frac{1}{5}''$ or 2 ds. = 1 ℔.



By the question 5 Kg. = 11·25 ℔. and 10 Kg. = 22·5 ℔. we plot points (remembering the scale chosen) (5, 11·25) and (10, 22·5). These two points and the origin must be in a straight line. OP is the graph.

The student must note that we don't plot (1, 2·25) direct from the relation 1 Kg. = 2·25 ℔., for if we represent 1 ℔. by 1 in. there will be no room in the paper to represent a decent number of Kg.; for 5 Kg. = 11 ℔., nearly and 11 in. will be required. Again if we represent 1 ℔. by $\frac{1}{10}''$ or $\frac{1}{5}''$ accurate plotting of 2·25 ℔. will not be possible.

(a) If you are told to draw the graph of a cyclist who rides at the rate of $9\frac{1}{4}$ miles per hour, plot the time 4 hours and the corresponding distance 37 miles.

(b) Given that 1 mile = 1·6 Km., how will you proceed? Evidently by plotting 5 miles and 8 Km.

(ii) 1 Re. = 1 s.-4 d. = 16 d.

Scale along X-axis $\frac{1}{10}''$ or 1 dn. = Re. 1
,, ,, Y ,, $\frac{1}{10}''$ or 1 dn. = 8 d.

OQ is the graph.

(iii) 1 metre = $1\frac{1}{11}$ yds. ∴ 11 metres = 12 yds.
22 ,, = 24 yds.

Scale along X-axis, $\frac{1}{10}''$ or 1 dn. = 1 metre.
Y-axis $\frac{1}{10}$ or 1 dn. = 1 yd.

OR is the graph.

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The ready reckoner and its usefulness must be taught at this stage.—See section on Profit and Loss.

All the diagrams, (especially those on squared paper) printed in this book or for the matter of that, in every *printed* book, can at their best, give only a clear idea of the method of procedure. The students must draw big diagrams for themselves.

11. Before concluding this lesson we should request the teacher to give a lesson to the students on 'Statistics' and its importance in everyday life.

Notes.—(a) Meaning of statistics—record of numerical facts observed.

(b) Follow no regular or established laws.

(c) Greater the number of observations, the more accurate the expectation, though not altogether reliable.

(d) i. Doctor treating a typhoid patient—Temperature chart—how useful for future reference in treating another case of typhoid.

ii. The height of barometer noted at different intervals and the temperature chart.—To forecast weather in different seasons.

iii. The preparation of the Budget from previous records.

(e) The value of a visual representation of these figures, as giving an idea of the figures between two consecutive observations.

N.B.—In fact the visual representation of statistics is the only practical application of knowledge of graphs. No one solves simultaneous equations graphically or compares fractions by means of graphs.

The Ready-reckoner and the statistical graphs and a few examples on Time and Distance—these only need graphical treatment.

12. If $(-3, 2)$ is a point on a line, $(-3, 2)$ must satisfy the relation $y = mx + c$, i.e., $2 = -3m + c$. (*Vide* 7 Lesson III.)

Again if $(5, -3)$ is also a point on the line, $-3 = 5m + c$. Two points determine a line.

Solving for m and c we have $m = -\frac{5}{8}$ and $c = \frac{1}{8}$ *i.e.*, the equation of the line passing through $(-3, 2)$ and $(5, -3)$ is $y = -\frac{5}{8}x + \frac{1}{8}$ *i.e.*, $8y = -5x + 1$.

i.e., $5x + 8y = 1$.

13. (a) Draw the graph of $2x - 3y = 5$. In the same figure draw the graph of $3x + 2y = 7$. How are the lines placed with reference to each other?

(b) Also draw in another figure the graph of $y = \frac{2}{3}x + \frac{1}{3}$ and the graph of $y = -1\frac{1}{2}x + \frac{3}{4}$. See that these lines are perpendicular.

(c) The student should note that a linear equation in x and y viz., $y = mx + c$ is perpendicular to the line $y = -\dfrac{1}{m}x + d$.

(d) As there may be any number of lines perpendicular to a line, the lines will be obtained by giving different values to d.

(e) Take the equation, $y=3x-2$. Any line perpendicular to this is $y=-\frac{1}{3}x+d$.

If this line is to pass through another given point, say $(6, -1)$

(Note that two points determine a line), $-1=-2+d$ or, $d=1$.

i.e. The line perpendicular to $y=3x-2$, through $(6,-1)$ is $y=-\frac{1}{3}x+1$ i.e. $3y+x=1$.

(f) If the 'm's of the two lines be the same, the st. lines will be parallel. Draw graphs of $2x+3y=7$ and $2x+3y=3$. Again $2x+3y=d$ is also parallel to the given line and d is to be determined by any other condition given.

(g) Draw the graphs of $y=3x$ and $y=3x-3$. The graph of the former passes through the origin while that of the latter does not. If the absolute term be wanting, the graph passes through the origin. If the equation be thrown in the form $y=mx+c$, c gives in magnitude and direction the length of the intercept of the line on the Y-axis.

EXAMPLES XVI.

To be done in Form V.

1. Determine the equations of the lines passing through each of the following pairs of points :—

i. $(3, -1), (2, -3)$
ii. $(-2, -3), (3, -1)$
iii. $(-7, 11), (-11, -7)$
iv. $(6, -2), (-2, 6)$
v. $(-23, 8)$ $(0, -3)$
vi. $(0, -23)$ $(23, 0)$
vii. $(5, 0)$ $(0, -5)$
viii. $(7, -3)$, $(0, -11)$

2. (a) Determine the equation of the line passing through $(5, -3)$ and perpendicular to line (i) *supra.*

(b) Find the equation of the line passing through $(5, -8)$ and perpendicular to each of vi, vii, viii *supra.*

(c) Find the equation of the line parallel to iii, v *supra* and passing through $(10, -8)$ and $(0, 0)$ respectively.

3. A man motors at the rate of 13 miles an hour. If y be the distance travelled in x hrs., $y=13x$ is the equation connecting the hrs. and the distance. Draw the graph of $y=13x$. For a given abca. the corresponding ordinate gives the distance travelled and *vice versa*.

(a) Draw the graph of a man who travels at 6 miles an hr.

(b) " " Cyclist " 8 miles an hr.

4. The graphs may be drawn without writing out the equations.

Distances travelled may be measured along X-axis and the time taken along Y-axis.

A man has to go to a place 18 miles distant. Find graphically when he will reach his destination if he walks at 4½ miles an hour. Represent the distance of 18 miles on the X-axis and time along Y-axis.

In 2 hrs. the man travels 9 miles.

Scale in X-axis $\frac{3}{10}''$ to represent one mile.
Y-axis $\frac{3}{10}''$ " one hour.

5. A man is able to save Rs. 10 a month. How much would he have saved in 13 months?

6. 1 cub. ft. of water weighs 1000 oz. By choosing suitable scale draw the graph showing the relation between the volume of water in cub. ft. and its weight in oz. From the graph find:—

(1) the weight of 3·4 cub. ft. of water.

(2) the volume of 4500 oz. of water.

7. 1 Re. =16d. Draw a graph to convert English money to Indian money by simply reading the co-ordinates.

From the graph find the equivalent in
Eng. money of Rs. 24.
" " Indian money of £ 2.

8. 5 inches = 12·7 cms. Draw a graph for expressing inches in cms. Find from the graph the number of inches in 10 cms. and the no. of cms. in 3·5 in.

9. The European mercantile weight-equivalent is 8 viss = 25 lbs. Find from a graph the equivalents in lbs. of $3\frac{1}{2}$ viss, $4\frac{1}{2}$ viss, 10 viss and the equivalents in viss of 16 lbs., 30 lbs.

10. 'A pint of pure water
Weighs a pound and a quarter'

From a graph find the equivalent in lbs. of $3\frac{1}{2}$ pints, 2 gallons.

11. One *naligai* = 24 min. Find from a graph the number of *naligais* in 2 hrs., 1½ hrs., ¾ hr.

12. The number representing miles per hour multiplied by $\frac{22}{15}$ gives so many feet per second, so that 15 miles per hr. is equal to a rate of 22 ft. per second. Draw a graph from which the equivalent of miles per hr. to feet per second may be read off. From the graph find the number of feet per second for rates of 20 miles per hour, 18 miles per hour, and the rate in miles per hour of a body which moves through 66 ft. per second.

13. A man goes from P to Q a distance of 23 miles at 4 miles an hour. Find graphically when he reaches his destination.

14. A student cycles from a town to a village 18 miles distant at 9 miles an hour. At the 15th mile he takes rest for 10 min. Find when he will reach the village.

15. Incomes are taxed at 4 pies in the rupee. Draw a graph from which taxes to an income of Rs. 300 may be read off. If a person pays a tax of Rs. 2-9-8, what is his income and what is the tax on an income of Rs. 350 ?

LESSON IV.

Some Arithmetical Examples worked out.

[*N.B.*—More examples of arithmetical graphs will be found under each topic in the other sections.]

Ex. 1. A person A starts from a place P to Q, 16 miles distant and walks at 4 miles an hr.; 40 min. later a cyclist leaves P for Q and overtakes A in 20 min. Cycling at the same rate he meets, another cyclist riding at 24 miles an hr. from Q to P, half-way between P and Q. Find graphically—

1. when each will reach his destination;
2. the rate of the first cyclist;
3. the time of starting of the second cyclist from Q.

To fix the scale—We will have to represent

(a) i. 16 miles. ii. 4 hrs.

(b) 40 min. 20 min. and 20 min. *i.e.* the time taken by the second cyclist to go half-way—are all multiples of 10.

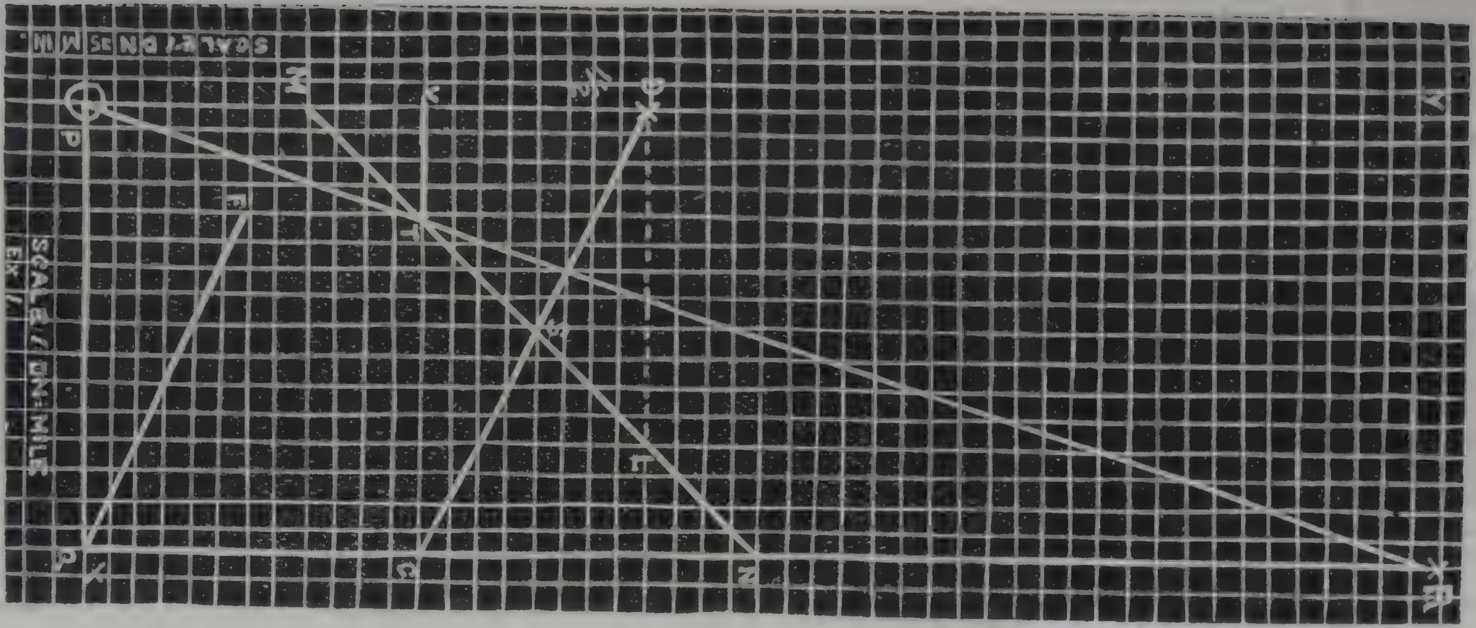
∴ $\frac{1}{10}''$ or 1 dn. in the figure may represent 1 mile and $\frac{1}{10}''$ or 1 dn. along the other axis may represent 10 min.

But then the figure will be very small. Therefore to get a decent figure the student may take $\frac{1}{2}''$ to represent one mile along the X-axis and $\frac{1}{10}''$ to represent 5 min. along the Y-axis.

We however choose $\frac{1}{10}''$ to represent one mile and $\frac{1}{10}''$ to represent 5 min. The size of the page permits only a small diagram.

The points (4, 12), (8, 24), (0,0) must be in a line and this line is the graph of the man walking at 4 miles an hour. As the time of reaching the destination is asked, see that the line goes through (16, 48) also. PR is the graph of the man.

Scale along X-axis is 1 dn. = 1 mile and along Y-axis, 1 dn. = 5 min.



The length PM represents 40 min. The two are together 20 min. later. Therefore through V (where MV represents 20 min.,) draw a parallel to X-axis to meet the graph of the man. Join MT which is graph of the cyclist, and his rate is given by DL = 12 miles an hour. The two cyclists are together at S.

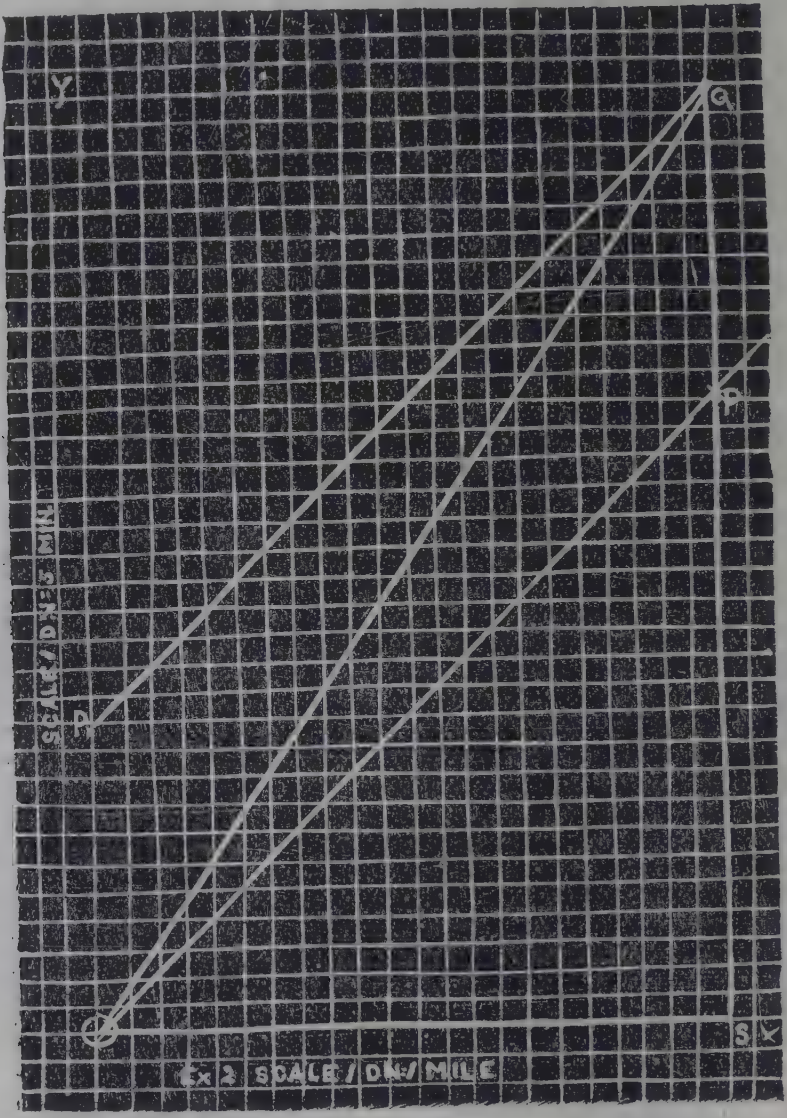
QF is the graph of a cyclist riding at 24 miles an hour starting from Q. Through S draw a parallel to FQ to meet QR at C. The cyclist should have left Q, 20 min. later than the other cyclist who had left P.

QR, QN, PD give the times of reaching the destination.

Ex. 2. I find that I shall be 45 min. late in reaching my destination if I bike at 8 miles an hour and that I shall be 10 min. too soon if I bike at 12 miles an hour. Find from a graph (1) the distance of my destination from the starting place, (2) the rate at which I should bike to be there in time.

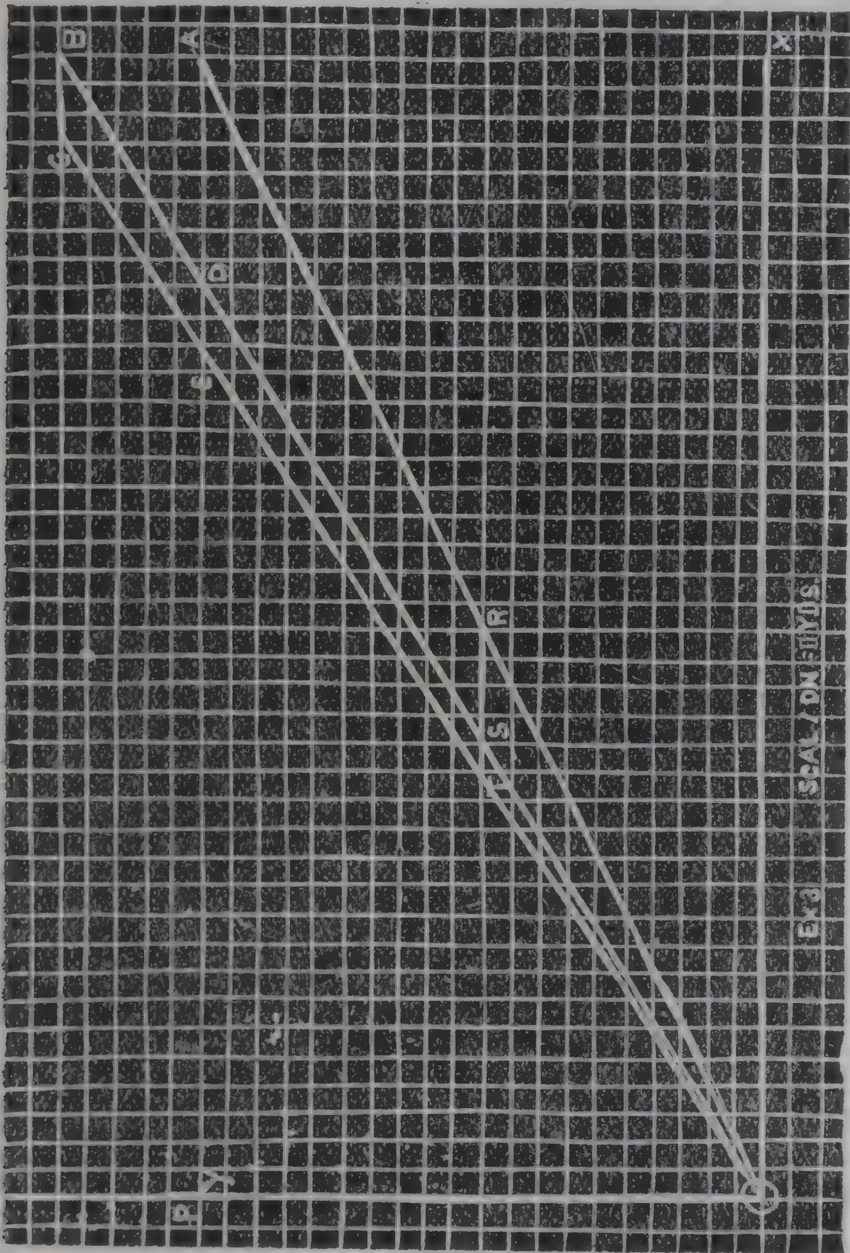
OP is the graph of 12 miles an hour.
OQ ,, ,, 8 miles an hour.

The distance measured along an ordinate between OP and OQ must be 11 units representing the difference in time i.e. 55 minutes, as at the quicker rate the man will be at the destination 55 min. earlier than at the slower rate.



Through R where OR represents 55 min. draw RQ a parallel to OP to meet OQ in Q. Then ORPQ is a parallelogram, and QP = 11 units. Therefore QP produced to cut OX at S gives OS, distance of destination.

Ex. 3. In a quarter mile race A can beat B by 88 yds. and C by 110 yds. By how many yds. can B beat C in the same race? In a furlong race, supposing they run at the same rate as before find how many yds. start, A can give each of B and C, to run a dead heat.



In time represented by OP, A is at the destination, B is 88 yds. behind and C is 110 yds. behind. Therefore OA, OD, OE are their graphs respectively.

If B continues, when B is at winning post C is 27½ yds. behind, represented in the graph by BC.

Similarly in a furlong race RS and RT give the starts A can give B and C respectively to run a dead heat.

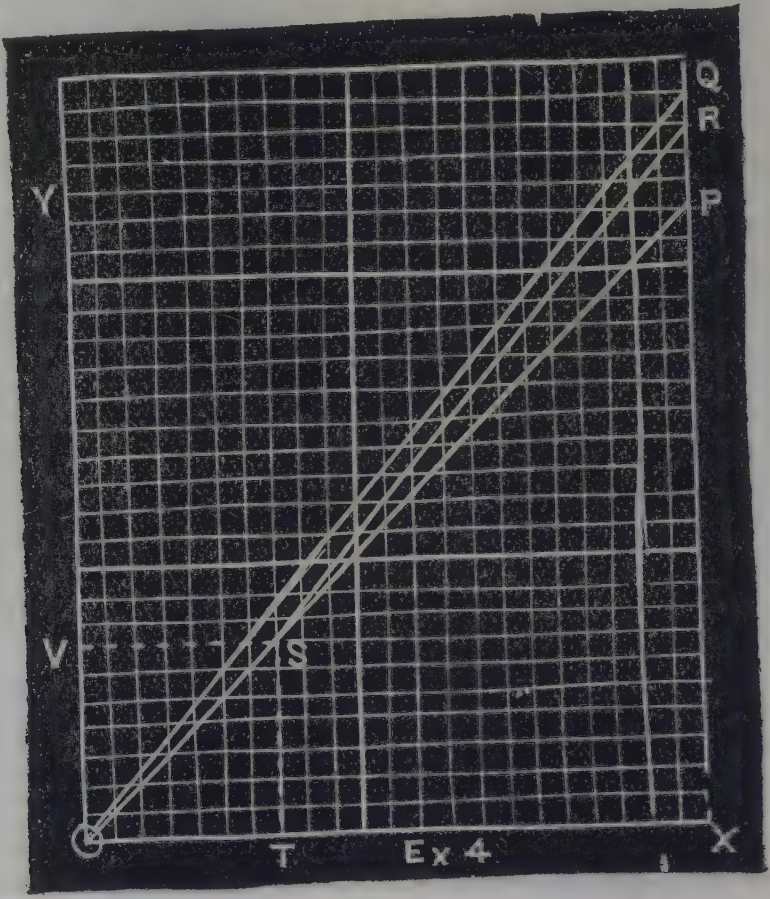
Ex. 4. A cyclist started on a twenty-two mile journey. He covered a certain distance at 12 miles an hour and then being tired reduced his speed to 10 miles an hour, and thus completed the journey in 2 hrs. 5 min. Find when and where he changed his speed.

Solution 1.—Graphical,

OP is the graph of 12 miles an hour.

OQ " " 10 "

XR is the actual time taken. Through R draw a parallel to OQ to meet PO in S: OT gives the place where he changed his speed, OV the time when he changed his speed.



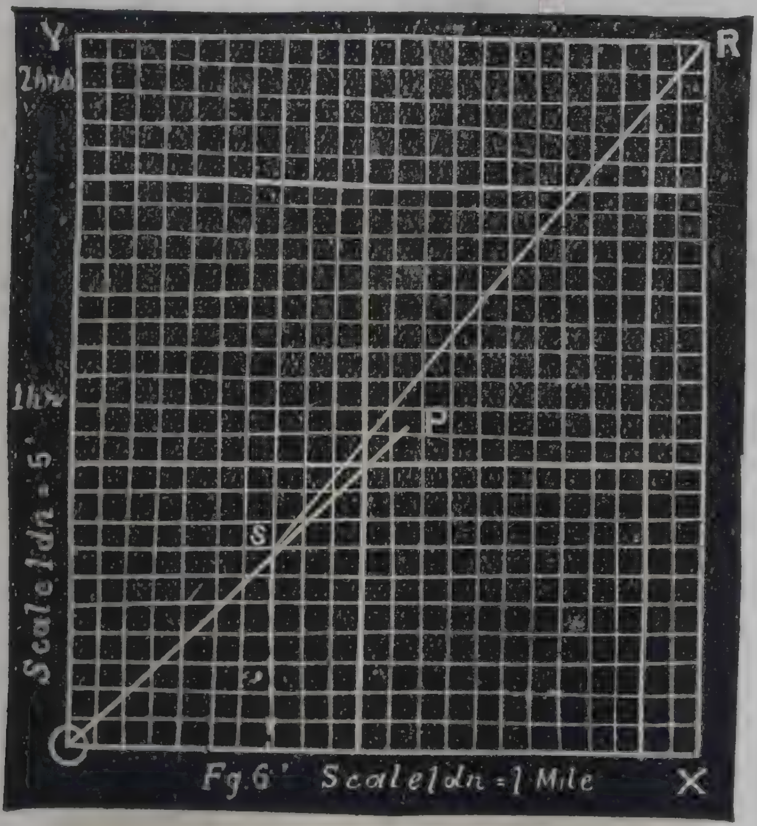
Ex 4

From the place where he starts, the graph is in the direction OS; and from the place where he reduces his speed, the graph is in the direction SR. To find S, we fix the direction by OQ and the time being given, through R we draw a parallel to the direction.

Sol. 2. Graphical.

In 2 hrs. 5 min. the cyclist has reached his destination.

∴ As seen from O, the direction of the graph is OP, and as seen from R the direction of the graph is RS.



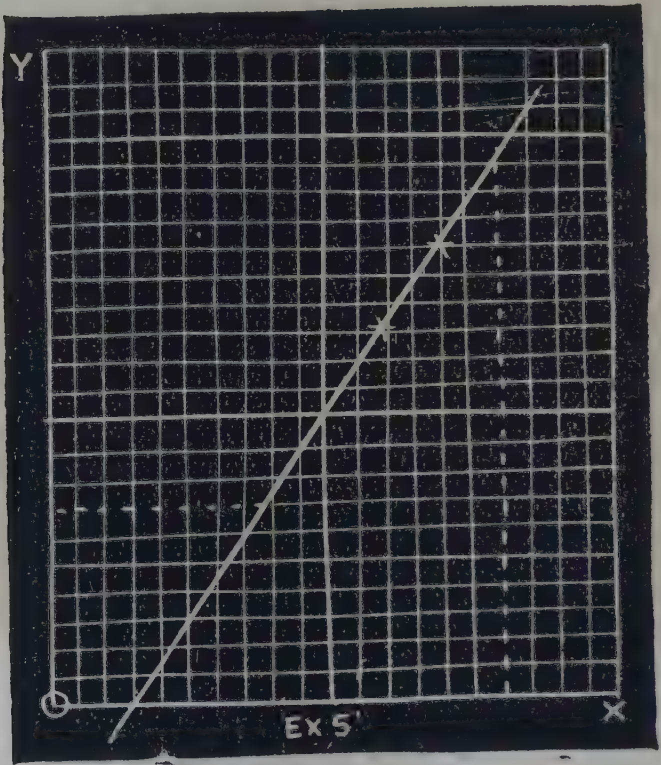
RS is drawn with R as origin, RY as X-axis and RX as Y-axis and it is the graph of 10 miles an hr.

The two graphs meet at S, the place of change of speed.

The above exercise may be profitably preceded by the following exercise, viz., A cyclist rides a distance of 7 miles at 12 miles an hour and then reduces his speed to 10 miles an hour. When will he reach the destination 22 miles distant ?

Ex. 5. The marks obtained by students in an examination had to be revised. Boys who had secured 12, 14 originally got in the revised list 13, 16 respectively. Find graphically the original marks secured by a boy who had 7 in the revised list and the revised marks of a boy who had secured 16.

X-axis : Original marks } 1 dn. in each co-ordin-
Y-axis : Revised marks } ate represents 1 mark.



Plot points (12, 14) & (13, 16). Join them. Read others from the graph.

Ex. 6. At the time of the German reverse (1914), a body of Germans began their retreat at the rate of 24 Kilometres a day from a place 36 Kilometres distant from the Allies' Battle Front. The allies began their pursuit a day later, at the rate of 32 Kilometres per day. After three days' marching they were delayed in their pursuit for a day to construct a bridge across a river; they then continued the march for three days more, and were again delayed for half a day to repair another bridge blown up by the enemy. After how many days from the beginning of the retreat were the Germans overtaken?

Draw a graph to illustrate the movements of the armies.

Find from the graph, how far the armies were apart seven days after the beginning of the retreat. (S. S. L. C. 1921.)

Solution.

O is the camp of the Allies.

G " " Germans.

The Germans go away from G.

GT is the graph of the retreat of the Germans.

OB denotes 1 day. The Allies began pursuit a day later.

Ba is the graph of the pursuit.

After three days' pursuit there is delay of a day,

(i.e., the day advances and no progress is made in the journey, hence the graph goes vertically up two divisions; (ab) shows the delay of a day. The x-co-ordinate of a and the x-co-ordinate of b are the same showing that no advance in distance is made.)

bc is the progress.

cd delay of ½ day.

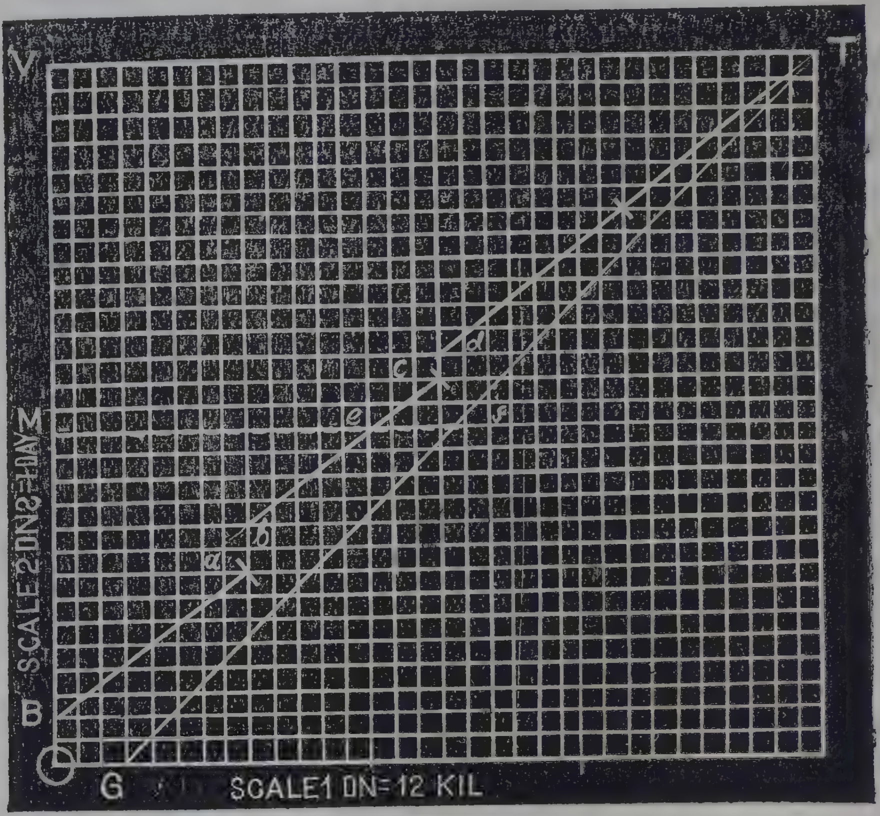
dT. Graph of journey continued.

The two graphs meet at T, shewing time and place of overtaking.

$TV = 32$ *divisions*; The Germans were overtaken at a distance of 384 Km. from the Allies' camp.

Again $VO = 29$ *divisions* i.e., the Germans were overtaken 14½ days after the beginning of the retreat.

$OM = 14$ divisions. Draw *Mef.* (*ef*) gives distance between armies on the 7th day after the retreat $= 4$ *divisions* $= 48$ *Km.*



EXAMPLES XVII.

N.B.—A few questions on statistical graphs only need be done in Form IV.

Boys may be advised to use squared-paper of foolscap size.

The examples in this exercise must, wherever possible, be solved both theoretically and graphically.

1. P and Q are two places 27 miles apart. A and B leave P and Q respectively at the same time and travel towards each other to Q and P, A at 9 miles and B at 6 miles per hour. Find when and where they will meet.

2. P and Q are two places 18 miles apart. A leaves P at 8 a.m. and travels to Q at 10 miles an hour. He takes rest at the 12th mile for 10 min. and then continues his journey at the same rate. B leaves Q at 7-10 a.m. and proceeds to P walking at 4½ miles an hour. Where will B meet A and when will each reach his destination?

3. One day I started on my bicycle to a place 12 miles distant. I rode at 8 miles an hr. and the machine gave some trouble when I had proceeded 6 miles, and I was therefore delayed there 10 min. Starting again I had to bike only at three-fourths of my original rate. How many minutes later did I reach the place?

4. A motor-bus left a station at 9 a.m. and ran at 15 miles an hour. 10 min. later, another bus left the same station and ran at 20 miles an hour. When and where will the latter overtake the former?

5. A cyclist started from a town and rode at 9 miles an hour. 20 min. later a motorist left the same town, and travelling along the same road as the cyclist, overtook him in twenty minutes. Find the speed of the motorist.

6. A villager was directed by his master to go to a town. The former went on foot at 4 miles an hr.; 1 hr. 30 min. later the master wanted to give him some information and forthwith started on his bike and overtook him at the 9th mile. What was the speed of the master?

7. Two persons A and B set out together on a journey. They walked at the rate of 3 miles an hour, and after they had proceeded for ¾ of a mile, B returned walking at the same rate to his place of starting. Here he was detained ¾ of an hour. Setting out again he overtook A, who had been walking all the time, at the end of 2½ hours from the second time of starting. At what rate did he walk?

8. A man walks from a village to a place at 3 miles an hour. 1 hr. 20 min. later another leaves the same village and walks to the same place at 4 miles an hour and both reach the destination at the same time. How far is the place from the village?

9. A train starts from A and runs towards C which is 100 miles distant, at the rate of 30 miles an hour ; at the same time the mail cart starts for C from B half-way between A and C, and is overtaken by the train at the 25th mile from C. At what rate does the mail cart go ?

10. A man rode a bicycle from A to B, a distance of 54 miles, at an average rate of 8 miles an hour. Another man started from A on horseback half an hour after the cyclist and arrived at B, 15 min. before. Find the rate of the rider.

11. P starts from C and travels towards D at the rate of 6 miles per hour ; two hours afterwards Q starts from C and going 10 miles per hr. reaches D, 4 hours before P. Find the distance from C to D. Also find where and when Q will overtake P.

12. Two men A and B start from P, at 4 and 5 A.M. respectively to walk to Q, a distance of 50 miles ; B passes A at the 20th milestone and reaches Q at 5 P.M. When will A arrive there ? What is the rate of travelling of each ?

13. An express train starts from a station A at one o'clock for a station B at 30 miles an hour, and in 15 minutes is followed by an ordinary train at 20 miles an hour. A train from B to A at 25 miles an hour, after travelling one hour, meets the express and in 20 minutes more meets the ordinary train. At what time did the train leave B ?

14. A cyclist started from P on a 36-mile journey to Q. He rode the first hour at 12 miles an hour and reduced his speed by 2 miles an hour at the end of every hour of cycling. A motorist who left P 2½ hrs. later reached Q just when the cyclist reached it. At what rate did the motorist travel ?

15. Twenty plays were staged by a Dramatic Company and the cost of a season ticket was Rs. 12. The option was also given to purchase tickets at 12 annas each per day. I however purchased a season ticket and having been unable to attend a few performances calculated that I had literally paid at the 12 annas rate for the performances I attended. For how many performances was I absent ?

16. (a) A man has to go a distance of 30 miles. If he rides on his bicycle at 8 miles an hour, in how many hours will he reach the destination?

(b) If he bikes at 12 miles an hour, how many minutes would he save?

(c) If he goes ⅔ of the distance at the former rate and the remaining distance at the higher rate, how many minutes would he save?

17. A motorist drives for a time at 15 miles an hour and then reduces his speed to 12 miles an hour and reaches his destination 50 miles distant in 3 hrs. 39 min. Find when and where he changed his speed.

18. A motorist's rate uphill along a slope is 8 miles an hour and downhill 12 miles an hour. If he goes up and returns to the starting place in 2 hrs. 55 min., find the height of the hill.

19. I bought 42 yards of cloth some at 8 as. a yard and the rest at 6 as. a yard, and the total cost came up to Rs. 19-2 as. Find how many yds. of each variety I bought.

20. A labourer was engaged for 30 days on condition that he should receive Re. 1-2 as. for every day he worked and pay a fine of 6 as. for every day he was absent. He received at the end of the stipulated time Rs. 8-4 as. Find for how many days he worked.

21. A cyclist has to ride 68 miles. He rides for a time at 10 miles an hour and then reduces his speed to 6 miles an hour and covers the distance in 8 hrs. 16 min. Find when and where he reduced his speed.

22. A sum of Rs. 420 is lent partly at 4 p. c. and partly at 7 p. c. and the total annual interest is Rs. 24. Find the sum lent at 7 p. c.

23. (a) In what proportion must sugar at 14 as. a viss be mixed with sugar at Re. 1-8-0 a viss, that the mixture may be sold at Re. 1-4-0 a viss?

(b) If 4½ viss of the first sort be taken, find how many of the second sort must be taken and also find the total quantity of the mixture.

24. A permanent officer draws a starting salary of Rs. 120. He is given an increment of Rs. 5 per annum. When will he draw a salary of Rs. 150? In how many years from the date of his entering service will he draw Rs. 250?

25. A teacher is appointed on a salary of Rs. 90 per month and he is given an annual increment of a few rupees. If at the beginning of a year his salary is Rs. 115 and he has worked for 5 years, find the annual increment.

26. A teacher who gets an annual increment on his pay draws Rs. 104 at the beginning of the 7th year of his service and he drew Rs. 96 at the beginning of the 5th year of his service. Find his initial salary and annual increment.

27. The marks obtained by students in a certain examination had to be raised. In so doing the marks of boys who secured originally 21 marks and 42 marks were raised to 33 and 61 respectively. Find the revised marks of boys who secured 33 and 60 in the original list and the original marks of boys whose revised marks were 53 and 69.

28, 29, 30, 31. See Qs. 109, 110, 111, 112 on page 48.

32. In a furlong race A beats B by 40 yards, and C by 60 yards. By how many yds. can B beat C in the same race?

33. In a half-mile race A gives B a start of 180 yds. and both run a dead heat. If they both run from scratch and run a quarter-mile race, by how many yds. can A beat B, supposing their rates of running to be the same and uniform throughout?

34. When will the hands of a clock be together, at rt. angles, and in a st. line, between 4 and 5 o'clock?

35. Passengers by a motor-bus service are allowed a certain weight of luggage free and the rest is charged. If a passenger carrying 28 lbs. is charged 8 as. and another carrying 32 lbs. is charged 10 as., find the weight of luggage allowed free and the rate of charge per lb.

36. For late attendance beyond 3 days a student has to pay a fine of 6 ps. for each late attendance. He has also to pay the monthly school-fees of Rs. 1-12-0. If he paid Re. 1-15 as. for a certain month, for how many days was he late?

37. One boy paid Rs. 2 for school-fees and fine for five days' late attendance, and another pupil in the same class paid Rs. 2-3 as. for the fees and fine for late attendance on eight days. Find the amount of school-fees and the rate of fine for each late attendance.

38. An army of 5000 men costs a country £ 800,000 per annum to maintain it; an army of 10.000 men costs £ 1,300,000 per annum to maintain it; what is the annual cost of an army of 8000 men? Take the simplest law which is consistent with the figures given. Find the cost both theoretically and graphically.

39. (a) A boy walked to his school at the rate of 55 yds. per minute and was 9 min. late. Next day he walked at the rate of 66 yds. per min. and was 5′ late. Find (i) the distance of the school from his house, (ii) what his rate must be to reach the school in time.

(b) I had to go to a place. I found that if I were to travel by a bus just then starting and running at an average speed of 15 miles per hour, I would be late by 12 min., and if I were to engage a taxi which could be driven at 20 miles an hour, I would be 21 min. too early. Find the distance of my destination from where I started.

40. A gramaphone with 20 plates is sold for Rs. 250, while the same gramaphone with 28 such plates is offered for Rs. 276.

i. Find the price of the gramaphone alone from arithmetical considerations.

ii. If x denote the price of the machine in rupees and y the price of each plate, frame two equations in x and y and draw their graphs. What do the co-ordinates of the intersection point give?

iii. Can you draw a single straight line graph to read off the price of the gramaphone with a given number of records? Find from the graph the price of a gramaphone with 24 records.

41. Two thermometers A and B are both graduated, but on *A*, the freezing point is marked 32° and the boiling point 212°, while on B neither of these points is indicated. The following pairs of corresponding readings of the two thermometers are observed:—

$\left.\begin{array}{lll} \mathrm{A} & 59^\circ & 77^\circ \\ \mathrm{B} & 35^\circ & 45^\circ \end{array}\right\}$ Show by means of a graph the readings of B corresponding to those of A between 25° and 212°, and from your graph ascertain,

(1) the freezing point and the boiling point on B;

(2) the reading of each thermometer corresponding to zero on the other;

(3) the temperature for which the readings of A and B are the same.

42. The marks x secured by each student are revised to y such that $y = ax - b$; a and b being numbers. A boy who obtained 25 marks has thus his marks raised to 37, while another's which was 32 is raised to 51. Find the values of the constants a and b. Draw the graph of the equation and from the graph find (i) the boy who is not at all benefited by this rule, (ii) the boys to whom the rule is prejudicial, and (iii) the original marks of a boy who by this enhancement has secured 43 marks.

43. In a certain school teachers who enter service are started with a salary of Rs. 90 per month and are given an increment of Rs. 3 per annum. In another school the initial salary is Rs. 80 per month, but the annual increment is Rs. 5.

Find from purely graphical considerations when the salaries of two teachers (one employed in each school) will be the same, supposing they have started service together.

44. To become a member of the Tennis Section of a certain club, a person must pay Re. 1-8 as. per month for membership and 1½ annas for every tennis set he plays, or pay a consolidated sum of Rs. 4-8 as. per month and play as many sets as he can, though he is not allowed to play two consecutive sets when players are waiting, so that he cannot play more than two sets a day. He can go to the club only on four days in the week. (a) Shew graphically that he may adopt either method of payment if he goes to the club only for tennis. (b) How much more will a person who plays 40 sets in the month have to pay if he adopts the former method of payment ?

45. (a) When there were 18 students in a hostel the total expenses of the hostel in a month came to Rs. 243, and when there were 30 students the expenses came up to Rs. 375. If x denote the constant expenditure that should be incurred whatever the strength of the hostel be, find that constant expenditure and the total cost if there were 45 students in the hostel.

(b) Illustrate graphically.

46. (a) A certain printer charges Rs. 2 for the first thousand copies of a notice and Re. 1-4 as. for every additional thousand, while another charges Re. 1-8 as. for every thousand, no matter how many copies are printed. I found that for the copies I had to print, it cost me the same if I gave the work to the one or the other. How many thousands had I to print ?

(b) Draw two graphs with the same axes and scale, to read off the costs. Find from the graph when the costs will be the same.

47. A village schoolmaster's income is derived from a fixed monthly allowance of Rs. 15 subscribed by the village community and school-fees at 12 as. per pupil. Frame an equation connecting the total income and the number of pupils and draw its graph.

A schoolmaster in an adjoining village gets his income only from school-fees and charges each student Re. 1-4 as. The strength of each school is the same and emoluments are also the same. Find graphically the number of students in each school.

48. Passengers by a bus are allowed 10 lbs. of luggage free and the excess is charged at 8 ps. per lb. Another allows no luggage free and charges 6 ps. per lb. Draw two graphs with one and the same axis and find from the graph,

1. when the two charges come to the same amount ;
2. when the charges differ by an anna.

49. Passengers on the railway are allowed a certain weight of luggage free, and the rest is paid for at 5 s. 4 d. per cwt. If a traveller with a luggage weighing 256 ℔. had to pay 8 s. for his luggage, find the weight of luggage allowed free, and draw a graph connecting the amount of luggage carried and the charges paid.

50. A commercial traveller's income consists of a fixed monthly pay of Rs. 40 and a commission of 5 per cent. on the sales he effects. Frame an equation connecting his total income 'y' and the sales in hundreds of rupees 'x' and draw its graph.

(a) Find the amount of sales effected if his income in a month be Rs. 75.

(b) Find the total income if he had secured sales for Rs. 480.

51. The weight of a diamond in carats and the corresponding price are given below. Draw a graph from which you can read off the price of a diamond of given weight.

Wt. in carats	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
Price in Rs.	25	45	75	100	450	550	800

(1) What will be the approximate price of a diamond weighing $\frac{3}{8}$ carat?

(2) If a diamond of the same quality is priced Rs. 500, what should it weigh?

52. The number of degrees y in an angle of a regular polygon of x sides is given by the formula

$$y = \frac{180\,(x-2)}{x}$$

From this formula find the number of degrees in each angle of a regular polygon of 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, and 20 sides. Tabulate the number of sides and the magnitude of the corresponding angle, plot them on squared paper and draw a *smooth* curve to pass through the points. From the graph read off the no. of degrees in an angle of a regular polygon of 7 sides and the number of sides of a regular polygon each of whose angles is $157\frac{1}{2}$ degrees.

53. A train starting at 12 o'clock on a 25-mile journey passes the points given in the annexed table at the stated times :

Time ..	12	h. m. 12-4	h. m. 12-8	h. m. 12-13	Arrives h. 12-20 Stops 3′.	h. m. 12-27	h. m. 12-31	h. m. 12-38 Stop.
Distance in miles..	0	2	5	10	15	17	20	25

Plotting times horizontally and distances vertically, draw the curve shewing the speed of the train throughout.

54. By choosing convenient scale, determine graphically which of the following trains is the fastest ?

(1) 36 miles in 39 minutes.
(2) 25½ ,, 28 ,,
(3) 48½ ,, 55 ,,
(4) 44½ ,, 36 ,,

55. The time of starting of each train, the distance it runs and the time of its arrival at its destination are given below :—

Distance from Madras Central	Stations.	No. 1	No. 2	No. 3	No. 4
302 miles	Podanur	2-40	7-8	9-15	17-50
328 miles	Mettupalayam	4-55	8-10	10-25	20-5

Find graphically which of these four trains is the fastest.

56. The places and times of departure, the destinations and times of arrivals and the distances between a few stations are given below. Find which train is the fastest and arrange them in descending order of rapidity.

No. of Train.	Place of Departure.	Time of D.	Destination.	Distance between.	Time of Arrival.
No. 29 P. ..	Madras Eg. ..	13- 0	Villupuram ..	102	21- 5
No. 1 C.B.M.	,, ..	20-15	Trichy Jn. ..	252	8- 4
No. 3 R.E...	,, ..	21-20	Mayavaram J.	177	10-50
No. 23 P. ..	Madura Jn. ..	1-10	Maniyachi Jn.	79	7- 0

[Time is calculated at 24 hours from midnight to midnight.]

57. (*a*) A distance of 48 miles is travelled partly by coach and partly by train in 4 hrs. Rate of coach being 4 miles an hour and that of the train 15 miles an hour, find from a graph as accurately as you can, when and where I left the coach and took to the train?

(*b*) Frame equations in x and y and solve the system and check your result.

58. On squared paper draw carefully two lines at rt. angles and another line through their intersection making an angle of 60° with one of the perpendicular lines. Prove that this line can be treated as a graph for the multiplication or division of a quantity by $\sqrt{3}$ without the necessity of calculation. From the graph find $\sqrt{3}$ times 8 and $\sqrt{3} \times 4·5$.

59. Given that 10 seers = 20·6 lb., draw a graph to read off the equivalent in lb. of Indian weights up to 100 seers. From the graph find as accurately as you can the equivalent of (1) 57 seers in lb., (2) 192 lb. in seers.

60. The co-ordinates of the 4 points P, Q, R, S, are $(0, -5)$, $(12, 0)$, $(4, 6)$, and $(-8, -3)$. The points are joined in the order given. Calculate the lengths of the sides and find the areas of the portions of the figure lying in the first and fourth quadrants.

61. The following table shows the average number of years, persons live after the ages indicated; i.e., those who reach the age of 40 may be expected to live or rather live on an average 27·6 years more. Make a graph showing the 'expectancy of life' at different ages.

Find from the graph, how many years more a person, may be expected to live, who is now 38 years of age.

Age in years.	0	1	2	3	4	5	6	7	8	9	10	20	30
Expectancy of life in years.	38·7	44·7	47·6	49·8	50·8	51·25	51·17	50·8	50·2	49·6	48·3	41·5	34·2

Age in years.	40	50	60	70	80	90	100
Expectancy of life in years.	27·6	21·1	14·3	9·2	5·2	3·3	2·3

62. A river is 3 furlongs wide at a point and the following soundings of depths have been mapped across the river from the point.

DISTANCE FROM BANK.	DEPTH.
10 yds.	1 ft. 3 in.
40 yds.	2 ft. 4 in.
100 yds.	4 ft. 3 in.
150 yds.	4 ft. 6 in.
200 yds.	4 ft. 7 in.
250 yds.	5 ft. 1 in.
300 yds.	6 ft.
350 yds.	6 ft. 2 in.
400 yds.	5 ft. 9 in.
500 yds.	4 ft. 11 in.
600 yds.	3 ft. 8 in.
640 yds.	2 ft. 3 in.
650 yds.	1 ft. 2 in.

(1) How far can a man 5 ft. 4 in. high venture from either shore?

(2) If the level of water falls 10 in., how far can the same man venture from either shore?

(3) If the water rises by one foot, how far can he then venture?

63. The discharge in pounds per minute through an orifice in the bottom of a tank and the corresponding head of water in the tank are as follows:—

Head in ft.	0·5	1·0	1·5	2	2·5	3	3·5	4	4·5	5
Discharge in lb. per minute.	12·1	18·4	22·5	25·5	28·0	30·5	33·0	35·0	36·2	39·0

Plot on squared paper and find from the curve, the discharge for a head of 4·25 feet and the probable discharge for a head of 6 feet.

64. The length of a degree of longitude varies with latitudes. The subjoined table shews the length of a degree of longitude corresponding to the degree of latitude noted above.

Latitude	0	10	20	30	40	50	60	70	80	90
Length in miles of a degree of longitude ...	69	68	65	60	53	44½	34½	24	12	0

Exhibit these results graphically and estimate approximately the length of a degree of longitude at latitudes (1) 46°, (2) 75°.

65. We know that the day-time is not always of the same length. The following figures give the times of the rising and the setting of the sun on certain days of the year in the vicinity of London.

DATE.		SUNRISE.		SUNSET.	
		H.	M.	H.	M.
Jan.	1	8	8	3	58
March	1	6	49	5	36
March	21	6	5	6	11
May	1	4	35	7	19
June	1	3	50	8	3
June	21	3	44	8	19
Aug.	1	4	23	7	49
Sep.	1	5	13	6	48
Sep.	23	5	44	6	2
Nov.	1	6	54	4	34
Dec.	1	7	45	3	53
Dec.	21	8	5	3	51

Find the duration of day-time for the days noted. Plot them on squared paper and draw a graph. From the graph find the length of the day on the first day of the months omitted in the list given.

66. The maximum and minimum temperature of each month in the year 1923 as recorded at Greenwich are given below:—

	Jan.	Feb.	Mar.	Apl.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
MAX.	46·8	48·6	51·6	56·0	61·5	65·1	78·4	74·1	67·5	55·0	47·4	46·9
MIN.	35·4	38·0	38·1	39.3	43·2	47·4	56·3	53·0	47·4	41·0	36·4	38·8

Find the mean temperature for each month and plot the figures on squared paper and draw a smooth curve passing through the points.

67. A rifle, sighted to 1000 yards, rests on a support 5 feet from the ground and is fired. The height of the bullet above the support is given by the following table.

Distance in yards from firing point—	100	200	300	400	500	600	700	800	900	1000
Vertical height above support in feet—	7·3	11·2	15·0	18·5	21·0	23·3	25·0	22·5	16·5	0.

On squared paper give a representation (a smooth curve) of the path of the bullet, showing also the ground level and height of support, and find in what positions a butt of height 20 feet, would stop the bullet.

68. The temperature of a patient taken every two hours on a day showed :—

6 a.m.	103°
8 ,,	103·5°
10 ,,	103·6°
12 Noon	103·6°
1 p.m.	103·4°
4 ,,	103·2°
6 ,,	103°
8 ,,	103·2°
10 ,,	103·7°
12 Midnight	103·5°
2 a.m.	103·4°
4 ,,	103·3°
6 ,,	103·4°

Indicate these changes by means of a smoothly drawn graph and find when (1) the temperature was highest, (2) the temperature was lowest.

69. The price of printing paper steadily increased during the last twelve months in a certain year. The price of each pound as quoted in the various months are given below :—

May—July	7 as. per pound
Aug.—Oct.	8½ as. ,,
Nov.—Jan.	10 as. ,,
Feb.—April	11 as. ,,

Exhibit by means of a graph the rate of increase in the price of a ream of paper which weighs 32 pounds.

Supposing that the price increased every day gradually, estimate from your graph what ought to have been the price on the 15th of September and when the price would have been 10 as. 6 ps. per pound.

70. The following telegraph rates (inland) have been in force at different times.

(i) 8 as. for the first twelve words and ½ anna for each extra word.
(ii) 6 as. do. do. 1 anna ,,
(iii) 4 as. first eight do. 1 anna ,,

(a) How many words should be telegraphed that rates (i) and (ii) may cost him the same ?

(b) Draw the three rates and find how much will be charged under each rate for sending 20 words by wire.

71. The annual premiums of Rs. 1000 payable at the end of 20 years from the time of insurance or on death if earlier, are given in the following table. The amount to be paid as premium depends on the age of the person who insures his life i.e. a person aged 20, should pay annually Rs. 51-7-0 to entitle him to Rs. 1000, twenty years hence or on death if it happens earlier.

Age next Birthday.	Amount. Rs.	A.
20	51	7
21	51	10
22	51	15
23	52	3
24	52	7
25	52	12
27	53	2
29	53	9
31	54	4
33	55	2
35	56	0
37	56	14
40	58	5

Draw a smooth curve for the graph and read off from the graph the premiums for persons aged 30, 34 and 38 respectively.

72. A man sells kettles. He has made them of only three sizes as yet and has fixed the following prices :—

12 pint.	Kettle.	Each.	Rs. 4—4 as.
6 ,,	,,	,,	,, 3—2 as.
2 ,,	,,	,,	,, 1—6 as.

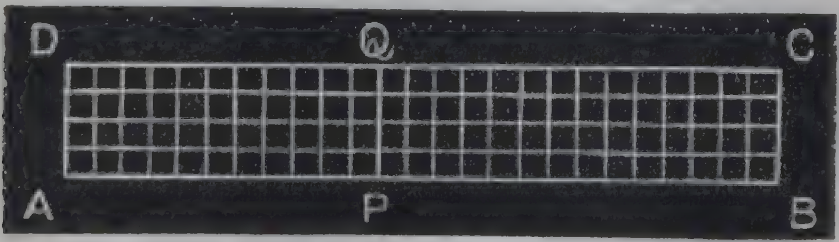
He knows that other sizes will be wanted and he wants to publish at once a price list for many sizes. Find graphically the probably correct list prices of his 4 and 8 pint kettles.

SECTION VI.

Percentages, Profit and Loss.

1. Consider the fraction $\frac{11}{25}$.

(*a*) The students might know that out of a whole, which is divided into 25 equal parts, 11 parts are taken.



ABCD in the figure representing the whole, APQD represents $\frac{11}{25}$.

(*b*) The students know also that the fraction does not change in value, if both the numerator and the denominator be multiplied or divided by the same quantity.

If the whole figure ABCD be divided into 100 equal parts, APQD represents 44 such parts i.e., APQD $=\frac{44}{100}$ of ABCD.

But we have stated that APQD $=\frac{11}{25}$ of ABCD.

$\therefore \frac{11}{25}=\frac{44}{100}$.

(*c*) A boy has secured 11 marks out of a maximum of 25. If the maximum be raised to 50, his marks must be put down as 22 and if the maximum be raised to 100 his marks must be put down as 44. Instead of saying that the boy has secured 11 out of 25, we may say he has secured 44 out of 100. 'Out of 100' is otherwise stated as 'per cent.' The boy has secured we say 44 per cent.

(Note the words—century, centigrade, centenary, the coin 'cent.') Hence 44 per cent. or as generally written down 44%, means 44 out of 100 i.e., 44 parts out of a hundred equal parts or $\frac{44}{100}$ or $\frac{22}{50}$ or $\frac{11}{25}$.

(*d*) Every percentage is a fraction. If the denominator of the fraction be made 100, the numerator expresses the percentage.

$$\tfrac{3}{4} = \frac{3\times 25}{4\times 25} = \frac{75}{100} \qquad \text{i.e. 75 per cent.}$$

$$\tfrac{11}{20} = \frac{11\times 5}{20\times 5} = \frac{55}{100} \qquad \text{i.e. } 55\ \%$$

$$\tfrac{14}{35} = \frac{14\times \frac{20}{7}}{35\times \frac{20}{7}} = \frac{40}{100} \qquad \text{i.e. } 40\ \%$$

$$\tfrac{2}{3} = \frac{3\times 33\frac{1}{3}}{3\times 33\frac{1}{3}} = \frac{66\frac{2}{3}}{100} \qquad \text{i.e. } 66\tfrac{2}{3}\ \%$$

2. The same answer paper is valued by 4 examiners. The maximum set by each examiner and the marks awarded by each are given below. Express each as a percentage. Find also, which examiner has given the highest marks and which the lowest. If the marks have to be set down to a uniform maximum of 80, how will, the marks awarded by each, stand ?

Name of examiner.	Maximum.	Marks Awarded.
A	20	8
B	40	18
C	50	19
D	75	33

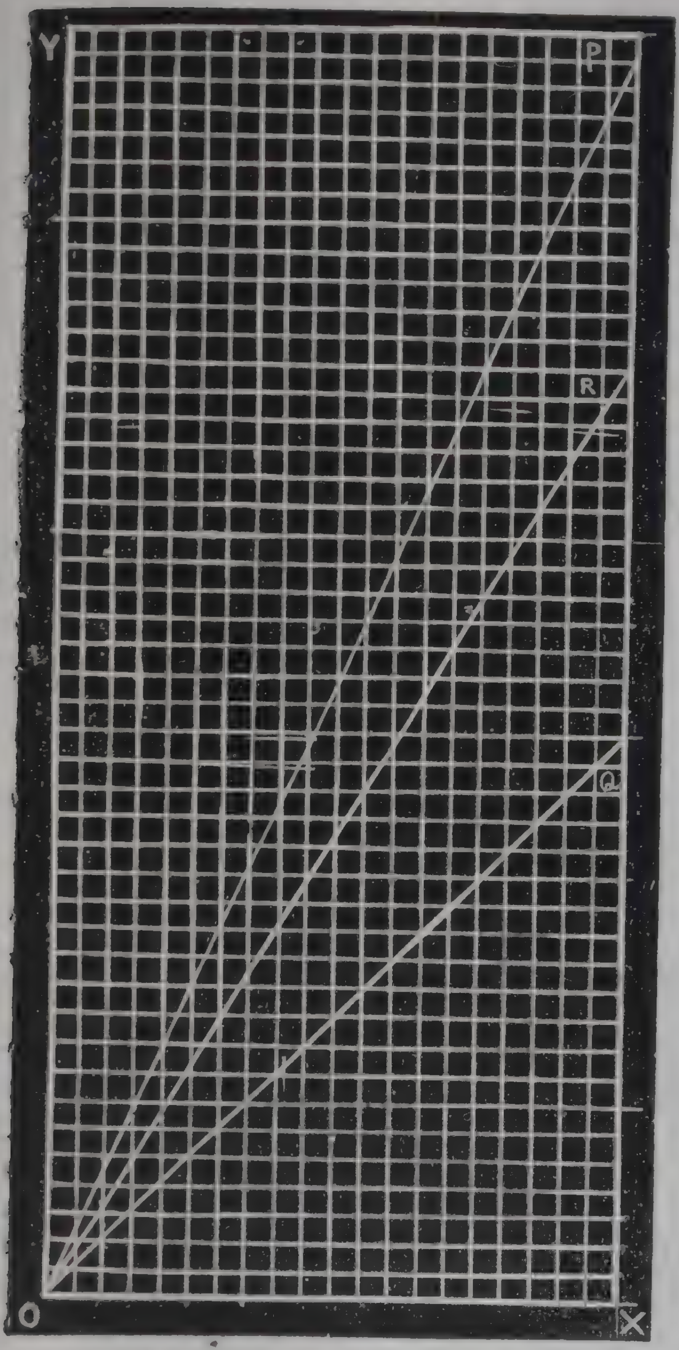
Examiner A has awarded $\frac{8}{20}$ of the Maximum
i.e. $\frac{40}{100}$ of the Max. i.e. 40%

B ,, $\frac{18}{40}$ of the Maximum
i.e. $\frac{45}{100}$ of the Max. i.e. 45%

C ,, $\frac{19}{50}$ of the Maximum
i.e. $\frac{38}{100}$ of the Max. i.e. 38%

D ,, $\frac{33}{75}$ of the Maximum
i.e. $\frac{44}{100}$ of the Max. i.e. 44%

Examiner B has awarded the highest 45 % and C the lowest 38%. If the marks have to be set down to a maximum of 80, the boy must get 32, 36, 30·4, 35·2 respectively.



3. On three different occasions a boy secured 11, 7 and 33 marks in Ele. math., the maxima being 25,35 and 100 respectively. Find graphically the percentage of marks he secured on each occasion. In the first paper the boy secured $\frac{11}{25}$ of the maximum, i.e., if y be the mks. he secured and, the maximum be x, $y = \frac{11}{25}$. Similarly the other relations are $y = \frac{7}{35}x$ and $y = \frac{33}{100}x$. We draw the graphs of these three equations.

Generally however we plot the points direct and draw the graphs.

In the figure, the maximum is represented along x-axis $\frac{1}{10}''$ or 1 dn. = 5 mks. and the mks. secured along y-axis $\frac{1}{10}''$ or 1 dn. = 1 mk. OP, OQ, OR are the respective graphs. Taking a length OX to represent 100, XP, XQ and XR give corresponding percentage 44, 20 and 33.

N.B.—The teacher will also point out that the fractions have here been compared graphically and that this is also the graphical method of reducing fractions to any common denominator.

4. Goods have to be sold at a profit of 2 as., on each rupee of cost price. (a) Find the selling prices of articles purchased for Re. 1-2 as.; Re. 1-4 as.; Re. 0-14 as.; Rs. 2-8.

(b) What is the percentage gain?

Solution.—

(a) Profit on 1 Re. is 2 as.

∴ on 2 as. is 3 ps.

∴ S P. of an article whose C. P is Re. 1-2 as. is (Re. 1-2 as.) + (2 as. 3 ps.)
= Re. 1-4-3 ps.

∴ „ Re. 1-4 as. is (Re. 1-4 as.) + (2 as. 6 ps.)
= Re. 1-6-6 ps.

∴ „ Rs. 2-8 as. is (Rs. 2-8 as.) + (5 as.)
= Rs. 2-13 as.

(b) The percentage gain is 12½.

5. In addition to the cost price and the selling price, there are yet two other prices called, 'The *marked price*' 'The *cash price*.'

Tickets are attached to the articles in the shop and the price of that particular article is noted in the ticket; or, a number is attached to the article and the price of that article is quoted in the catalogue against that number. The *marked price* is sometimes also called, The *catalogue price*. This price *generally* includes, the cost price, the profit and the interest on these two together, for a certain fixed period, generally, six months. Those who pay cash are allowed to deduct a certain per cent. from the catalogue price, for these have not got to pay the interest, as they buy for cash. (In the catalogues of certain firms students might have noted at the foot of each page some such statement, '10% discount for cash or V.P.P.').

The *marked* price is generally, the credit price, the price for those who buy on credit. *Discount* is the portion allowed to be deducted from the price.

'The marked price of an article is Rs. 20. A discount of 10% is allowed for cash. Find the cash price.' The cash price is evidently (Rs. 20 − Rs. 2) = Rs. 18; but this amount includes C.P. and Profit.

'The marked price of an article is Rs. 20. A discount of 10% is allowed for cash and the dealer still gains 20%. Find the C. P.'

C. P. + $\frac{1}{5}$ C. P. = Rs. 18; ∴ C. P. = Rs. 15.

6. The students must also be told that the things they purchase, pass through the hands of many dealers generally, the *manufacturer*, the *wholesale dealer*, the *retailer* and the purchaser, (in this case, the student) who is also generally called the *consumer*.

EXAMPLES XVIII.

Oral.

To be done in Form IV. The more involved questions need not be done orally.

1. Express the following fractions as percentages:—
$\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{5}$, $\frac{7}{10}$, $\frac{9}{25}$, $\frac{18}{25}$, $\frac{3}{24}$, $\frac{9}{45}$, $\frac{39}{75}$

2. i What percentage of a rupee is an anna?
ii ,, an anna is a pie?
iii ,, a rupee is two annas?
iv What is the value of 6¼% of a rupee?

3. The rates of school fees have to be enhanced by 6¼%. How much must a student pay who hitherto was paying Rs. 4?

4. The published price of a copy of Modern Elementary Mathematics is Re. 1-12 as. If a discount of $9\frac{3}{8}$% is allowed in case more than 50 copies be bought at a time, what will 64 copies cost?

5. A few years ago the railway fare between T and K was 4 as. 6 ps. Now it is 7 as. 6 ps. What is the increase per cent.?

6. In the year 1907, four measures of rice could be had for a rupee. Now rice sells at 2¼ measures per rupee. What is the increase per cent. in price?

7. A few years ago, a family of 4 members could be maintained with Rs. 32. Now owing to high prices of foodstuffs, the same family requires Rs. 50. What is the increase per cent. in the cost of living?

8. Thirty-six students sat for the Intermediate examination. 4 passed with distinction and 16 passed in the second class. Find the percentage of failures.

9. A person weighed 120 lb. last year. A few months ago he fell ill and now he weighs 114 lb. What per cent. of the original weight has he lost?

10. The strength of a high school last year was 630. Another school was started and the strength of the school consequently fell to 420. What is the percentage fall in the strength?

11. A house is purchased for Rs. 3500. 40% of the money is paid at once. Find the amount still due.

12. The population of a country is 30,000 and it is said that only 30% can read and write. How many cannot read or write?

13. A tin of kerosine contains 24 bottles of oil. If half a bottle is lost by leakage, what per cent. of the full tin still remains?

14. During a certain reorganisation, the pay of the officers and menials in an office was enhanced by $16\frac{2}{3}\%$, with the result that Rs. 350 had to be disbursed. How much did the office get before the reorganisation?

15. A cawnie was leased for Rs. 180 last year. If the same cawnie be let for Rs. 190 this year, what is the percentage increase?

16. A sheet of paper is 10 in. long and 5 in. broad. It is ruled into square inches. If the four corners be cut off along the diagonals of the corner squares and the strips (rt. $\angle d$ $\triangle s$) be removed, what percentage of the original area will remain?

17. A marriage could be celebrated on a decent scale, a decade ago, for Rs. a. Now on the same scale, the marriage costs Rs. b. What is the increase per cent. given that $b > a$?

18. The price of coffee seeds is Rs. 2-4 as. a viss. This is due to a fall of 4% in the usual price. What was the usual price?

19. The owner of a house was not willing to dispose of his house last year. The same house is now estimated at Rs. 3080, which is 12% below last year's estimate. What was that estimate?

20. Two papers were set in English. The maximum for the first was 80 and that for the second 60. A boy secured 45% in the first paper and 35% in the second paper. How many marks did he secure on the whole? What percentage of the total did he get in the two papers together?

21. There are two subjects under the B. Group. Boys must get 30% of the maximum, for promotion, in the two papers together. In the first paper a boy secured 8 mks. to a maximum of 50. What percentage at least of the maximum of 70 must he get in the second paper to entitle him to promotion.

22. A merchant sold ⅓ of his lot of goods at a reduction of 20% on the cost price and the remainder at a gain of 15% on the cost price. What was his total gain or loss per cent.?

23. In a school 10% are Christian, 15% other Non-Brahmins and the rest 420 Brahmins. Find the strength of each community in the school and the total strength of the school.

24. Boys must get 40% of the maximum, in each subject to entitle them to promotion; but if they get less in a single subject, they are promoted, provided the decrease is not more than 6¼% of the minimum required. A boy gets 45 mks. which is just the lower minimum. What is the maximum?

25. The current price of sugar is cent. per cent. higher than what it was five years ago and 12½% less than what it was a year ago. If the current rate is 10 as. 6 ps. per viss, what was it a year ago and five years ago?

26. Seven hundred and fifty people voted, 60% for and the remainder against, a certain candidate. An illegal proceeding was suspected and in the re-election 100 of the party that voted for the candidate went over to the opposition. Find the percentage increase in the opposition.

27. 15% of a certain sum is Rs. 5-4 as. Take aliquot parts and find its (i) 37½% (ii) 22½% (iii) 10% (iv) 42½%.

28. Write down as accurately as you can without working:—

(a) 5% of 75·46
(b) a% of Rs. 3435-12 as.
(c) 99% of Rs. 7000
(d) 49% of Rs. 7000.
(e) 102% of Rs. 53-2 as.
(f) 33⅓% of Rs. 12-8 as.
(g) 6¼% of a certain amount, 50% of which is 35% of Rs. 200.
(h) 17½% of a sum, 15% of which is Rs. 7-8 as.

29. An article is purchased for Rs. 20 and sold for Rs. 24. (i) What is the gain? (ii) What is the gain per cent?

30. An article is purchased for Rs. a and sold for Rs. $(a+5)$. What is the gain per cent?

31. Sugar is purchased at a annas a viss and sold at $(a+2)$ as. a viss. Find the gain per cent?

32. (a) Sugar is purchased at $8x$ annas a maund and sold at b annas a viss and the merchant gains c annas on every maund sold. Find the relation between x, b and c.

(b) (i) If $x=10\frac{1}{2}$ and $b=11$, find c.
(ii) If $x=11\frac{1}{2}$, $c=12$ find b.

33. (a) If an article is purchased for Rs. x and sold at a gain of Rs. b, what is the selling price?

(b) If the same article is sold at a gain of b%, what is the selling price?

34. A horse bought for Rs. 75 is afterwards sold at a gain of 28 per cent. Find the selling price.

35. (a) By selling a cycle for Rs. 250, a dealer gains Rs. 25. What dit it cost him?

(b) By selling a cycle for Rs. 250, a dealer gains 25%. What did the cycle cost him?

(c) By selling a cycle for Rs. 116, a dealer loses 25%. Find the prime cost of the machine.

EXAMPLES XIX.

Written.

Examples marked with an asterik must be done in Form IV. Double asterlks indicate harder problems which however must be done in Form IV during the last term preferably.

*__1.__ Express the following numbers as percentages correct to one decimal place.

$$\frac{2}{7},\ 6\frac{1}{3},\ \frac{11}{273},\ \frac{10}{11},\ \frac{41}{72},\ 0{\cdot}3,\ 0{\cdot}17,\ 0{\cdot}32,\ 1{\cdot}75.$$

*__2.__ Of the 2250 inhabitants of a village, 900 are school children. What percentage of the inhabitants are school children?

*__3.__ Milk and water are mixed so that a measure of the mixture contains an ollock of water. What percentage of the mixture is water?

*__4.__ In a year of 365 days, 180 days are working days. The rest are holidays. What percentage of the whole year do the holidays form?

*__5.__ To 2½ measures of milk, 1 measure of water is added. What percentage of the mixture is water?

*__6.__ To 'a' measures of milk 'b' measures of water are added. What percentage of the mixture is milk?

*__7.__ A merchant buys cocoanuts at Rs. 6-4 per hundred and sells them at 3 for 4 as. Find his gain per cent.

*__8.__ A horse is bought for Rs. 75 and a carriage for Rs. 120. The former is sold at a gain of 28 per cent. and the latter at a loss of 35 per cent. Find the amount realised and the total gain or loss.

*__9.__ I bought a bicycle for Rs. 275. I sold it at a gain of 20 per cent. The purchaser sold it at a loss of 20 per cent. on what it cost him. Find this last amount.

*__10.__ In a certain firm goods are marked for sale at an advance of 30 per cent. on the cost price.

(a) Find the marked prices of articles which have cost the firm Rs. 75 and Rs. 175.

(b) Find the cost price of articles marked at Rs. 42 and Rs. 17-14 as.

*__11.__ From a certain school 88 boys were sent in for an examination. Three did not sit for the examination and 20 per cent. of those that answered, came out successful. Find how many passed.

*__12.__ A person bought 100 kalams of paddy at Rs. 2-8 per kalam. At what price per kalam must he sell to gain 25 per cent on his outlay? If he sold at Rs. 2-12 per kalam, what is his gain per cent?

*__13.__ Two places A and B are connected by a gravel road and by a railway. The railway route is 20 per cent. shorter than the gravel road, If the railway route be 17 miles 4 fur. what is the length of the gravel road?

14. A bungalow was advertised for sale at a profit of 10% on the cost price. No offer of purchase forthcoming, a sum of Rs. 250 was deducted from the advertised price; and as still the bungalow remained unsold a further reduction of 5% was made on this price. The bungalow was sold and the owner realised a profit of 2·6% on his cost price. Find the amount for which it was originally advertised for sale.

15. A retail dealer purchased 5 reams of paper at Rs. 4-4 as. per ream. The top and bottom quires in each ream were a little bit soiled and could be sold only at 6 ps. per quire less than the price of the other quires. On the whole he gained Rs. 6-9 as. At what price per quire of good paper did he sell?

*__16.__ A gentleman bought a coach for Rs. 350 and a horse for Rs. 250. He sold the coach at a gain of 20% and the horse at a loss of 20%. What was the percentage gain or loss on the whole?

****17.** A house and its furniture are estimated at Rs. 7500. The furniture alone is sold at 20% above the price at which it has been estimated and the house fetched 5% less than the estimated amount. On the whole they were thus disposed of for Rs. 250 more than the estimated amount. What was the estimated cost of the house?

18. A shopkeeper marks his goods at an advance of 20% on the cost price. If on cash sales he wants only 12½% gain, what rate per. cent discount on the prices marked must he give his cutomers who buy for cash?

19. The catalogue price of a bicycle is Rs. 275 and the company allow a discount of 5% for cash. If the company even after allowing this discount make a profit of Rs. 51-4 as., what did it cost the company?

20. A maund of tamarind costs Re. 1-2 as. The seeds have then to be removed and in the process the weight of each maund is reduced by 40%. At what price per maund must the seedless tamarind be sold. so as to gain 12½?

21. The manufacturers of a certain brand of fountain pens quote their manufacture to wholesale dealers at a profit of 20% on the cost of manufacture. The wholesale dealer sells it to a retailer at 20% profit on his cost. The retailer sells each pen at a profit of 2 as. on each rupee paid by him. What is the cost of manufacture of a pen bought of the retailer for Rs. 3-6 as.

22. I bought a pair of bulls which I sold at a loss of 20%. I put in Rs. 80 more and bought another pair which again I had to sell at a loss of 12½% and having added Rs. 37½ to the proceeds I bought a third pair for Rs. 300. What was the cost of the first and of the second pairs respectively?

If I sold this last pair for Rs. 430 what is my gain or loss per cent. on the whole?

***23.** A banker had in his drawer 48 piles of Rupees, each pile consisting of Rs. 20. One of his clerks stole away the rupees and in their places placed 48 piles of half-annas, each pile consisting of 19 such coins and a rupee at the top thus making 20 coins in each pile. By how much % of the original amount was the banker defrauded?

***24.** The number of candidates that sat up for an examination in the several centres, and the number that passed in each, are given below :—

CENTRE.	NO. APPLIED.	PASSED.
A	164	83
B	61	30
C	110	37
D	213	71
E	301	60
F	269	54

Find the percentage of passes at the best and at the least successful centres.

25. A dealer bought a bicycle for Rs. 120 and sold it at a gain of 10% to a purchaser. who again sold it to his friend at a loss of 10% on his purchase price. How much did he realise ?

***26.** The population of a town is 43000. Next year the population increased by 12 % and in the succeeding year, the population fell by 10 % of the increased population. What was the total increase or decrease per cent ?

27. A merchant marked his goods at a profit of 30 %. He sold a third of his goods at this rate and had to sell the remainder at a reduction 25 % on the marked price. What was his total gain per cent ?

***28.** I bought a bicycle for Rs. 105. I exchanged the saddle and paid an excess of Rs. 12. I also bought a light for Rs. 18. If I then sold the bike for Rs. 150. what rate per cent did I gain ?

29. A person owned 800 acres of land and this yielded 24,000 kalams of paddy in a certain year, which he sold at Rs. 4-6 per kalam. Next year 10 % of the lands was unfit for cultivation, while the yield of each acre of the remainder was on an average 5 % more than the average yield per acre, in the preceding year, If the price per kalam fell by 5 %, what was the total increase or decrease in the income ?

***30.** The strength of the IV, V and VI forms of a High School, the current rate of school fees per pupil in each form and the respective enhancements in the school-fees proposed are given below. Find the increase in the collections.

IV.	120	Rs. 4-0-0	6¼ %
V.	100	Rs. 4-6-0	10 %
VI.	110	Rs. 5-0-0	12½ %

***31.** A merchant buys mangoes wholesale at Rs 15 per thousand. He has to pay Rs. 2-8-0 per thousand for railway and other charges and he finds that 60 per cent. of the mangoes, get so much damaged in transit, that they should be thrown away. At what price must he sell his mangoes so as to make a profit of 25 % on his outlay?

32. A merchant advertises goods at a price that would give him 40 % profit. At this price he succeeds in selling ¾ of his stock; but to dispose of the remainder he has to lower his prices by 40%. What is his percentage profit on the transaction?

33. A manufacturer sells his goods at 60 % over the cost of manufacture to a retail dealer, allowing him however 3 % off this price for cash. The dealer puts his selling price up 40 % over his net outlay, but allows his customers 10 % discount for cash. By how much is this selling price greater than the cost of manufacture?

***34.** The manufacturer of a machine makes 10 % profit, the middleman 8 % and the shopkeeper 10 %. Find the cost to the manufacturer, when the shopkeeper sells it for Rs. 816-12 as.

35. A publisher brings out, at a cost of Rs. 1500, an edition of 750 copies of a book and agrees to give the author 60 % of the profits. He allows the bookseller a discount of 25 % of the selling price of the book and delivers 13 copies as 12. If 50 copies are given away as presentation copies and 50 remain unsold, and if the publisher's share of profits is Rs. 300, find, (1) the selling price of the book, and (2) the gain per cent made by the bookseller.

36. A college orders for its library books from London whose cost, as given in the catalogues, amounts to £ 100. Two fifths of this sum is net, but for net books, the purchaser has no further expense. On the others a discount of 25% is allowed, but £ 2 is charged for packing and carriage. A local bookseller would have charged net books at 13 as. and others at 10 as. to the shilling. Find how much is gained or lost by sending the order to London. £ 1 = Rs. 15.

***37.** A merchant buys in Madras 210 bags of rice at Rs. 10-12. per bag of 164 pounds. He sends them by rail 320 miles at 6½ pies, per ton, per mile, but during the journey 7¾ pounds are stolen from each bag. Find at how may measures per rupee he must sell the remainder in order to clear Rs. 95-15 as. by the transaction. (One measure = 3⅛ pounds).

38. A person sold 86 measures of rice for Rs. 13-7-0 thus gaining 25 %; and 154 measures at a profit of 10 %. Supposing he sold the whole at a profit of 16%, how much more would he have gained?

39. (a) Which is more profitable :—to throw off a ready money discount of 2½ per cent. or allow six month's credit, the general rate of interest being 8 %?

(b) If a tradesman's goods be marked at prices 30% above cost price, how much per cent. more woud he make on his outlay, by adopting the more favourable of the above ways, than he would by adopting the other?

****40.** A merchant buys 1,260 maunds of corn. One fifth of which he sells at a gain of 5%, one third at a gain of 8% and the remainder at a gain of 12 %. If he had sold the whole at a gain of 10% he would have obtained £ 22-13-0 more. What was the cost price per maund?

41. A Madras merchant bought 900 yards of silk in London at 5 s. 6 d. a yard, when the exchange value of a rupee was 1 s. $3\frac{11}{16}$ d. The total cost of importation was 11% of the purchase prices. He sold 200 yds. which were damaged in transit at Rs. 2-8-0 a yard and the remainder at Rs. 6-8-0 a yard. What percentage did he gain or lose on his outlay?

42. A tradesman having purchased a bankrupt's stock at an average reduction of 33⅓ % on the marked prices, sold one-half of the stock at a reduction of 20% and the remainder at a reduction of 25 % on the marked prices. If the expenses of the sale amounted to 7½ % of the receipts, what was the tradesman's gain per cent. on the price he paid for the stock?

43. A vessel contains milk 10 % of which is water. If a quarter of a measure of this be mixed with half-a-measure of another variety, 15 % of which is water, what percentage of the mixture is water?

44. A vessel contains a measure of milk, 12 % of which is water. How much pure milk must be added to this that the proportion of water may be reduced to 8 %?

45. In an engagement, 10 % of the strength of an army was killed and 7 % was wounded; 80 % of the wounded survived and then it was found that they had lost 285 men. What was the original strength of the army?

***46.** Seventy-five per cent of the area of a farm is fit for cultivation, of the remainder 85 % is pasture and the rest is waste; the area of the waste is 3 ac. 20 poles. What is the area of the farm?

***47.** The populations of the upper and lower parts of a town were equal and after the former had fallen 20 % and the latter risen 15 %, the total number of inhabitants was 39390. What was the population of each part at first?

48. An ore contains two metals and their combined weight is 80 % of that of the ore. In the process of extraction only 90 % of the metals could be obtained. If, of this, 45 % consists of gold and the rest of the inferior metal, find what weight of each metal is obtained from a ton of ore.

49. The gross receipts of a co-operative society are apportioned as follows. 42 % to pay working expenses, 18 % to pay interest on the capital borrowed and of the balance 25 % to be taken to the reserve fund. The balance in a certain year justified a dividend of 8 % to the shareholders. If the paid-up share capital be Rs. 6,000; find,

1. The gross receipts.
2. The amount borrowed at 6 p. c.

****50,** A man's monthly income in a certain year was Rs. 350. He saved 45 % of this income. Next year his income rose by 15 %, but still he was able to save the same amount he had saved the previous year. By how much per cent did his expenses increase?

****51.** The school fees were collected a couple of years ago in 8 instalments. Last year the fees were enhanced by 6¼ %, the number of instalments being the same. If it is proposed to raise the number of instalments to 10, the rate of each instalment being the same as enhanced last year, find the aggregate increase per cent of the original rates.

***52.** The strength of a school consisting of boys and girls is 630. The number of boys is 2½ times the number of girls; 40 % of the boys and 60 % of the girls came out successful in an examination. What percentage of the total passed in the examination?

****53.** A merchant buys 4,000 maunds of rice, one-fith of which he sells at a gain of 5 %, one-fourth at a gain of 10 %, one half at a gain of 12 %, and the remainder at a gain of 16 %. If he had sold them at a gain of 11 % he would have made Rs. 728 more. What was the cost of the rice per maund?

54. One measure of milk which contains 11 % of water is added to 5 measures containing 7 % of water and to this mixture half-a-measure of water is added. Find the percentage of water in the mixture.

55. A firm advertise a 'Grand Clearance sale' and say that they have reduced their prices by 22½ p. c. of the catalogue rates; and thereby sell their articles at 7 % loss. At what rate per cent profit were the catalogue prices fixed?

(a) If x denote the catalogue price and y the cost price of an article, establish a relation between x and y.

(b) Draw the graph of the relation and from the graph find the catalogue price of an article, which had cost the firm Rs. 2—8 as. and the cost price of an article whose catalogue price is Rs. 2—10 as.

56. A timber merchant purchases a big log of teak-wood 18 ft. by 20 in. by 15 in. at Rs. 210 a ton; and it is taken to a place 4 miles distant from the shop, the rate of carriage-fare being 10 as. per mile. He saws out of it two beams 14 ft. by 9 in. by 14 in. and the rest is sawn into planks; If in the process of sawing 5 % of the wood be wasted and he sells the beams at Rs. 6 a cub. ft.

and the planks at Rs. 6-12 a cub. ft. find the merchant's gain. A log of wood 50′ × 1′ × 1′ weighs a ton.

57. The price of a Safety Fountain Pen is Rs. 6-4. If a dozen pens were bought at a time, the company allows a discount of 10 p. c.; and if cash be paid a further reduction of 5 p. c. on this price is allowed. A dealer buys 2 dozen pens and pays cash and sells each pen at Rs. 6-10 as. What profit per cent does he make? Find his total gain.

58. A municipality invites tenders for the supply of 10½ lacs of bricks. A contractor agrees to supply at Rs 5 per thousand. 12½ % of the bricks supplied are rejected. He however supplies the 10½ lacs and makes a profit of 25 per. cent. on his outlay. How many bricks does he make and what does each thousand actually cost him?

****59.** A contractor bought, 2250 pharas of unslaked lime at Madras at the rate of Rs. 45 for 100 pharas. On slaking it every phara gave 8 cub. ft. of lime, but of this $\frac{1}{15}$th was unserviceable; the carriage of the remainder to the place where it was required (distant 18 miles) cost 4 annas per 100 cub. ft. per mile. At what rate per cub ft. must he sell it there, in order to gain Rs. 90 on his outlay.

60. (a) A rice-dealer bought 3 cart-loads of paddy at Rs. 20—8 as. a pothi. He paid 5 annas for every kalam for grinding the paddy in a rice-mill and each kalam yielded only 14 measures of rice. He sold the husk for Rs. 8—9 as. and sold the rice at 3 measures per rupee. What is his gain? (A cart load is 3 pothis. A pothi = 5 kalams and each kalam = 30 measures.)

(b) Which is profitable to a consumer?—to buy rice of the dealer, or in the bazaar at Rs. 16-4 per bag which is said to contain 50 measures but does actually contain 47½ measures. The bazaar-man includes the price of the bag also and would pay six annas back if the bag be returned.

61. (i) I ordered 120 lb. of ice and this was supplied at 6 pies per ℔. In the transit a certain portion melted away and I sold the remainder at one anna a lb. and gained 60 per cent on my outlay. How many lb. of ice had melted away?

(ii) At what price per lb. ought I to have sold the remainder had I wanted to make cent per cent gain?

62. A contractor calculates to make 25 per. cent. on the actual cost of a house, and the engineer who prepares the design charges 5 per cent on the money paid to the contractor. If the owner after paying the contractor and the engineer, sells the house at a profit of 15 per cent and thereby gains Rs. 945, what did the house cost the contractor?

63. Five thousand copies are issued of a book, the price of which is Rs. 3 per copy; the cost of printing is 8 as. per copy, binding Rs. 2 per dozen, and carriage, advertising etc. Re. 1 per dozen; the publisher disposes of them to the retail book-seller charging 25 copies as 24 and 30 per cent less than the selling price, and upon the whole receipts takes 10 per cent commission for himself and gives the balance to the author. How much does the publisher, author and the retail bookseller respectively receive on this edition?

64. I publish 2,000 copies of a book at a cost of Rs. 300. I reserve 11 copies for my use and sell the rest to a bookseller at 9 as. each copy and count 13 copies in the dozen. How much per cent have I gained? If the book-seller sells at 9 as. each copy, find his gain per cent.

***65.** The pre-war and post-war prices of certain varieties of Tennis Bats are given below. Find the percentage increase in each case.

	Pre-War Price.		Post-War Price.	
	Rs.	As.	Rs.	As.
Tropical Doherty	32	8	42	4
Spalding Gold-medal	35	8	44	6
Prosser's Improved Phenomenon	35	0	49	0
Davis Risely Hexagon	34	8	38	13

(a) If a club orders for one of each variety and the company supplies them at the enhanced rates allowing 20 % discount on the bill, find what per cent of the pre-war rates, the club paid in excess for the four bats together.

****66.** The price of pure silver per Rupee weight is Rs. 1-3-6. The charges for making a silver cup come to two annas and a half for every Rupee weight in the finished cup. If I buy one seer and a half of silver and if 2½ % of the silver is wasted in the making, what will be the final cost of the cup, given that a seer weight is equal to that of Rs. 24?

67. A manufacturer sells his goods 60% over the cost of manufacture, to a retail dealer allowing him 5% of this price for cash. The dealer puts his selling price 40% over his outlay, but allows his customers 10% discount for cash. By how much per cent. is this selling price greater than the cost of manufacture?

68. A person purchased a lot of season goods and marked the prices at an advance of 35 per cent. on the cost price. He was able to sell at this rate only a third of the goods and the remainder was sold at a reduction of 12½ per cent. on the prices marked. The gain on the whole was Rs. 190. Find the amount of his investment.

***69.** A market woman bought a sackful of brinjals for Rs. 12-8 as. She sold half the quantity at 5 for 6 pies and the remaining half at 1 pie each, and gained 10 per cent. on the whole. How many were there in the sack?

70. i. A cloth-merchant marks his goods at an advance of 12½ per cent. on the cost price and uses a false yard measure which is 2¼ inches too short. What is his actual gain per cent.?

ii. Having detected the shortage in the length, by how much per cent. of itself should the selling price be cut off, that the dealer may get his usual profit of 12½%?

71. By selling an article for Rs. p, a merchant lost q per cent: he would have gained r per cent., if he sold it for Rs. t. Establish a relation between p, q, r, t; if $p = 78$, $q = 2\frac{1}{2}$, $r = 5$, find t.

****72.** A bookseller bought copies of a certain book at Re. 1 each. He was able to sell 100 of the number at a gain of 12½ per cent. and the rest at a loss of 9⅜ per cent. and gained Rs. 6-14-0 on the whole. Find the total number of copies.

73. The published price of a book is 25 per cent. greater than the price charged to a bookseller for a single copy. When taking a quantity the bookseller gets 13 copies as 12. If the bookseller allows to purchasers a discount of 2 as. on each rupee of the published price, find his gain per cent. (1) if he buys a single copy and sells it, (ii) if he buys a quantity and retails it.

****74.** The receipts of a company amounted in one year to Rs. 1,77,000, and the expenses to Rs. 1,25,000. In the subsequent year, the receipts fell by 2 per cent. and the expenses increased by 1 per cent. Find by what percentage of itself the profit in the first year exceeded the profit in the succeeding year.

75. (i) A wholesale dealer supplied goods to a retailer at a profit of 30 % on what the goods had cost him. The retailer became a bankrupt and the court decreed a dividend of 11 as. 6 ps. in the rupee. How much did the wholesale dealer lose in the transaction ?

(ii) If another wholesale dealer who had also supplied to the same retailer at the same rate, had done so having received an advance of half the amount, what was his gain or loss per cent. ?

76. A milkman adulterates his milk to such an extent that by selling this milk at a profit of 20 p.c. he may gain 38%. The customer agrees as to the price but dislikes the quality ; whereupon the milkman mixes some pure milk and finds that by selling this new mixture at the price already quoted he can gain only 32 per cent. Find (i) the percentage of water added originally, (ii) the percentage of pure milk added afterwards.

77. It had been found by experience that of the wood given in sizes to a carpenter for making furniture, 25 % is wasted in the first process of cutting, sawing, chiselling etc. and in the next process of planing 5 per cent. of what is left after the first process, is wasted ; but the furniture made is polished and glazed finally, which adds 2 per cent. of the weight which the furniture had when the polishing was begun. A set of furniture fresh from the carpenter's shop weighed 1500 ℔s. Find how many cubic feet of the wood the carpenter began his work with, if 50 cubic ft. weigh a ton. Give the answer correct to the nearest cub. ft.

78. (1) A coffee merchant was selling coffee at Rs. 3-12-0 a viss and thereby was gaining 25%. Owing to a fall in the price of the commodity due to a large produce in the estates, he had to cut down his price and reduced it so as to gain only 5 as. on each viss sold. In what ratio must his sale increase that he may make as much as he did before?

(2) If at the dearer rate he was selling per month Rs. 1500 worth of coffee seeds, how many viss per month must he sell at the latter reduced rate?

79. A cycle dealer allowed a discount of 10 per cent. on the catalogue prices of certain makes of cycles and a further discount of 5 per cent. on this amount for cash and still made a profit of 18¾ per cent. The wholesale prices of these cycles fell by 8⅓ per cent. and the dealer therefore cut down the catalogue prices by 5 per cent. and offered the same concessions and discounts as before. What profit per cent. did he make?

80. A manufacturer had marked his goods at an advance of 40 per cent. on the cost price, but allowed a discount of 10 per cent. on this price and a further reduction of 6¼ per cent. on this last amount for cash. A retailer bought goods of the dealer for cash, and in his turn marked his goods at a price that would give him 25 per cent. profit and allowed a discount of 12½ per cent. for cash. Find correct to one decimal place the excess per cent. of the price paid by a cash customer on the cost of manufacture.

81. A publishing company offered an author 25 per cent. of the sales or 35 per cent. of the profits. If a copies of the book were printed and sold at Rs. b each and if the total cost of printing, advertising etc. be Rs. c and if the author gets the same amount by either method find the relation between a, b and c. If $a=1500$, $b=2\frac{1}{2}$, find c.

82. A dealer marks his goods at such a price that would give him 20% profit, even though he deducts 1 anna from the marked price. If x denote the cost price and y the marked price in annas, write down the equation connecting x and y.

Draw the graph of the equation; and from the graph, find the marked price of an article which cost him 5 as. and the cost price of an article marked for annas 13.

****83.** At a clearance sale books were sold 10% below, glass-ware 15% below, and piece-goods 25% below their catalogued prices. The amounts realised under the respective heads were Rs. 202-8 as., Rs. 518-8 as. and Rs. 461-4 as. If the catalogue prices were fixed at a uniform profit of 25% on the actual cost price find (1) the total catalogue prices of goods sold (2) the total cost price of goods sold, and (3) the net gain or loss in the clearance sale.

***84.** A set of 120 letter-papers with envelopes to match, costs Rs. 4-8. The letter-papers alone cost Rs. 3. A person had to post 100 invitation letters on a marriage occasion. By how much per cent., will buying embossed envelopes at one-anna each be cheaper than buying 100 envelopes and letter-papers at the rate of Rs. 4-8 per set of 120 and affix one-anna stamp to each?

85. A dealer bought two bicycles of two different firms, the price of one being 10 per cent. cheaper than the other. He sold the dearer for Rs. 282 and made 20 per cent. thereby. If he should gain 25 per cent. on the whole, what profit should he get on the other? Express this profit as a percentage of the cost of that machine, correct to two decimal places.

86. In a certain district there are two schools. In the first 20 per cent. are Mohammedans, 50 per cent. Christians and the remainder Hindus. The second school contains only Christians and Hindus. If the two schools were amalgamated there would be 8 per cent. Mohammedans and 56 per cent. Christians. Find the percentages of the Christians and Hindus in the second school.

87. The Waterman's Ideal Fountain Pen 'Regular Type' is quoted at 12s. 6d. by the company in England. The importing company has to pay 22½% of the cost for customs duty, packing etc. Exchange is at 1s. 5⅛d. per Rupee. The company sells it in retail at Rs. 8-8 as. per pen and makes 12½% profit. What percentage discount is allowed for trade?

88. A cabinet maker undertook to supply 1800 chairs to a club and fixed a price that would give him 18¾ % profit even though 5 % of the number supplied failed to stand the test and would not be paid for. At the trial 12 % failed to satisfy the management and these chairs were not paid for. The cabinet maker however gained Rs. 855. What did each chair actually cost him and at what price did he quote ?

***89.** A manufactory turns out 50 tons of iron goods weekly, using up for the purpose 51 tons of iron at £ 6-15 s. a ton, 100 tons of coal at 11 s. 6 d. per ton, and £ 45 worth of other materials; rent, rates and taxes amount to £ 219 annually; wages, incidental expenses, £ 75 per week. At what price per cwt. must the iron goods be sold in order that the works may gain 8 per cent. per annum on a capital cf £ 35000 ?

90. Two persons A and B speculated in different ventures with capitals of Rs. 13,000 and Rs. 9,000 respectively. The former had been more fortunate and gained 3 % while the latter only one per cent. If the latter has doubled his capital, how much has the former got ?

91. A tradesman has been accustomed to give his customers six months credit and has marked his goods at such a price that would give him 12 % interest on the selling price. He however wants to introduce the ready-money system into his business. By what percentage of the marked price should he cut down the prices ? What would be the cash price of articles marked Rs. 13-4, Rs. 66-4, respectively ?

92. It appeared in a daily news paper that the annual expenses of Suez Canal might be estimated at about 178,00000 francs exclusive of 10,000,000 francs needed for payment of interest on the shares. Assuming that all the receipts over and above the sum of these amounts are to be divided as profits among the shareholders, the founders and the Egyptian Government, the holders of shares taking 70 per cent. and each of the latter parties 15 per cent; what must be the yearly receipts in order that an English holder of 20 shares, out of the 400,000 representing the whole stock, may receive £ 73-10s. as his proportion of the profits ? (£ 1 = 25 francs.)

93. At what rate per cent. profit must a tradesman mark his goods that he may allow 4 per cent. of the sales for bad debts, 7¼ per cent of the cost of goods for expenses, an average credit of 8 months and yet make a profit of 12½ per cent. money being worth 15 per cent.

94. A dramatic troupe is organised among educated people on co-operative lines with 600 shares of Rs. 10 each, fully paid up. During a season four plays are staged and the total expenses incidental are Rs. 300 towards theatre-rent, Rs. 150 for lighting, Rs. 130 for boarding and Refreshments, Rs. 100 towards advertisement and Rs. 50 coming under miscellaneous expenditure. If of the total collections 25% be set apart for reserve fund, 5% for charities, what should be the total average collection for every performance that, taking this as an average, the shareholders may receive a dividend of 9% supposing that the members can only stage 8 plays in the year, in two seasons of four plays each?

95. Two retail dealers in the same town get things from the same wholesale firm. One tradesman states his profit as 15% on the cost price; the other as 15% on the selling price. Find the difference in their actual profits on goods sold for Rs. 3220. If the difference in their profits amount to Rs. 94-8 as. find the value of goods sold by each and the price paid by either for the goods.

96. The net prices of goods described in a Catalogue are 15% less than the Catalogue prices, while a further discount of 5% on the net prices is given for cash payments. A customer, however, higgles and obtains 20% off the Catalogue price and finds that by so doing he has gained Rs. 4-10-6. What was the Catalogue value of the goods he bought?

97. To make an article takes 2½ hours of a workman's time paid at 7½ as. an hour. The material 10% of which is wasted in the working, costs 10 as. a pound. The final weight of the article is 20 ℔. At what price must it be sold to gain 8% on the total cost?

98. A contractor undertook to supply 1000 bags of rice, each bag containing 50 measures of rice at 3¾ measures for a rupee. The bags were supplied and it was agreed that ten bags chosen at random should be measured and the average of these taken as the quantity in each bag. These bags had 50, 50, 49, $49\frac{3}{4}$, $49\frac{7}{8}$, $49\frac{3}{8}$, $48\frac{15}{16}$, $49\frac{13}{16}$, $48\frac{7}{8}$, 49, $46\frac{5}{8}$ measures respectively. If the contractor was paid as agreed, what % of the profit he expected did he get, if his estimated profit was $16\frac{2}{3}$ %?

99. (*a*) A certain article is 20 % dearer at a firm A than at a firm B, while it is 10 % cheaper in firm A than in a third firm C.

An article is bought of the cheapest firm for Rs. 22-8 as. What would it have cost if bought at either of the other two firms?

(*b*) If this article had cost each firm only Rs. 20, what profit % did each firm expect at the prices quoted?

100. A merchant marks his goods at an advance of 15 % on the cost price. The wholesale price of the goods fell down and therefore the merchant had to offer a reduction of 15 % on his marked price. If he had stocked his goods at the higher rate how much would he lose on an investment of Rs. 15,000?

101. (*a*) A publisher estimates the actual cost of bringing out 100 copies of a book at Rs. 75. Find the retail price for each copy that he must fix so that, after spending 7½ % on the retail price on advertising, and giving the author 10 % of, and the retail dealer a discount of 20 % on, the retail price, the publisher may gain 25 % on the cost of publishing.

(*b*) If the publisher brings out 2000 copies of the book and 250 remain unsold, how much would the author get?

102. During a certain reorganisation of the pay of officers, those who were drawing below Rs. 50 were given an enhancement at an average rate of Rs. 5 each. The amount that would be spent at this rate was distributed at Rs. 6-8 as. for each officer in the town and Rs. 3-12 as. for each officer in the village. What percentage do the town officers form of the total number of officers drawing a pay of less than Rs. 50?

103. A motor-car is advertised in England for £ 300 freight, landing charges and chiefly customs come to 40 % of this advertised price. Exchange is at 1 s. 5½ d to the rupee. The same car is offered by a company at Madras for Rs. 4620 and the company is said to gain 10 % thereby. If the company has to pay freight, landing charges and customs duty on the same scale as any one else should, find what discount is allowed to trade.

104. A tradesman marks his goods at a price that would give him 25 % profit, even after allowing a reduction of 1 anna in the rupee off the marked price. If x denote the cost price in rupees and y the marked price in rupees, establish the relation between x and y.

Draw the graph of the relation and from the graph find the marked price of an article bought for Rs. 4-8 as. and the cost price of another marked Rs. 3-5-4.

105. A merchant who sold his goods at a profit of 10 % found that when he allowed 2½ % discount off his selling price, his business increased by ⅓. Find whether his total profits were increased or diminished by adopting this plan and in what proportion.

106. A merchant sells toys for children. The cheapest is bought by him for 8 pies and the dearest for 10 as. He wants 25 % gain on each article. Draw a graph (ready reckoner) from which he can read off the S. P. of every toy he has to sell. Find from the graph

(i) the S.P. of articles purchased for 1 a. 8 ps.; 2 as. 8 ps.; 3 as.; and 7 as.

(ii) the S.P. of articles which he sells for 6 as.; 5 as. 6 ps.; 3 as. 6 ps.; and 2 as.

SECTION VII.

Decimals—Approximations.

Contracted methods.

1. To multiply or divide a decimal by 10 or powers of 10.

$36{\cdot}732 \times 10 = 367{\cdot}32$; Reason—(a) If we multiply 36 by 10, the digit in the tens place becomes so many hundreds and the digit in the units place becomes so many tens and the digit in the units place is 0, for $36 = 36{\cdot}0$ and the digit in the first decimal place becomes the digit in the units place.

(b) Similarly when the digits in the whole number are respectively promoted to the next higher place, the digits in the decimal part are likewise promoted; so that in the product of $36{\cdot}732$ and 10, 7 in the first decimal place occupies the units place, 3 in the second decimal place occupies the tenths or the second decimal place.

Similarly if we multiply $36{\cdot}732$ by 100, each digit occupies the next but one higher place in the product.

Rule.—(i) To multiply a decimal by 10 or powers of ten, remove the decimal point to as many places to the right as the power indicates.

(ii) The student will at once understand that to ***divide by 10 or powers of ten, the decimal point must be removed to as many places to the left as the power indicates.***

Ex:—(a) $376{\cdot}432 \times 1000$ (i.e. 10^3) $= 376432$

(b) $0{\cdot}73467 \times 100$ (i.e. 10^2) $= 73{\cdot}467$

(c) $1{\cdot}734 \times 10000$ (i.e. 10^4) $= 17340$

(d) $441\,734 \div 100$ (i.e. 10^2) $= 4{\cdot}41734$

(e) $1{\cdot}734 \div 10$ (i.e. 10^1) $= {\cdot}1734$

(f) $21{\cdot}645 \div 1000$ (i.e. 10^3) $= {\cdot}021645$

(g) $\cdot0467 \div 10000$ (i.e. 10^4) $= \cdot00000467$

(h) $73\cdot4675 \times 10000 \div 100000 = 7\cdot34675$

2. (a) $48 \times \cdot36 = 144 \times 12$; If one of the factors be multiplied by 3 and the other divided by 3, the product remains unchanged.

$4\cdot8 \times 36 = 1\cdot6 \times 1\cdot08$; $73 \times 100 = 730 \times 10$; $14\cdot73 \times 200 = 1473 \times 2$; $\cdot7345 \times \cdot4842 = \cdot07345 \times 4\cdot842$.

(b) If both the numerator and the denominator of a fraction be multiplied or divided by the same number, the fraction is unaltered.

Hence (i) $76 \div 12 = 19 \div 3$

(ii) $34\cdot783 \div \cdot1234 = 347\cdot83 \div 1\cdot234$

(iii) $\cdot08756 \div 11\cdot346 = \cdot008756 \div 1\cdot1346$.

3. If a decimal, by multiplication or division with a suitable power of ten, be so changed as to have only one digit (i.e. the units place alone) in the integral part, the decimal so converted is said to be in the ***Standard form.***

(i) $37\cdot3476 = 3\cdot73476 \times 10$

(ii) $348\cdot1234 = 3\cdot481234 \times 10^2$

(iii) $\cdot034678 = 3\cdot4668 \div 10^2$

4. Approximations.—An idea of approximation is best given by getting answers to questions of the type given below:—

i. What is the distance between your house and the railway station?
ii. What, do you think, is the strength of the school?
iii. What is the height of the table?
iv. What is the length of the pencil you have?
v. What is the worth of your property?
vi. How many teachers are there in the school?

The student should then, by actual measurement of the height of the table, find out that he is or is not far wrong. One student will generally be more accurate than the other. Approaching as far as possible, the actual value is called *Approximation.* Generally we have only approximate ideas of the value or the nature of everything we see and observe. The more carefully we observe, the more accurate the result, the closer the approximation.

The student will be asked to turn to a particular page of a certain book and to state roughly how many lines of print, there are in the page. Suppose one boy says that there are 36 lines, another says that there are 37 lines, another 39 and another 41. Whose result is the most accurate if there are 40 lines actually, in the page?

We call the boys *A*, *B*, *C*, and *D* respectively.

The difference between the real and estimated no. as given by *A* is 4.
,, ,, ,, *B* is 3.
,, ,, ,, *C* is 1.
,, ,, ,, *D* is 1.

The 4, 3, 1 and, 1—the differences between the actual number of lines and the estimated numbers are called, **Absolute errors.**

Again, the ratio of the absolute error to the actual value is called the

Relative error—

A's relative error is $\frac{4}{40} = {\cdot}1$
B's ,, is $\frac{3}{40} = {\cdot}075$
C's ,, is $\frac{1}{40} = {\cdot}025$
D's ,, is also $\frac{1}{40} = {\cdot}025$

so that C's and D's answers are the most accurate.

The student should know that every ratio can be expressed as a percentage. $\left.\begin{array}{l}\frac{4}{40}=10\% \\ \frac{3}{40}=7\frac{1}{2}\% \\ \frac{1}{40}=2\frac{1}{2}\%\end{array}\right\}$ If the relative error be given in the form of a percentage, the error is called the **Percentage error.**

5. The length of a pencil if measured very carefully is actually 5·47 in. You are asked to give the length to the nearest tenth of an inch; Is it 5·4 or 5·5?:—

The Relative error in giving the result 5·4 is $\frac{{\cdot}07}{5{\cdot}47}$

and ,, ,, ,, 5·5 is $\frac{{\cdot}03}{5{\cdot}47}$

As the second fraction is the less, the less will be the error in giving the result 5·5.

Generally therefore if we are asked to give a result to the nearest n^{th} decimal place, we look at the $(n+1)^{\text{th}}$ place and increase the digit in the n^{th} place by 1 or leave it as it is, according as the digit in the $(n+1)^{\text{th}}$ place is 5 and greater than 5, or less than 5.

For example :—

73·730467 is 73·73 correct to two decimal places
64·647867 is 64·65 ,, ,, ,,
110·143769 is 110·144 ,, three ,,
·046789 is ·04679 ,, five ,,
·76799 is ·7680 ,, four ,,
(the 0 must be put down)
734786 is 735000 correct to the nearest thousand
646434 is 646400 ,, ,, hundred
756748 is 800000 ,, ,, one hundred thousand

6. Significant figures.

(a) $\pi = 3·141592$..............

The value of

π is 3 Correct to *one significant figure*.
π is 3·1 ,, *two* ,, ,,
π is 3·142 ,, *four* ,, ,,
π is 3·14159 ,, *six* ,, ,,

(b) Take the decimal 3·04607018

The decimal correct to one significant figure is 3
two ,, 3·0
three ,, 3·05
five ,, 3·0461

(c) Again take the decimal, 0·00304678

This decimal is ·003 correct to one significant figure
·0030 ,, two ,, figures
·00305 ,, three ,, ,,

Rule—Take that figure, in the given number, which is to the extreme left. If it is not a zero, that figure is the first significant figure.

Ex. Write down ·00046078 correct to three significant figures.

The figure to the extreme left is zero—neglect.
Again the next extreme figure ,, ,, ,,
Again ,, ,, ,, ,, ,,
Now ,, ,, ,, 4; This is the first significant figure. Begin counting from this; The answer must be ·000461 (for the last figure must be corrected).

An apology to Masters.

Educationists may object to the formulation of the routine method given here for finding the number of significant figures. Much as we should like to give in detail, the *rationale*, our experience of 15 years and over with so many sets of students, dictates that this routine method is to be preferred even in a *text book* and our conviction is corroborated by a like treatment in each of the many modern treatises English and American, we have consulted. We have tried *the reasoning out* of the process of finding the significant figures in the class room and while very few are responsive, considerable relief is exhibited as soon as the rule is given. When students have clearly understood how to write a given number correct to a number of significant figures and when they have a working knowledge of "significant figures," the rationale may be explained more profitably and the results speaking from our experience, have not been discouraging. Therefore it is, that we give below the teaching notes.

(i) Questions of the type given in Art 4 *supra* and the answers thereto should be made the bases of giving the students an idea of what we mean by '*unit.*'

(ii) Approximations depend on the units employed (*a*) The distance between the Sun and the Earth is given to the nearest million miles; the unit being the million (*b*) The height of the barometric column is read to the nearest tenth of an inch. (*c*) The length of a piece of cloth is measured to the nearest inch.

(iii) Therefore the absolute error may vary from $\frac{1}{10}''$ of an inch to thousands of miles; but it will be noted that the *relative error is the same so long as the significant figures are the same:*—

The significant figures tell us how many of the units employed are taken into consideration.

The radius of the earth is 3959 miles; A certain rod is measured accurately and is found to be 3·959 yds.

long. Expressing each number correct to three significant figures we have 396 tens of miles and 3·96 yds. In the former case the absolute error is 1 mile and in the latter case it is only ·001 yd; but the relative error in each case is $\frac{1}{3959}$

7. (i) ·0 × ·1 = ·001

(ii) ·02 × ·03 = ·0006

(iii) ·004 × ·001 = ·000004

(iv) ·4 × ·00002 = ·000008

(v) ·000004 × 2 = ·000008

(1) A digit in the 1st decimal place multiplied with a digit in the 2nd decimal place gives the result in the 3rd decimal place. or more generally a digit in the nth decimal place multiplied with a digit in the mth decimal place gives the result in the $(m+n)$th decimal place.

(2) A digit in the units place multiplied with a digit in the mth decimal place gives the result in the mth decimal place.

8. (i) Multiply 4·768342 by 7·234062 correct to three decimal places.

We must have the answer correct to three decimal places.

∴ We must have the fourth decimal place also, in the product, to correct the third decimal place.

We dont want therefore decimal places in the answer beyond the 4th.

7 in the multiplier must therefore be multiplied with 3 in the 4th decimal place to begin with; and the partial product will be the result of multiplying 7 and 4·7683. [It should be noted however, that as a portion of this product beyond the 4th decimal place is rejected as not necessary to our purpose, the carry arising from the product of 7 and 4, the fifth decimal place) must be added to the product of 7 and 3.]

The complete work stands thus—

$$\begin{array}{r} \cancel{4}\cancel{7}\cancel{0}\cancel{8}\cancel{3}42 \\ \cancel{7}\cancel{2}\cancel{3}\cancel{4}\cancel{0}62 \end{array}$$

	3 3 3 7 8 4	7×4=28, carry 3. Strike off 7 & 3
In the next partial product multiply 2 in the tenths place (mul^{r}.) with 8 in the 3rd decl place (mul^{d}.)	9 5 3 7	2×3=6, carry 1. Strike off 2 & 8.
Now, multiply 3 in the 2nd dec. place (mul^{r}.) with 6 in the 2nd dec^{l}. place (mul^{d}.)	1 4 3 0	3×8=24 carry 2. Strike off 3 & 6
Multiply 4 with 7 in the (mul^{r}.)	1 9 0	4×6=24, carry 2. Strike off 4 & 7
„ 0 with 4 „ „	0	Strike off 0 & 4.
	2	Though there is no dight to the left in the mult^{d}, we have digits in the mul^{r}., therefore the last partial product is the carry 6×4=24, i. e. 2.
	3 4 4 9 4 3	
The answer is	3 4 4 9 4 3	Mark off 4 decimal places
	3 4·4 9 4	Correct 3 decimal places.

(ii) Multiply 73.076324 by ·003467 and give the answer correot to 3 decimal places.

(*a*) Converting the multiplier into the *standard form* and making the compensating change in the multiplicand. we have 73·076324 × ·003467 = ·073076324 × 3·467.

The work may be begun now and it stand thus :

$$\begin{array}{r} 0\cancel{7}\cancel{3}\overset{*}{\cancel{0}}76324 \\ \cancel{3}\cdot\cancel{4}\cancel{6}7 \\ \hline 2192 \\ 292 \\ 44 \\ 5 \\ \hline \end{array}$$

The answer is ·253

The Rule. (i) Since the answer is required correct to 3 decimal places, put a mark over the 4th decimal figure in the multiplicand.

(ii) Just underneath the marked figure put the whole number in the multiplier and write down the rest in order.

(iii) Multiply the first figure in the multiplier with the one just above it, taking care to add with the product 'the carry'. (In the question, 3×7 gives a carry 2, and $3 \times 0 + 2 = 2$). Put down **2**.

(iv) Continue the multiplication with all the figures in the multiplicand to the left of the marked figure:

Thus $3 \times 3 = 9$ Put down **9**.
$3 \times 7 = 21$,, ,, **1** and carry 2.
$3 \times 0 + 2 = 2$,, **2**

(v) Strike off 3 in multiplier and 0 (multd.) Take the next figure in the multiplier and that is 4, and the next partial prouct will be that which arises from the multiplication of 4 with the remaining figures to the left of that figure (in the multiplicand) just struck off. Take 'the carry' as before:

$4 \times 0 = 0$, carry nil.
$4 \times 3 = 12$; Put down **2** just beneath the last figure in the first partial product (so that all the partial products will be in a line on the right-hand side) carry **1**.
$4 \times 7 + 1 = 29$ Put down **9**; Carry 2
$4 \times 0 + 2 = 2$,, ,, **2**

(vi) Strike off 4 in the multiplier and the 3 in the multiplicand. Begin with 6 multr. and proceed as before and remember 'the carry.'

$6 \times 3 = 18$ Carry **2**
$6 \times 7 + 2 = 44$ Put down 4 Carry 4.
$6 \times 0 + 4 = 4$,, ,, **4**

(vii) Strike off 6 in the multiplier and 7 in the multiplicand:

$7 \times 7 = 49$ Carry 5
$7 \times 0 + 5 = 5$ Put down **5**

(viii) Add the four partial products and the result is 2533. We have taken four places in the decimal part of the multiplicand, and the answer is therefore ·2533. Correcting the 3rd figure (since the answer is required only to 3 decimal places) we have ·253, which is the answer required.

(iii) Multiply ·076432 by $73\cdot3\dot{4}$ and give the answer correct to 4 decimal places.

$$\cdot076432 \times 73\cdot3\dot{4} = \cdot76432 \times 7\cdot33\dot{4}.$$

$$\cdot\cancel{7}\cancel{6}\cancel{4}\cancel{3}\overset{*}{\dot{\cancel{2}}}0$$

$$\cancel{7}\cancel{3}\cancel{3}\cancel{4}\cancel{4}44 \quad \ldots \quad \ldots$$

535024	... ...	7 × 0	Carry	Nil
22930	... ...	3 × 2 = 6	Carry	1
2293	... ...	3 × 3 = 9	Carry	1
306	... ...	4 × 4 = 16	Carry	2
30	... ...	4 × 6 = 24	Carry	2
3	... ...	4 × 7 = 28	Carry	3*
560586		5·6059	Answer.	

*NOTE.—*If the figures in the multiplicand are exhausted before those in the mdltiplier, the last partial product will be 'the carry' alone.*

(iv) Multiply $33\cdot\dot{7}\dot{6}$ by 73·046 and give the answer correct to 3 decimal places.

$$33\cdot\dot{7}\dot{6} \times 73\cdot046 = 337\cdot67676767 \times 7\cdot3046\ldots$$

$$33\cancel{7}\cdot\cancel{6}\cancel{7}\cancel{6}\overset{*}{\dot{\cancel{7}}}676$$

$$\cancel{7}\cancel{3}\cancel{0}\cancel{4}\cancel{6}$$

23637373	7 × 6 = 42, carry 4
1013030	3 × 7 = 21, ,, 2
	Multiplication by 0 will give 0.
13507	∴ strike off 0 and 7
2026	4 × 7 = 28, carry 3
	6 × 6 = 36, carry 4
24665936	

2466·594 is the answer.

9. (i) Divide:—·730764 by ·34687 and give the answer correct to 2 decimal places,

$$·730764 \div ·34687 = 7·30764 \div 3·4687.$$

(Our purpose is to throw the divisor into the *standard form.* It is evident that there will be one digit in the integral part, so that we want the answer to four figures, *viz.*)

integer	1
decimal places	2
1 place for correction	1
	4

Take 4 places in the divisor and as many places in the dividend as are necessary for one division.

$3\cancel{4}\cancel{6}\cancel{8}/7$) 7308 (2107	7308 contains 3468 nearly 2 times.
6937	2×7 carry 1. Strike off 8, the last figure in the Divr.)
381	371 contains 346 once, 1×9, carry 1 (Strike off 6.)
347	
24	24 is less than 34. Put 0 in the quot. (Strike off 4.)
24	24 contains 3, 8 times; but there is the carry. ∴ take 7 times. 7×4=28, carry 3.

From our estimate we should have one integer.

∴ 2·107 is the answer.

2·11 is the corrected answer,—because only two decimal places are required.

Note.—In the ordinary process of division as each partial division is done, we bring one figure from the dividend; but here, instead of doing so, we strike off one figure in the divisor. Further, if after bringing one figure from the dividend the divisor happens to be greater, we put zero in the quotient and bring another figure. Similarly, here if, after striking a figure in the divisor, the divisor happens to be greater, we put a zero in the quotient and *strike* another figure.

(ii) Divide 11·076348 by 231·0462 and give the answer correct to 3 decimal places.

$$11{\cdot}076348 \div 231{\cdot}0462 = {\cdot}11076348 \div 2{\cdot}310462.$$

We put W to show the number of digits in the integral part of the answer, and D decimal places required, and C the place required for correction.

W. D. C.

Nil + 3 + 1 = 4 places.

The answer will begin with a zero. ∴ it is sufficient, if 3 places are worked out.

$$\begin{array}{rl}
2\cancel{3}\cancel{1}/0462\;)\,1108\,(\;479 & \\
924 & \\
\hline
184 & \\
162 & \\
\hline
22 & \\
21 & \\
\hline
\end{array}$$

0·048 is the answer

(iii) Simplify:

$$\frac{73{\cdot}2762 \times 14{\cdot}7432}{29{\cdot}7631}$$

oorrect to 2 decimal places.

Here we have combined multiplication and division. Convert each operating factor into the standard form. Note carefully whether the compensating changes are made.

$$\text{The given fraction} = \frac{73{\cdot}2762 \times 1{\cdot}47432}{2{\cdot}97631}$$

$$\text{Rough estimate } \frac{73 \times 1{\cdot}5}{3} = \frac{109{\cdot}5}{3}$$

W. D. C.
2 2 1

We should have five places in the answer.

∴ there should be five places in the divisor.

∴ there should be five or six places in the dividend.

∴ the numerator should have 6 places,

the integral part of the numerator will contain 3 digits,

∴ The multiplication should be done correct to 3 decimal places.

(Note the argument carefully).

The work should be done thus :

$$\begin{array}{r}
\cancel{7}\cancel{3}{\cdot}\cancel{2}\cancel{7}\cancel{6}\cancel{2} \\
\cancel{1}{\cdot}\cancel{4}\cancel{7}\cancel{4}\cancel{3}\cancel{2} \\
\hline
732762 \\
293105 \\
51293 \\
2931 \\
220 \\
15 \\
\hline
1080326 \\
\hline
\end{array}
\qquad \underline{108{\cdot}033}\text{, when corrected}$$

$$\begin{array}{rl}
2\cancel{9}\cancel{7}\cancel{6}\cancel{3}/1\ )\ 108033 & (\ 36298 \\
89289 & \\
\hline
18744 & \\
17858 & \\
\hline
886 & \\
595 & \\
\hline
291 & \\
267 & \\
\hline
24 & \\
23 & \\
\hline
\end{array}$$

36·30 is the answer.

10. (i) The time of the swing of a pendulum may be calculated from the formula

$$t = \pi\sqrt{\frac{l}{g}}$$

where t is the time of swing in seconds, l the length of the pendulum in feet, $\pi = 3{\cdot}14159$, and g is a number which is the same for all pendulums. If the length of a pendulum which swings once a second is 39·1386 inches, find g correct to 4 significant figures.

From the formula $t = \pi \sqrt{\dfrac{l}{g}}$

$$\text{or } \frac{t}{\pi} = \sqrt{\frac{l}{g}}$$

$$\text{or } \frac{t^2}{\pi^2} = \frac{l}{g}$$

$$\text{or } g = \frac{\pi^2 l}{t^2}$$

Substituting the value.

$$g = \frac{3{\cdot}14159 \times 3{\cdot}14159 \times 39{\cdot}1386}{12} \text{ feet.}$$

(since $t^2 = 1$ from the value given, also dividing by 12, to have the answer in *feet*.)

$$\therefore\ g = 3{\cdot}14159 \times 3{\cdot}14159 \times 3{\cdot}26155.$$

Rough Estimate:—$3 \times 3 \times 3 = 27$. We want two decimal places in the answer. Perform the first multiplication correct to 3 decimal places. The work is shown below.

$$\begin{array}{rl}
\cancel{3}{\cdot}\cancel{1}\cancel{4}\cancel{1}\cancel{5}9 & \\
\cancel{3}\cancel{1}\cancel{4}\cancel{1}\cancel{5}9 & \\
\hline
94248 & \\
3142 & \\
1256 & \\
31 & \\
16 & \\
3 & \\
\hline
98696 & \quad 9{\cdot}870 \text{ when corrected.} \\
\cancel{9}{\cdot}\cancel{8}\cancel{7}\cancel{0} & \\
\cancel{3}{\cdot}\cancel{2}\cancel{6}\cancel{1}55 & \\
\hline
29610 & \\
1974 & \\
592 & \\
10 & \\
5 & \\
\hline
32{\cdot}19\cancel{1} & \quad \textit{Ans.}
\end{array}$$

(ii) Divide ·730461 by ·38·7324 correct to 3 significant figures.

·730461 ÷ 38·7324 = ·0730461 ÷ 3·87324. The answer consists of decimal portion only and the decimal part begins with zero. Therefore the answer is required to four decimal places. Now proceed.

11. Decimalisation of money:

English Money; Reduce as the decimal of a £,

(*a*) £ 73-8*s*.-4½*d*.

Multiply the number of shillings by 5 and prefix the decimal point - - ·40

Reduce the pence to farthings and mark off three decimal places $\cdot018\frac{18}{24}$

Add together $\cdot418\frac{3}{4}$

∴ £ 73-8-4½d. = £ 73·41875 = 73·419.

If the decimal part be correct to 3 places it will be true to the nearest pence.

(*b*) £ 81-7-11½.

= £ 81·398 to the nearest pence.

7 s. = £ ·35

11½ d. = 46 f. = $\cdot46\,\frac{4\cancel{6}23}{\cancel{2}\cancel{4}12}$

= ·046
19...

= ·048
Adding ·35

·398

Indian Money.

The following equivalents should be remembered.

8 annas = ·5 of a Re.
4 ,, = ·25 ,,
2 ,, = ·125 ,,
1 ,, = ·0625 ,,

Decimalize Rs. 37-7-3.

4 annas = ·25 of a Re.
2 „ = ·125 „
1 „ = ·0625 „
3 pies = ·015$\frac{15}{24}$ = ·015625

Multiply the number of pies by 5 and mark off 3 [decimal places

Adding, we have 37·453125.

If the answer be written correct to 3 decimal places, the answer will be correct to the nearest pie.

Ex. Find to the *nearest pie* the value of

$$\text{Rs.}\,\frac{73\cdot6478\times35\cdot6731}{441\cdot846}=\frac{7\cdot36478\times3\cdot56731}{4\cdot41846}$$

R. E. $\frac{7\times4}{4}=7$

W. D. C.
1 3 1 = 5.

Argument:—We want 5 places in the answer. We should have five places in the divisor. We should have 6 places in the dividend. We should work the numerator correct to 4 decimal places. The whole working stands thus:—

$$\begin{array}{r}
\cancel{7}\cancel{3}\cancel{6}\cancel{4}\overset{*}{\cancel{7}}\cancel{8} \\
\cancel{3}\cancel{5}\cancel{6}\cancel{7}\cancel{3}\cancel{1} \\
\hline
2209434 \\
368239 \\
44188 \\
5155 \\
221 \\
7 \\
\hline
\end{array}$$

$$\begin{array}{r}
\cancel{4}\cancel{4}\cancel{1}\cancel{8}\cancel{4}\cancel{6}/6)\;2627244\;(59460 \\
220923 \\
\hline
41801 \\
39766 \\
\hline
2035 \\
1767 \\
\hline
268 \\
265 \\
\hline
3
\end{array}$$

Rs. 5·946 = Rs. 5-15-2 nearly

12. Here is an extract from Indian Railway Statistics

Year.	Miles of State Railways.	Gross earnings in Rupees.
1903	21,131	36,00,82,035
1912	25,125	61,65,07,005

Calculate—

(1) The earning capacity of the Railways per mile in the two cases, corect to the nearest rupee.

(2) The percentage of increase in that capacity for the period 1903-12, correct to the first decimal place.

(S.S.L.C. 1922.)

(1) In the first case the earning capacity per mile is Rs. $\dfrac{36,00,82,035}{21,131}$

Performing contracted division by taking six places in the divisor; the resulting quotient is 17040.4

```
211310) 36,00,82 (170404
        148772
           855
            10
```

∴ The earning capacity in the first case is Rs. 17040 per mile.

Similarly in the second case the earning capacity is Rs. 24538 per mile.

```
251250) 616507 (245375
        114007
         13507
           944
           190
            14
             1
```

(2) The percentage of increase in that capacity is

$$\frac{(24538-17040)100}{17040}$$

$$=\frac{749800}{17040}=\frac{74980}{1704}$$

$=44{\cdot}0$ correct to the first decimal place.

```
1704) 7498 (44
       682
         0
```

13. **The Arithmetical square root—contracted method.**

If the student is asked to find the square root of any number to $2n+1$ figures, if $(n+1)$ figures have been obtained by the ordinary method, the remaining n figures may be obtained by contracted division.

Find the square root of $1767{\cdot}\dot{3}42\dot{3}$ correct to 4 decimal places.

Here we have to find 7 places, the seventh for correcting the sixth.

$$\begin{array}{r|l|l}
 & 17673423423423\text{ etc.} & 42{\cdot}03977 \\
 & 16 & \\ \cline{2-2}
82 & 167 & \\
 & 164 & \\ \cline{2-2}
8\cancel{4}\cancel{0}\cancel{3} & 33423 & \\
 & 25209 & 42{\cdot}0398\text{ Answer.} \\
 & 8214 & \\
 & 7563 & \\
 & 651 & \\
 & 588 & \\
 & 63 & \\
 & 59 &
\end{array}$$

The first four figures are obtained by the ordinary method for the extraction of sq. root and the remaining three by plain contracted division.

EXAMPLES XX.

N.B.—As considerble practice alone gives facility in working problems on contracted methods of multiplication and division, the teacher must begin this portion of the subject in Form IV, where direct questions may be proposed, more involved questions being reserved for Forms V & VI.

Questions marked with an asterisk may be done in Form IV.

Contracted methods.

1. The population of Great Britain and Ireland may be taken as 42,767,530. From these 615 members are to be elected for the House of Commons. How many members must be elected by a country whose population is 671,473?

*2. The population of India in the year 1921 was 318,942,480 and the following remarks are made:—

i. The males outnumbered the females in the proportion of 1000 : 945.

ii. There were 26,834,000 widows in India.

What percentage of the female population do the widows form?

*3. (a) The Fifth General Census of India was taken on the 18th March 1921. The final census returns gave a population of 318,942,480 as compared with 315,156,396 in 1911. Find the increase per cent.

(b) Of the total population 247,003,298 were in Br. India and the rest in Native states etc. What per cent. of the total population lived in Br. India?

*4. The area of the Pudukotah State is 1179 sq. miles and the total population is 426,813. How many on an average are there in each sq. mile?

*5. The output of various metals in 1921 and the output of the Br. Empire alone are given below in tons:—

Metal.	**Output of the World.**	**Output of the Br. Empire**
Iron ore	73,000,000	5,752,000
Pig Iron	36,500,000	3,957,000
Steel ignots and castings	38,700,000	4,719,000
Copper ore (metal)	537,000	46,000
Lead ore (metal)	869,000	199,000

What percentage of the total output of each variety is from the Br. Empire?

*6. In 1922, the Union of South Africa exported 7,009,859 oz. of gold valued at £ 29,775,984. Find to the nearest tenth of a pence the value of an oz. of gold.

*7. The United States Geological Survey has estimated the total output of coal for the world in 1922 as follows.

Country.	Produce in metric tons
United States	417,000,000
Germany	278,000,000
Great Britain	256,000,000
Minor Countries	95,000,000

Find for each country the percentage of the World's Production correct to one decimal place

8. A sovereign may not weigh more than 123·474 grains and has to be withdrawn from circulation when it weighs less than 122·5 grains. A heap of sovereigns weighs 12 ℔. Troy to the nearest grain. State the greatest and least possible number in the heap. A pound Troy contains 5760 grains.

9. A box containing only rupees weighs 20 ℔s. The weight of the box alone is a quarter of a quarter. If a rupee may weigh from 180 to 178·7 grains (Troy) find the greatest and least possible number of rupees in the box.

*__10.__ If the assessable value of the country of London is £ 40,142,274 find what rate per pound must be levied in order to raise £ 2,500,000.

Also, by how much must the assessable value increase that an increase in the rate of 1d. per pound should add £ 20,000 to the amount raised?

*__11.__ Given that 1 metre = 39·371 inches, and 1 kilogramme = 2·2046 ℔. and 1 cub. metre of water weighs 1000 kilograms, find the weight of a cub. ft. of water in ℔. correct to the nearest tenth of an ounce.

*__12.__ A cub. ft. of water weighs approximately 62½ ℔. A litre of water weighs a Kilogram. An inch = 2·54 cms. From these data calculate to one decimal place the number of grams in 1 ℔.

*__13.__ A sovereign weighs 123·27 grains and an American gold dollar 33·2199 grains. Find the value, in British money to the nearest penny, of an American dollar.

14. A man buys 500 yds. of copper wire at 2d. a yard and sells it in France at 30 centimes per metre. Find his profit in English money correct to a penny, having given £ 1 = 25·34 francs and 1 yd. = ·914 metre.

15. Lace which can be bought in Caen at 7·63 francs per metre incurs an additional value of 65% by transit to England. How much will it cost in England per yard? Given 1 metre = 39·3 in. and 55 francs = 43 shillings.

*__16.__ Find to the nearest franc, the value of Rs. 10,000; when 1 rupee = 1s. 4d. and £ 1 = 25·24 francs.

17. If gold can be beaten out so thin that a grain will form a leaf of 56 sq. in., how many of these leaves will be required to make up the thickness of a sheet of paper, the weight of a cub. ft. of gold being 14,580 oz. and 400 sheets of paper being 1 in. thick ?

***18.** Find to the nearest pie the value of

$$\text{Rs. } \frac{21{\cdot}407 \times 15{\cdot}231}{225{\cdot}456}$$

***19.** If $x \times 108{\cdot}432 = 6008{\cdot}125 \times {\cdot}34751$, find the value of x correct to three significant figures.

20. 355·27 oz. Troy is the weight of 35 cub. in. of gold of which the S. G. is 19·258. Find the weight of 49 cub. in. of silver, of which the S. G. is 10·474 (to 3 places of decimals).

21. A metre is equal to 39·37 inches and a cub. decimetre of water weighs a Kilogramme. If a cub. in. of water weighs 252·45 grains, find correct to two decimal places, the number of pounds avoir. in a Kilogramme.

22. Given that 1 cub. ft. of water weighs 62½ lbs. and that 1 ft. equals 30·48 centimetres, express a pound as a decimal of a Kilogramme, correct to 2 decimal places.

23. "The opening of the Otira tunnel, in August 1923 completed the railway connection between the east and west coasts of the South Island of New Zealand. Its whole length is 5⅓ miles. It has a gradient of 1 in 33 and it has cost £ 1,000,000."

Find from these data,

i. The height of one end of the tunnel above the other.

ii. The cost to the nearest pence of a furlong of the tunnel.

***24.** The London and N. Western Company owns a railway line of 2708 miles laid at a cost of £ 197,905,293. The income in the year ending Dec. 1922 was £ 45,496,005 and the working expenses came to £ 36,517,287. Find i. the cost of laying each mile of the railway, ii. the income per mile, iii. the percentage of working expenses to the income correct to one decimal place.

*25. The fastest running, without stoppage, is made by the companies as under:—

Great Western	77¼ miles	1 hr. 15 min.
London Midland and Scottish	72 ,,	1 hr. 17 ,,
London and North Eastern	44⅛ ,,	43 ,,
Southern	26½ ,,	28 ,,

Find the rate of running in miles per hr., to the nearest tenth of a mile and arrange them in order of rapidity.

*26. The total earnings of all lines of Indian Railways were Rs. 92·89 crores in 1921–22 as compared with Rs. 91·98 crores in 1920–21. Find the increase per cent. The working expenses were 76·22 per cent. of gross earnings in the year 1921–22. Find the net earnings.

*27. The circumference of the earth is 40,000,000 metres and the length of a metre is 39·3708 in. If the ratio of the circumference to the diameter be 355 : 113 find the diameter of the earth in miles.

*28. A yard measure is too long by 0·35 of an inch. If it is used to measure the sides of a rectangle, find by how much per cent. the true area differs from the calculated area.

29. The returns for land in Madras are:—

(1) Forests ... ... 12,923,395 acres.
(2) Lands not available for cultivation ... ... 13,556,677 acres.
(3) Waste land suitable for cultivation ... ... 6,468,312 acres.
(4) Current fallow ... ... 7,961,143 acres.
(5) Net area under crops ... 25,525,978 acres.

Without using unnecessary figures, calculate these as percentages of the whole, correct to one decimal place.

*__30.__ The following table shows the distribution of the Indian private sea-borne trade in 1922—23. Rupee figures converted at Rs. 15 = £ 1. The values of imports are given in thousands of pounds.

Countries.	Imports. £ 155,060	
United Kingdom	60·1	per cent.
Belgium	2·7	„
China	1·2	„
Germany	5·1	„
Japan	6·3	„
Java and Borneo	5·8	„
United States	5·6	„
All other Countries	?	„

Find, in rupees, the imports from each of the countries given. Give the answer to the nearest thousand rupees.

*__31.__ The planet Mercury describes its orbit round the sun in 87·96926 days; the Earth in 365·2564 days; Neptune in 60181·11 days. Find to two significant figures, the number of our years in (1) Mercury's year, (2) Neptune's year.

*__32.__ (1) A metric ton = 1000 kg. If 1 kg. = 2·2046 ℔. find in pounds, the difference between a metric ton and an English ton.

(2) If $x \times 108{\cdot}432 = 6008{\cdot}124 \times {\cdot}34751$, find correct to three significant figures, the value of x.

*__33.__ (a) The velocity of light is given as "$2{\cdot}9986 \times 10^{10}$cm. per second or 186,330 miles per second." From these data find, correct to 4 significant figures, how many inches go to a metre.

(b) "To convert sq. in to sq. centimetres multiply by 6·451." Verify the accuracy of this rule.

*__34.__ From the following table find (to the nearest shilling) the value per cwt. of coffee in 1903, 1904 and 1905. Find also the percentage falling off in 1905 in quantity and in total value as compared with 1904, each percentage to the nearest integer and reckoned on the 1904 numbers.

Year.	Quantity imported in cwts.	Value in £.
1903	1,143,526	3,134,924
1904	1,055,866	3,329,598
1905	956,266	2,707,169

*35. (a) Taing $\pi = 3\frac{1}{7}$, show that

$$\frac{1}{\pi} = \frac{1}{4} + \frac{1}{16} + \frac{1}{176}$$

Use this to divide 57·6 by π correct to two places of decimals.

(b) If π hours = 3 hrs 8 minutes 30 seconds, find the value of π as (i) a decimal fraction (to four decimal places). (ii) as a vulgar fraction.

*36. A shilling weighs 127·27272 grains and contains 92·5% of pure silver; one-rupee weight of silver is worth only 11 as. 6 ps. If a rupee weighs 180 grains, find the value of the silver in the shilling to the nearest pie.

*37. Find from the following table which variety produces the most butter per gallon of milk:

VARIETY.	MILK USED.	BUTTER PRODUCED.
A	132 gals.	$22\frac{4}{5}$ lb,
B	794½ gals.	139 lb.
C	61 gals.	12 lb.
D	632 gals.	112 lb.
E	118 gals.	20¼ lb.

*38. The contribution to the second war loan ending fourth June 1918, from the various important divisions of India are given below; Calculate what percentage (correct to one decimal place) of the total, each division has contributed and arrange them in descending order of magnitude.

Bengal	Rs.	3,67,08,500
Bombay	,,	1,82,15,500
Madras	,,	86,23,500
Punjab	,,	34,85,000
Burma	,,	26,32,500
Other Provinces ...	,,	37,58,300

*39. The approximate gross earnings of the Mysore State Railways for the week ending 18th May 1918 are given below:

Mysore—Arisikere Railway	Rs. 7753 or Rs. 75-4-11	per mile.
Kolar—Bangalore Railway	Rs. 7055 or Rs. 69-0-6	,,
Tarikare-Narasimharaja pur Railway (Tram)	Rs. 417 or Rs. 18-4-11	,,

Find to the nearest mile the number of miles of each railway.

*40. In calculating the safe eccentric load on a column the following expression was arrived at:—

$$1+\frac{\dfrac{64}{3{\cdot}25\times5}}{2{\cdot}82\times2{\cdot}87}\text{ tons:}$$

Complete the calculation to the nearest hundredth of a ton.

*41. "In the year 1906, the United States had 36,212 miles of railway and the total capital invested was 15,804,087,423 dollars. The gross earnings were 2,346,640,286 dollars and the net earnings 790,187,712 dollars."

Find from these data;

(i) The earnings per mile of railway correct to the nearest hundred dollars.

(ii) The expenses ,, ,, ,,

(iii) The percentage rate of interest earned by the railway correct to three significant figures.

*42. The total working expenses on all Indian Railways amounted in 1906 to Rs. 22,00,74,000 or 49·88 per cent of the gross earnings as compared with Rs. 19,94,00,000 or 47·84 per cent in 1905. Find by what percentage of the gross earnings of 1905, the gross earnings increased in the year 1906, correct to four significant figures.

*43. Prof. Less-Smith takes the statistics of Indian Reports 1906—1907 and thus analyses them:—

Total imports of private merchandise into Br. India	£ 72,205,000
Imports from the United Kingdom	£ 48,199,000
" " British Colonies	£ 5,558,000
" " foreign countries	£ 18,448,000

Express each of the above as a percentage of the total imports of private merchandise into Br. India.

44. Writing on the Problem of High Prices Prof. Kale remarks:—"Cocoanut kernel or copra as it is called, has nowadays become dear. Some 148 thousand cwt. of this article were exported in 1902—03 and fetched nearly 122 thousand pounds while 1906—07, a smaller quantity, *viz.*, 126 thousand cwts. brought an equal number of pounds."

Taking Rs. 15 as the value of £ 1 find to four significant figures the price in Rupees of 1 ℔. of copra in each of the periods and hence find the percentage increase in the price in the latter period.

45. An extract from the D. C. B. Statement of the Forests Department of a State for Fasli 1332 is given below.

ITEMS.	DEMAND. RS. AS. PS.	COLLECTION. RS. AS. PS.	BALANCE. RS. AS. PS.
Reserve Forests.	21125-1-11	20762-13-11	
Plantations.	17628-5- 4	17480- 9-10	
Quarries.	31430-0- 0	31000- 0- 0	
Miscellaneous.	3816-6- 6	3816- 6- 6	

Find in each item (*a*) the balance (*b*) the percentage of the collection to the demand, (*c*) the percentage of the total of collection to the total of the demand.

46. The following extract from Hazell's Annual gives the distances of a few stations from London and the time taken by the fastest trains.

Time.	Miles.	Journey Time. H.	Journey Time. M.
Aberdeen	539½	11	15
Birmingham	113	2	0
Bradford	192½	3	40
Carslisle	299¼	5	50
Edinburgh	392½	7	45

Find the speed of the trains in miles per hour each result correct to the tenth of a mile.

47. When as a consequence of the German defeat exchange stood at 38,500 marks per £ a gentleman in India speculated and bought marks for 125 thousands of rupees. Exchange between England and India was at 1 s. $4\frac{5}{16}$ d. to the rupee. Again when exchange was at 82,000 marks per £ and there was a fear of a worse state of affairs, he reconverted his marks into Rupees, the rate of exchange then being 1 s. 4 d. per rupee. Find his gain or loss to the nearest rupee, supposing that he had to spend Rs. 14,500 extra towards discount, postage, correspondence, loss of interest etc.

48. "The most prevelent religion in India is that of the Hindus; their number being nearly three fourths of the total population. Together with the Muhammadans who number 62,458,077 they comprise nearly 92 % of the whole population." There are 9,476,759 Buddhists.

Find from the statements above (1) the total population of India to the nearest million. (2) The percentage of Buddhists to this population.

49. Extract from the address delivered by Sir M. Visvesvaraya. at the VII Indian Economic conference.

"The wealth of India before the war was estimated at Rs. 5,400 crores. This meant an average property or wealth amounting to Rs. 180 per head of population; on account of the general low level of occupations and low standard of living the death rate in

India is very high being over 30 per thousand. Education is extremely meagre, less than 6% of the population being literate."

From these data find correct to three significant figures :—

(i) the population of India (ii) the number of deaths per year (iii) the strength of the literate population.

*50. "There were in India at the end of the year 1921—22, 47,879 co-operative societies with a membership of 2,983,534 and a working capital of Rs. 28,98,32,000 and they advanced loans to the extent of Rs. 16,11,49,710" Find (1) the number of members on an average in each society correct to the nearest integer, (2) the average working capital of each society to the nearest anna (3) the amount of loans advanced by each society to the nearest anna and (4) the average loan raised by each member to the nearest pie.

51. Extract from the introduction to the 18th Edition of Pears' Cyclopaedia :—With the present edition the total issue of Pears' Cyclopaedia is brought up to 1,200,000 copies representing a popularity exceeding all previous records...................... .... This single volumne consists of 1070 pages. The 1,200,000 copies weigh over 840 tons, and would require more than 37 miles of book-shelving to accomodate them. Taking the pages themselves and laying them down singly, end to end, we should get a length of 69,350¾ miles, or about three times round the world" Find from the figures given above.

i. The length to the nearest tenth of an inch of a page of the book.

ii. The weight in ozs. of a copy to the nearest hundredth of an oz.

iii. The thickness of a copy to the nearest hundredth of an inch.

iv. The diameter of the earth to the nearest hundred miles.

52. In the year 1907 the average share, per head of population of the United Kingdom, in the imports and the exports of British produce were £ 14-14 s. and £ 9-14 s. respectively. Given that the total imports amounted to £ 645,904,176; find (i) the population of the United kingdom and (ii) the total value of the exports in that year, each result correct to the nearest hundred.

53. Standard gold is formed by mixing 11 parts of fine gold with one part of copper; how many sovereigns will be coined from 4 ounces Avoir of fine gold, when 1 ℔ Troy of standard gold is coined into 46·725 soverigns.

54. A certain reef of quartz when crushed yields 0·0011% of gold. If the working expenses amount to 62·5% of the gross receipts and the profit on each 100 tons crushed amounts to £ 52–10 s. find the number of grains of pure gold in a sovereign.

55. If the interest on the national debt is 18 s. 6 d. per second, how much is that for a year of 365 days?

Supposing that Government pays on an average interest at 3½% per annum, find the amount of the debt to the nearest hundred-thousand.

56. (i) The Hanoverian mile is 25400 feet long, each foot beeing ·9542 English foot. Find to 4 places of decimals the fraction that an English mile is of a Hanoverian mile.

(ii) A square courtyard costs £ 38–10–5 d. to pave at 3 s. 9 d. per sq. yd. Find the length of its side.

57. From the observation of the eclipses of Jupiter's satellites, it is found that the time taken by light to travel from the Sun to the Earth is 499 seconds (with a possible error of 2 sec. either way). If the velocity of light is 186,330 miles per second, find the distance from the Sun to the Earth, and the possible error in your result. (It will be sufficient to give your result and the possible error to the nearest hundred thousand miles).

58. If the third satellite cf Jupiter be ·0000885 of the planet and ·02947 of the mass of the earth, find the mass of the earth in terms of the mass of Jupiter.

59. Express a horse power in Kilogram metres. A horse-power is 550 foot-pounds per second. 1 metre = 39·37 inches. 1 Kilogram = 2·2 lbs. The work of raising 1 pound through 1 foot is called a foot-pound and that of raising a kilogram through 1 metre, a kilogram metre.

60. Sugar being composed of 49·856 per cent of oxygen, 33·265 per cent of carbon, and the remainder of hydrogen: find how many pounds of each of these materials there are in one ton of sugar.

61. (a) The thickness of a book excluding the covers is an exact inch. How many pages of the same book will be a centimetre thick, if the whole book contains 454 pages; given that 1 metre = 39·37 inches?

(b) Find also the thickness of a sheet of which the book is made, correct to the nearest hundred-thousandth of an inch.

62. The following rule was used to convert tons to kilograms. Multiply the weight in tons by 1000 and add one-fiftieth of the product to the result. Find to the nearest kilogram the error in thus converting a weight of 58½ tons; a ton is 1016·05 kg.

63. I wish to ascertain how many sheets of paper there are in a bundle weighing 120 lbs. I find that 20 sheets weigh between ·73 and ·74 lb. Find the maximum and the minimum number of sheets in the bundle.

64. Taking the area of the United Kingdom 121,481 square miles as the unit, express in terms of this unit to 3 places of decimals the areas of the following countries—Portugal, 35,490 sq. miles, India, 1766,642 sq. miles.

65. (a) If one foot = ·3047945 metre, express the height of Mount Everest 29,002 ft. in metres to the nearest metre.

(b) If one can travel 264 kilometres by rail in France for 39 francs, find the fare in English money (to the nearest penny) for a distance of 77 miles at the same rate, supposing that 1 km. = 1092 yards and 25 francs = £1.

***66.** The subjoined table gives the total amount of the actual revenue and expenditure of India, distinguishing Indian and home expenditure, in the financial year ending March 31st 1906.

Revenue.	EXPENDITURE.		Total Expenditure
	In India.	In Great Britain.	
RS.	RS.	RS.	
1,09,99,19,355	78,92,79,579	27,92,61,975	

Find (i) the total expenditure and the balance.
(ii) what percentange of the total expenditure is the balance.
(iii) ,, ,, was incurred in (a) India (b) Great Britain.

67. " The total value of the silver, copper and bronze coined in British India from 1835—36 to 1906—07 inclusive, has been Rs. 5,17,23,35,522 including Rs. 35,39,12,584 the value of 155,971,088 British dollars, Rs. 8,02,68,091 the value of 35,374,555 straits dollars, Rs. 5,05,520 the value of 497,630 straits fifty cent. pieces etc., etc.

From these data find the value of a British dollar, of a straits dollar and of a straits fifty cent piece, to the nearest pie.

68. The weights in grammes of the gold coins in use in the various countries and their fineness are given in the subjoined table. If the value of the coin depends on the weight of gold in it, find how many coins of each country go to a sovereign.

England	Sovereign	7·9881	·916
Germany	20 mark piece	7·96495	·900
Russia	The Imperial	12·902	·900
France	20 franc piece	6·4516	·900
United States	Gold Dollar	1·6718	·900

Each answer must be correct to three significant figures.

69. The following table gives the periods of some of the planets about the sun. Given that a sidereal year is

365·256 days, express the periods in sidereal year correct to 4 significant figures :—

Mercury	...	87·969	days.
Venus	...	224·700	„
Mars	...	686·980	„
Jupiter	...	4332·585	„
Saturn	...	10759·220	„

70. The metre was originally determined so that forty million metres should be equal to the earth's circumference. A metre is 39·37 inches. Compute from these data the mean radius of the earth in miles, given that the circumference of a circle is to its diameter as 355 to 113.

EXAMPLES XXI.

SUBSTITUTION.

1. If $F = EI\pi^2 \div 4l^2$
and $I = bt^3 \div 12$ find F when $E = 3 \times 10^7$, $\pi = 3{\cdot}142$ $l = 62$, $b = 2$, $t = 0{\cdot}5$.

2. When x and y are small we take

$\dfrac{1+x}{1+y}$ as being very nearly equal to $1 + x - y$.

What is the error in this when $x = 0{\cdot}02$ and $y = 0{\cdot}03$?

3. The velocity v (measured in miles per hour) of a stream flowing in a straight channel of uniform breadth and depth is given by the formula :—

$\dfrac{1}{v^2} = a + \dfrac{b}{x}$, where the depth of the water is x inches. If $v = 2$, when $x = 4$ and $v = 3{\cdot}3$ when $x = 12$, find to one decimal place the value of v when $x = 8$.

4. The magnification m, produced by a lens of focal length, f cms., when the lens is held at a distance a cms. from the eye, the object being at a distance b cms. from the eye, is given by the formula,

$m = \dfrac{Df}{bf - a(b - a)}$ where D denotes distance of distinct vision.

Given that $m=6{\cdot}25$, $D=25$, $b=14{\cdot}5$, $a=4{\cdot}3$ find f, the focal length of the lens.

5. The weight W in pounds of a foot of iron piping is worked out from the relation $\mathrm{W}=\frac{\pi tw}{144}(b+t)$, where t denotes the thickness of the pipe in inches, w the weight of a cub foot of the iron, b the diameter of the bore in inches.

If $w=448$, $b=6$, $t=\frac{5}{8}$ find W.

If $\mathrm{W}=64{\cdot}3$, $w=441$, $t=\frac{3}{4}$ find b to the nearest tenth of an inch.

6. The volume of a segment h cm. high of a sphere r cm. radius is $\frac{\pi h}{3}(3rh-h^2)$ c. cm.

(*a*) How many litres of milk will a hemispherical silver cup of diameter 3·8″ contain ?

(*b*) If the milk level is at half the height, how many litres are there ?

7. The area of a circular ring is given by the formula $(D^2-d^2)\times 0{\cdot}7854$, where D and d are external and internal diameters respectively.

i. Find to the nearest tenth of a square foot the area of a path 14 feet wide, round a circular flower-bed 8·6 feet in diameter.

ii. Find the weight, to the nearest gramme, of a circular ring of metal cut from a plate weighing 6·7 grammes per sq. cm. the external and internal diameters of the ring being 7·3 cms. and 4·7 cms.

8. The energy stored in similar fly-wheels is given by $E=ad^5n^2$, where d is the diameter in feet, n is the speed in revolutions per minute and a is a constant. If when $d=4$, $n=60$, $E=18{,}250$, find a for that kind of fly-wheel.

9. A mixture of gold and silver weighs W gm. in air and w gm, in water. If S_1, S_2, is the specific gravity of gold and silver respectively the weight of gold in the mixture is,

$$\frac{W - S_2\,(W - w)}{S - \dfrac{S_2}{S_1}}\,\text{gm}$$

From this formula find the weight (correct to 4 significant figures) of gold in a mixture of gold and silver which weighs 160 gm. in air and 149·5 gm. in water. The specific gravity of gold is 19·36 and of silver 10·51·

10. The height H of a mountain in feet is given by

$$H = 49000\left(\frac{R - v}{R + v}\right)\left(1 + \frac{T + t}{900}\right)$$

where R, v are the observed heights of the barometer in inches at the bottom and top of the mountain and T, t, the observed temperatures at the bottom and top. The following observations are made at the bottom and top of the mountain.

	Barometer.	Thermometer.
At bottom ...	29·60 in	60°
At top ...	25·35 in	32°

Find the height of the mountain to the nearest foot.

11. A bar of copper is longer when hot than when cold; the length l at temperature t degree centigrade is given by the formula $l = a\,(1 + pt.)$ where a is the length of the bar at 0°C, and $p = 1{\cdot}708 \times 1/10^5$. By how much per cent does the length increase, if the bar is first put into freezing water at 0°C and then into boiling water at 100°C. Find also the increase in millimetres if $a = 20$ metres.

12. To prevent a train from leaving the rail as it passes round a curve, the outer rail is raised. The necessary elevation is determined by the formula,

$$\text{Elevation of outer rail in inches} = B \times \frac{V^2}{R \times 1{\cdot}25};$$

where B is the gauge of the line in *feet*, R is the radius of curve in *feet* and V is the speed of the train in *miles per hour*. The standard gauge being 4 feet 8⅓ in. find in tenths of an inch, the elevation of the outer rail when a train at 20 miles an hour goes round a curve of 425 feet radius.

13. The volume of a frustum of a cone is $\frac{1}{3}h(a + b + \sqrt{ab})$; h denoting the height of the frustum, a and b, the areas of the two ends. Calculate the volume to the nearst cub. in. when $h = 4{\cdot}4$ in., $a = 27{\cdot}5$ sq. in and $b = 70{\cdot}1$ sq. in.

14. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$; find f to 2 places, if $u = 34{\cdot}3$ and $v = 18{\cdot}9$.

SECTION VIII. (*A*).

Interest—Bankruptcy—Partnership.

1. **Rate per cent.**—A person who borrows money has generally to pay, in addition to the sum borrowed, a certain extra amount for the use of the money. This extra amount is called *Interest*. A person who lives in the house of another has to pay *rent* and interest is also the rent for the use of the money. At the time of borrowing, the interest that the borrower must pay is stipulated. It is generally fixed at so many rupees per annum on every hundred borrowed. This rate of interest per annum is called "rate per cent per annum." This rate per cent. varies considerably from 2 per cent. to even 300 or 400 per cent.

N.B.—The teacher will tell the students that the rate of interest demanded by the lender depends upon:—(i) the solvency of the borrower (ii) the value of the *mortgaged* property (iii) the nature of the money market and above all (iv) the mentality of the lender. Big sums are generally lent at a lower rate. Amounts below Rs. 1000 are lent at a higher rate. The greater the risk the lender takes, the greater the rate of interest. The greater the urgency of the borrower and the greater the necessity, the greater the rate of interest generally demanded. It is to counteract this avarice of the money-lenders that co-operative societies are being started everywhere. Incidentally the teacher will tell the student something on the various aims and objects of co-operative societies.

2. Unless otherwise stated, rate per cent, means rate per cent per annum.

The amount lent or borrowed is called the *Principal*.

" " given for the use of the money is called *Interest*.

The Principal and the interest are together called the *Amount*.

3. A person lends Rs. 4000 at 9 per cent per annum. What will the interest come to, at the end of two years?

Int. on Rs. 100 for 1 year is Rs. 9.
" 4000 or 40 hundreds for 1 year is Rs. 360.
" 4000 " 2 years is Rs. 720.

The interest Rs. 720 calculated above, is called *Simple Interest.*

But the student will note that the interest at the end of the 1st year is Rs. 360.

If this interest be paid to the lender, he may invest it elsewhere and get interest on the Rs. 360 also; so that he reasonably demands of the borrower, interest for the 2nd year on the principal, as well as on the interest portion. Interest is calculated on the *amount* at the end of each year, at the rate per cent agreed upon and this interest is called *Compound interest.*

Find the compound interest on Rs. 5000 for 2 years at 12 per cent per annum.

The work is generally exhibited as follows.

Rs. 5000	
600	Interest for 1 year.
5600	Amount at the end of the 1st year.
672	Interest for 2nd year.
	Amount to be paid at the end of 2 years.

The interest alone in this case is Rs. 1272; whereas if S. I. were calculated the interest would come to Rs. 1200 only. The interest also bears interest i.e. the interest is capitalised.

4. When accounts are settled between the borrower and the lender, it generally happens that interest has to be calculated for fractions of a month and for days also. If the amount lent be very large, interest for a day will be a few rupees which cannot be neglected. Complications in the calculation of interest arise when payments had been made off and on, not exactly at the end of each year. The calculation of interest in such cases is generally a tedious affair and this work is lightened by tabulated results called, 'Ready Reckoners.' (The teacher will produce a ready reckoner and tell boys, how to use it. Every office writer has a copy). The Indian system of ருசிக்கணக்கு must be

explained. Incidentally the Tamilian rate of intesest ஒருவட்டி, முக்கால்வட்டி, அரையே அரைக்கால் வட்டி, அரை வட்டி, etc., corresponding to 12, 9, 7½, and 6 per cent etc., should be explained. In some places we have such terms as 'ஒருகாசுவட்டி' which is 2 pies per rupee per month; 'அரைஅணாவட்டி' which is 6 pies per rupee per month.

5. *The third and tenth and tenth rule* for calculation of interest for a certain number of days.

The approximate division of a number by 73 can be effected by what is called tho *3rd-tenth-and-tenth rule.* The student may easily verify that

$\frac{1}{73} = \frac{1}{100}(1 + \frac{1}{3} + \frac{1}{30} + \frac{1}{300})$, very nearly. Hence to divide a number by 73, add to the number to be divided its $\frac{1}{3}$, $\frac{1}{10}$ of $\frac{1}{3}$ and $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{3}$ and divide by 100.

This rule is particularly useful in calculating interest on a given sum for a given number of days at a given rate % per annum.

Now if p be the principal, r the rate per cent. per annum and n the number of days for which interest is required, we have:

$$\text{Interest required} = \frac{n}{365} \times \frac{r}{100} \times p = \frac{nr}{36500}p$$

$= \frac{2nr}{73000}p$; i. e., multiply the principal by rate % and by twice the nuber of days and divide the product by 73000 to get the interest required.

Now

$$\frac{1}{73} = \frac{1}{100}\left(1 + \frac{1}{3} + \frac{1}{30} + \frac{1}{300}\right)\text{nearly}$$

$$\frac{1}{73000} = \frac{1}{100000}\left(1 + \frac{1}{3} + \frac{1}{30} + \frac{1}{300}\right)\text{nearly.}$$

Example

Find the interest on Rs. 40 for 51 days at 2½ per cent. per annum.

$$2nr = 2 \times 51 \times 2\tfrac{1}{2} = 255$$

$$p \times 2nr = 255 \times 40 = 10200$$

$$= \text{N (say)}.$$

$$\begin{array}{rcr} \text{N} & = & 10{,}200 \\ \frac{1}{3}\,\text{N} & = & 3{,}400 \\ \frac{1}{30}\,\text{N} & = & 340 \\ \frac{1}{300}\,\text{N} & = & 34 \\ \hline 100000 \,| & & 13974 \end{array}$$

∴ Interest = Rs. ·13974 = 2 as. 3 ps.

N.B.—The result will be more accurate if for every £ 10 in the interest we deduct ¼ d. and two pies for every Rs. 10 in the interest.

6. The following three formulae and their manipulations are often useful in the calculation of interest.

(i) $I = \frac{Pnr}{100}$; I = Interest, P = Principal,

n = No of years. r = rate per cent per annum (S,I.)

(ii) A = P + I; A = Amount

and because $I = \frac{Pnr}{100}$

$$A = P + \frac{Pnr}{100} = P \times \left(1 + \frac{nr}{100}\right) \ldots\ldots\ldots\ldots\ldots$$

(iii) $A = \left(1 + \frac{r}{100}\right)^n \times P \ldots\ldots\ldots\ldots$(C. I.)

7. If a person contracts debts recklessly, after a time he is unable to pay his debts, *or*, A merchant speculates and is ruined. He who is unable to pay off his debts is called an 'Insolvent.' The lenders file suits against him and the borrower puts in a petition and the Court decrees he is an 'Insolvent.' Receivers are appointed, who are empowered to collect all amounts due to the Insolvent (Assets) and divide the amount equally among the lenders. It always happens that the lenders receive only so many annas on each rupee lent. We take a concrete example.

A person has dues to the extent of say, Rs. 12,000.
His assets amount to Rs. 9000.
He can pay only 12 as. on each rupee.

The rate per rupee declared by the receiver is called the *Dividend*.

The amount due „ „ '*Liabilities*'

N.B.—The whole of the assets cannot be divided into the Liabilities as generally there are "*Goods debts* and *Bad debts*" and also the *expenses of Bankruptcy*. The receiver appointed by the court is to be paid for his work out of the assets of the insolvent. A merchant who fails in speculation is called a '*Bankrupt*'

8. The following points must be remembered in the calculation of Interest.

(i) 6¼ % per annum = 1 pie per rupee per month *inversely*.

(ii) 1½ pies per rupee per month = 9⅜ %
2 „ „ „ = 12½ % i.e.
annually ⅛ of the Principal ; and so on, for multiples.

(iii) 12 % is Re 1 per month per hundred.
9 % is 12 as „ „
7½ % is 10 as „ „
and so on.

EXAMPLES XXII.

To be done in Form IV.

Oral.

1. A person borrows Rs. 3500 for his daughter's marriage and agrees to pay back the Principal and interest at 12 % per annum at the end of a year from the date of borrowing. How much should he then pay?

2. From one person, a mirasdar borrows Rs. 1500 at 9 % per annum and from another Rs. 2500 at 7½ % per annum. How much will he have to pay to both at the end of a year?

3. I buy a house for Rs. 1800 and improve it at a cost of Rs. 400. What rent per month must I fix, to get a return of 6% on the capital invested?

4. A person borrows Rs. 4000 at 7½ % interest and immediately lends Rs. 2500 at 9 % and the balance at 12 %. How much per year will he gain?

5. A sum of money doubles itself in 8 years reckoning Simple interest. What is the rate per cent?

6. In how many years, calculating simple interest at 12½ % will a sum of money double itself?

7. Which is cheaper to the borrower: to pay an interest of 1 pie per rupee per month or to borrow at 6%? How much will he gain in a year by borrowing Rs. 1800 at the cheaper rate?

8. I borrowed a certain sum at 9% S. I. and had to pay Rs. 448 at the end of 1 year 4 months from the date of borrowing. Find the sum I borrowed?

9. A man pledged a jewel and borrowed Rs. 200. At the end of 3 years the jewel became the property of the lender who was entitled to a sum of Rs. 260. If simple interest were calculated, what was the rate of interest agreed upon?

10. A medal of the value of Rs. 30 to be awarded annually had to be instituted in memory of a famous teacher. How much money must be invested in the bank which allows 6% interest, that the interest alone might be utilised for the annual medal?

11. (*a*) The current rate of interest is 2 pies per Re. per month. How much is it per hundred per year?

(*b*) What rate per cent. per annum is,

(i) 3 ps. per Re. per month?
(ii) 0·75 pie „ „ week?
(iii) 9 pies per Rs. 100 per day?

(*c*) (i) Which of these three rates is the cheapest?
(ii) How much does that cheapest rate differ from 18 % per annum?

EXAMPLES XXIII.

N.B.—Compound Interest may with advantage be taught in Form IV. If the topic be reserved for Form V., *Qs.* marked with an asterisk may be done in Form IV.

The symbolical relation between P, I, n and r must be drilled in.

Written.

*__1.__ The interest on Rs. 100 for 1 year is Rs. r. Find the S.I. on Rs. P for n years. If I stands for the interest and A for the amount, express I and A in terms of P, n and r.

*__2.__ Find with the help of the above formulæ the interest and the amount on:—(a) Rs. 150 for 3 yrs. at 12%. (b) Rs. 275 for 2½ yrs. at 9%. (c) Rs. 420 for 1 yr. 5 mths. at 6¼%. (d) Rs. 666-10-8 for 9 mths. at 12½%. (e) Rs. 1250 for 20 mths. at 7½%.

*__3.__ From the formula obtained for I and A, find expressions for P, n and r in terms of the rest.

*__4.__ Use the expressions arrived at in the previous question to find out the following.:—

(a) the Rate per cent r, when (i) P = Rs. 240, $n = 2\frac{1}{2}$. and I = Rs. 48. (ii) P = Rs. 1250; $n = \frac{2}{3}$, A = Rs. 1312-8-0. (iii) $P=360$, $I=119$, $n=4\frac{1}{3}$.

(b) the Principal (P) when (i) I = Rs. 14-7-0, $n=3\frac{1}{2}$. $r=6$. (ii) A = Rs. 3750, $n = 5$ $r=5$. (iii) A = Rs. 731-8-0, $n=6$, $r=6\frac{1}{4}$.

(c) The time (n) when (i) $P = 1050$, $I=420$, $r=8$. (ii) P = Rs. 225, A = Rs. 256½, $r=3\frac{1}{2}$.

*__5.__ (a) A certain sum amounts at S.I. in 2 years to Rs. 240 and in 5 years to Rs. 300. Find the sum and the rate per cent.

(b) A certain sum, Rs. x amounts in m years to Rs. A and in n years to Rs. B. Express x in terms of the other quantities. Find them, when $m=2$, $n=3\frac{1}{2}$, $A=464$ and $B=512$.

*__6.__ A man borrows Rs. 500 for 9 months at 3% per annum and by immediately lending it out for the same period at a higher rate of interest makes a profit of Rs. 5—6—0. What rate of interest does he charge?

*__7.__ A chetty gets from a person a promissory note for Rs. 2,400 and deducting the interest for this amount for 3 years at 12%, simple interest pays him the balance. If at the end of 3 years the person pays back the sum borrowed, what rate of simple interest has the chettiar actually got?

*__8.__ A money lender gets from a bank a loan of Rs. 1600 at 8% per annum and lends it as follows:—Rs. 425 to *A* at 9%; Rs. 350 to *B* at 12%; Rs. 600 to *C* at 12½% and the rest at 15%. If at the end of 18 months he collects all the money and pays back the bank loan, find what net profit he would have made in the transaction. Calculate simple interest throughout.

*__9.__ A owes B, Rs. 5,000 payable at the end of a year and an equal sum payable at the end of 3 years. If the current rate of interest (supposed simple) be 3½ % per annum, what amount paid at the end of the second year will cancel both the debts. If *A* offers to give Rs. 10,000 at the end of the second year in payment of both the debts does *B* gain or lose by the offer, and by how much?

*__10.__ A sowcar lends Rs. 450 to a person at 8¾% S. I., per annum on the pledge of a jewel worth Rs. 750 on the understanding that if the loan be not cleared before it becomes equal to the worth of this jewel, the jewel will not be returned to him. How long should the sowcar wait to consider the jewel his own property?

*__11.__ A person saves every year a sum of Rs. 450 which he lends out forthwith at 9% per annum S. I. What will he be worth at the end 5 years?

*__12.__ I borrow Rs. 1,500 at 15% per annum and purchase a small motor bus to open service between 2 places. I get on an average a monthly income of Rs. 400. Estabishment charges come up to Rs. 75 a month and the cost of oil etc. to Rs. 150 a month. I deposit the balance every month in a bank which pays interest at 7½%. Supposing that no repairs occur in the middle to necessitate any withdrawal from the bank amount, what sum will stand to my credit at the end of 8 mths.? How much more should I add to pay off the sum borrowed?

13. The following is an advertisment in a newspaper. "Motor cars ready for sale. Rs. 1,500 cash or Rs. 250 cash and 14 monthly instalments of Rs. 100 each." What rate of interest does this bring to the company?

14. An officer in a Railway company gets on his retirement a bonus of Rs. 4,000 which he deposits in a Government bank giving an interest of 7% per annum S.I. Another officer of the same age and experience, and getting the same pay, is given a pension of Rs. 50 a month, which he sends forthwith to the same bank for the same rate of interest. Supposing both live for 10 years, who will have a greater sum to his credit in the Bank? By how much will that be greater? Sum of the first n natural numbers—$(1+2+3\ldots+n)=\frac{n\times(n+1)}{2}$

***15.** A loan of Rs. 250 is taken on personal surety from a certain Co-operative bank on the agreement that the sum should be returned in 10 monthly instalments of Rs. 25 each and that interest at 1¼ pies per Re. per month on the amount due, be paid with every instalment on or before the 15th of every month. If an instalment falls overdue, the bank would charge interest at 12% per annum on the sum overdue. The borrower pays 6 instalments regularly with interest and then stops payment. If at the end of the tenth month he is required to discharge the loan, how much will he have to pay to the bank?

***16.** I borrow Rs. 2,400 at 6% per annum S. I. and reserving Rs. 400 for my expenses lend the remainder at 12% per annum S I. When will the sum lent out with its interest be just sufficient to pay off my debt?

***17.** A person lends Rs. 300 to a customer at 10% per annum. What is the interest due from the latter at the end of the year? If he is unable to pay the interest and agrees to add the interest to the capital, how much more of interest should he pay next year? If interest is added like this to the capital every year, what will the customer owe to the person at the end of the 3rd year?

18. Fing the Compound Interest and Amount on:—

(i) Rs. 1350 at 9% for 2 years.

(ii) Rs. 2,800 at 7½% for 2 years.

(iii) Rs. 1800 at 9% for 2 years, interest payable half-yearly.

19. (a) If A denotes the amount at the end of n years reckoning compound interest express it in terms of P, n and r.

(b) i. Find r when P = Rs. 32000, $A = 34279{\cdot}2$ and $n = 2$.

ii. Find n when $P = 5000$, $A = 5832$ and $r = 8$.

iii. Find P when $A = 11576\frac{1}{4}$, $n = 3$ and $r = 5$.

20. Owing to the rise in the price of food stuffs during the war, the price of land during the war rose every year by 16% of its worth during the previous year. But when three years hence, peace was concluded, the price fell by 20% of the maximum value the land had reached. There has since been no fluctuation in the price. Find the present cost of an acre of land worth Rs. 2500 before the war.

21 A person borrows a sum of Rs. 1500 to purchase a diamond ear-ring, at 10% per annum C.I. The jewel deteriorates in value by 10% of its worth during the previous year. At the end of 3 years, he sells the jewel to pay off the debt. How much would he lose in the transaction?

22. The yearly income of a certain school was Rs. 12000 and the expenditure Rs. 10000. Owing to the increase in the salary of teachers, the expenditure rose every year for 3 years by 2½% of that in the previous year; but the income fell by 3½% every year, of the previous year's income owing to a decrease in the no. of admissions. Will the school be self supporting at the end of 3 years?

23. A share-holder in a business increases his capital annually by ⅓ of his capital at the beginning of the year. At the end of 3 years his capital is such that it can fetch him an annual interest of Rs. 512 when lent at 8% per annum. Find the capital he started with.

24. I bought a house for Rs. 6675 and got yearly a rent of Rs. 350 from it, which I invested at 8% C. I.

At the end of 3 years, I sold the house for Rs. 7200. What rate of simple interest had I been getting?

25. A person borrows Rs. 2500 at 9 % Compound Interest and pays at the end of every year Rs. 1000 towards the loan. Find how much he will have to pay at the end of the 3rd year.

26. I lay by, annually Rs. 600 which I deposit in a bank which pays me Compound interest at 8 % per annum. What amount will be due to me from the bank at the end of the 3rd year? If my daughter's marriage is to be celebrated now, find what amount I can borrow now at 8 % C. I. that I may at the end of 3 years liquidate the debt from my savings in the bank.

27. A person borrows Rs. 12,610 at 5 % per annum C. I. Show that he can discharge the loan by 3 equal annual instalments of Rs. 4630-8-0 each, the first payment being made at the end of the first year.

28. A spendthrift zemindar had during his lifetime incurred a debt which amounted to Rs. 46,410 when his son came to take possession of the estate. If the son desires to clear the loan in 3 equal annual payments find the amount of each payment reckoning C. I. at 10 % per annum.

29. Three brothers aged 18, 19 and 20 own a house in common, worth Rs. 3310. They propose to sell the house and divide the money realised, in such a way that they may all have equal shares at 21, their shares being improved meanwhile at 10 % per annum C. I. What will the elder brother get now according to the proposal? If he would like to keep the house for himself how much should he pay the other two now, in cash?

30. A person owns a property of Rs. 256,610. His two sons aged 18 and 19 respectively quarrel with him and insist on the partition being made. The father, finding all persuasion useless, agrees at last to the partition on condition that his share now and his sons' shares at 21 shall all be equal, the sons' shares being improved meanwhile at 5 % per annum C. I. Find the father's share.

31. (a) Show that the interest got by keeping Rs. x in a bank for y months, is the same as that got by keeping Rs. xy in the same bank for one month.

(b) A keeps Rs. x in a bank for y months and B, Rs. m for n months. What will be the ratio of the interest they will get from the bank ?

(c) A keeps in a business Rs. x for y months and B, Rs. m for n months. In what ratio should they divide the profits ?

32. Find, correct to the nearest rupee, the amount of the debt of a person from the following sums paid as annual interest.

	Rs.	A.	P..
To A borrowed at 6½ % interest	73	8	0
,, B ,, ,, 9 ,,	149	12	0
,, C ,, ,, 10½ ,,	4831	5	4
,, D ,, ,, 7½ ,,	981	10	8

33. There are 36 rooms in a building. These are let for tenants. On an average 2 rooms remain unoccupied the whole year, while the others are occupied only for 8 months in the year. Each tenant has to pay Rs. 6 a month. The rent is collected by a clerk who has to be paid Rs. 15 a month. The cost of repairing the building comes to Rs. 240 a year. The annual taxes come to 10 % of the total collections. If money be worth 7¼ % what price would the building reasonably fetch ?

34. A person owned two cawnies of land. A farmer wanted the lease of the land for 2 years. He offered Rs. 275 ready-money for the lease of 2 years for each cawnie or 30 kalams of paddy per cawnie at the end of each year. He lets one of the cawnies at the former rate and the other at the latter. The paddy he received at the end of the 1st year was sold at Rs. 5 per kalam and the paddy received at the end of the 2nd year at Rs. 5-12 per kalam. If every amount he received and raised was forthwith lent at 12 % compound interest, find the total income at the end of two years and also find which of the terms was more prfitable and by how much.

35. A bank advanced a sum of Rs. 1500 to a person on agreement that interest at 8 per cent shall be charged and that the interest was payable half-yearly. No payment was made and a suit was filed by the bank. The court declared at the end of 18 months from the date of borrowing, that compound interest shall be paid at the rate and on the terms agreed to, together with the cost of the suit. If either party had incurred an expenditure of Rs. 120, find how much had gone out of his purse on the whole. Also find what rate calculated at simple interest did his expenditure work up to.

*__36.__ The founder of a free high school incurs the following expenditure in maintaining the school :—

Pay of teachers Rs. 560 per month ; Repairs to school building Rs. 300 per annum ; school library and laboratory Rs. 500 per annum ; scholarships to boys Rs. 75 per month ; contingent expenses Rs. 15 per month ; salaries to peons Rs. 30 per month : stationery Rs. 500 per annum. How much should he invest at 5 per cent that the school work may go on peacefully for years to come ?

*__37.__ A man buys some property for Rs. 280,000 and lets it on lease at a rent of Rs. 15,500 per annum. If the property costs Rs. 1400 a year for repairs, what interest does he get on his investment ?

If the rent be collected by an agent for a commission of 5 per cent find the net interest obtained by the land-lord.

*__38.__ A gentleman owns an estate, yielding annually Rs. 9,500 worth of produce of which he spends 35 per cent on repairs, management, taxes etc. He sells the estate and realising Rs. 250,000 invests Rs. 175,000 in a bank paying 5 per cent. interest and the balance in another paying 6 per cent. He has however to pay income-tax at 9 pies per rupee. Find his increase per cent. in the income.

39. A person started a Bus service between two stations. He bought a car for Rs. 12,000 ; He engaged the services of two drivers at Rs. 40 each per month ; two cleaners on Rs. 10 each per month and two Booking clerks on Rs. 15 each per month. The car could be run only for 20 days in the month and two trips to and fro each day i.e., four journeys and carried 18 passengers, in each journey, on an average. On Service days petrol had to be bought for Rs. 20 a day and incidental expenses came to Rs. 5 each day. He wanted to lay by Rs. 650 per month towards capital and interest on the money invested, and a sum of Rs. 470 per month towards repairs unforseen. How much should he charge each passenger that he may get a net income of Rs. 50 per month for his maintenance ?

40. A person has lent Rs. 1500 at 9 per cent. and another sum of Rs. 1000 at 2 pies per rupee per month simple interest being calculated in both cases. He wants to consolidate these into a single loan bearing compound interest such that his income at the end of two years from the new investment may be equal to the interest he would otherwise draw, if the loans at simple interest had continued for the two years. Find correct to one decimal place, what rate per cent. compound interest he should charge.

41. A man sold his property for Rs. 12,000 and invested the money in a bank paying 5 per cent compound interest. At the end of the first year, the man withdrew Rs. 2200 and at the end of the 2nd year Rs. 4000. What amount will stand to his credit at the end of the third year ?

42. (a) A person charges by way of interest 2 pies per rupee per month ; What rate per cent. per annum is this equivalent to ?

(b) If at this rate Rs. 500 be borrowed on condition that if interest be not paid once in six months, the amount of interest will be added to the capital on that date, how much will the person have to pay at the end of 18 months, no payment having been made before ?

43. A press office was started on co-operative lines. A capital of Rs. 7,500 necessary, was raised by selling shares of Rs. 10 each and a further sum of Rs. 12,500 was borrowed at 6½%. The Directors passed the following resolution. " Out of the annual net profits the interest on the amount borrowed shall first be paid : 10 per cent.

of the balance be set aside as reserve fund, a dividend shall be declared for the shareholders and the balance if any, be utilised for the liquidation of the debts." If at the end of the first year, a sum of Rs 4,500 was paid towards the liquidation of debts after paying the maximum allowable dividend of 9 per cent. to the share-holders, what rate per cent. profit did the concern bring in?

44. A person had invested Rs. 2,500 in a Bank A and a further sum of Rs. 2000 in another Bank B. The former allowed interest at 6 per cent. and latter at 5 per cent, interest being added to the capital at the end of each year. Fearing the stability of Bank A, the person withdrew Rs. 1000 from the bank and invested it in the other. Likewise he did, at the beginning of the second year also. At the end of that year both the banks failed and while the former declared a dividend of 10 as. in the rupee, the latter declared a dividend of 12 as. 6 ps. in the rupee. By how much more would he have been the loser had he not transferred his accounts from the one to the other?

45. Two brothers divided a property of 44 lacs between them in the ratio of 6 : 5, the elder getting the bigger share. The elder's share at the end of a year was 5 per cent. less than what it was at the beginning, and at the end of the second year 4 per cent. less than at the end of the 1st year; while the younger steadily increased his capital by 5 per cent. of its value at the beginning of each year. Who was the richer at the end of two years, and by how much?

46. On Jan. 1st, 1922 a property was estimated at Rs. 17,500. On Jan. 1st, 1923, the price fell by 8 per cent. and on Jan. 1st, 1924 it rose by 12 per cent. of the recently estimated value. Money being worth 9 per cent. find by how much he would have been the gainer on Jan. 1st, 1924, had he sold the land on Jan. 1st 1922 and lent the amount for interest, the interest being added to the capital every year.

47. A man pledged a jewel worth Rs. 2900 and borrowed Rs. 2250 at 9 per cent. interest, on condition that interest should be paid at the end of each year, failing which, the rate of interest for each successive year shall be increased by 1 per cent., the interest being calculated on the amount-of the previous year. If no payment was made for two years and ths lender kept the jewel for himself, how much should the owner receive?

48. A and B both heirs to an estate of Rs. 64,000 divide it equally between them. A immediately sells his share and invests the sum at 6¼ per cent, interest being added to the capital annually. B lets his estate to a farmer who grows plantains therein and who is to pay a rental of Rs. 3500 at the end of two years. During this interval the estate has risen in value by 12½ per cent. Which of the two is richer at the end of the period and by how much?

49. Two brothers divided equally among them a sum of Rs. 1,20,000. The elder invested his share in a bank allowing 7½ per cent. Compound interest. The younger purchased a plot of ground and built 40 small houses and let each house on a monthly rent of Rs. 12. Two houses on an average remained unoccupied in the year and the other houses were occupied on an average only for 10 months in the year. Repairs, taxes, establishment, cost him 25% of the collections. He invested the balance in a bank. At the end of two years he sold the houses for Rs. 62,000 and interest allowed on his investment in the bank was only Rs. 200. Who was the richer at the end of the two years and how much had he in excess of the other's?

50. Which is better:—To borrow Rs. 7000 at 7½% compound interest for 3 years, or to borrow at 5% for the 1st year, 7 per cent. for the 2nd year, and 9 per cent for the third year, compound interest. By how much per cent of the former is the latter advantageous or disadvantageous to the lender?

51. Two private contractors offer to take on hand the construction of a bungalow. To build as per plan shown, one of them demands Rs. 24,150 to be paid at the end of two years when the work will be completed. The other demands Rs. 23,750 but stipulates that half the amount should be paid at the end of the first year and the remaining half on completion of the work, If money be worth 9% which tender is the cheaper and by how much?

52. A person had with him Rs. 8000. He invested one-half in a bank paying $6\frac{1}{2}\%$ compound interest and the other half in another bank which promised half-a-per-cent less but added the accumulated interest to the capital every half year. Find, to the nearest anna, which investment was the more profitable for him if he withdrew the whole amount in two years from the two banks.

53. A piece of land is offered Rs. 17,500 now. It brings in an income of Rs. 750 at the end of each year. Which will be more profitable :—To dispose of the land now for the amount offered and invest the proceeds at 9% compound interest or to wait for two years, invest the income alone at the above rate and sell it at the end of two years when it would fetch Rs. 18,000 ?

54. When sovereigns were selling at Rs. 15 each, a man borrowed Rs. 4,000 at 12 per cent. compound interest. He lent forthwith Rs. 1,000 at 14 as. per hundred per month simple interest and bought sovereigns for the remainder. He made them into jewels. At the end of two years from the date, the price of sovereigns rose to Rs. 20—4—0. He sold the jewels getting the price for the net weight of sovereigns in them, got back the amount he had advanced for interest and paid his debts. How much did he gain if the jewels weighed only 190 sovereigns.

55. Two men start on a salary of Rs. 120 each per mensem. One gets an increase of 5 per cent. (on the previous year's salary) only every two years, and the other a yearly increase of 4 per cent. Find the difference in the amounts they will be drawing at the end of the fifth year.

56. Each of two brothers gets Rs. 6350 ; one lends it at 10 per cent compound interest. The other buys property which yields Rs. 500 per annum, which he lends at 12 per cent compound interest. At the end of 3 years he sells the property for Rs. 6,500. Which is the more profitable business ? How many rupees has the one realised more than the other ? What rate per cent simple interest has each got upon his share ?

57. A company borrows from a bank Rs. 16,600 on the understanding that at the end of each year a portion of the principal is to be paid off with interest at 4 p. c. per annum on the amount standing unpaid during that year; and in two equal annual instalments the debt is cleared off. Find the amount of each instalment.

58. A person had to borrow Rs. 2,500. A money-lender demanded 9 per cent. interest (on the capital) which should be paid at the end of each year, failing which 12 per cent shall be charged by way of interest on the interest portion alone. Another money-lender would lend him at a uniform rate calculated at 1½ pies per rupee per month, the interest being added to the capital every year. If he could pay off his dues only at the end of two years, find which offer is more advantageous to the borrower and the amount he would gain thereby.

59. A wholesale diamond merchant sold diamonds to the value of Rs. 12,000 to a retail dealer and gained 20% on what the diamonds had cost him. For the amount the former received a promissory note at 9% interest. At the end of 12 months from the date, the retail dealer became a bankrupt and the court decreed a dividend of 13 as.—6 ps in the rupee. If the former computed interest at 9% on what the diamonds had cost him, find whether the bargain was on the whole profitable? Find the amount of profit or of loss. Also find what percentage of the expected amount he got at the end of the 12 months.

60. Being urgently in need of money, a person had to apply to a usurer for a loan. The lender managed to get from the borrower Rs. 45 at the time of lending, being three months interest on Rs. 1,500 advanced; and the borrower in haste dated the Promissory Note 1-3-23 instead of 1-8-23. At the end of a year from the actual date of borrowing the lender filed a suit for the Principal and interest at the rate, from the *date* of the note. What rate of interest was demanded and how much in excess of the reasonable amount had he to pay?

61. A business was started with a capital of Rs. 5,00,000 in shares of Rs. 100 each, fully paid up. A certain portion of the profits was placed in the reserve fund, another portion paid as dividend on the capital and the balance was brought forward for the current year and this balance amounted to Rs. 1,500-15-6. The current year's profits came to Rs. 30,423-12-9. Of this amount a sum of Rs. 6,000 was placed in the reserve fund; and to the balance, the balance at the end of the previous year was added. From this total amount a dividend is paid to the shareholders which is as great a whole number of rupees per share of capital as possible, and the remainder is carried forward. What is the amount carried forward?

62. A lends B, £ 182 for 60 days. B on another occasion lends A, £ 60 for 70 days. The party still under obligation proposes to grant the other a loan of £ 35 for such a time as shall cancel the obligation. What time must that be?

63. A father has to his credit Rs. 1,09,180 in a bank. How must he divide it among his two sons aged 18 & 19 so that each may draw the same amount when completing his twentieth year; The bank allows compound interest at 6%.

64. An elastic rubber ball after striking the ground bounces to $\frac{4}{7}$ of the height from which it fell. After striking the ground the third time it rises $3\frac{7}{8}$ inches: find from what height it fell at first.

65. The population of a town has increased 20% every ten years. How much per cent has it increased in 30 years? If it is now 1,00,000, find what it was thirty years ago to the nearest hundred.

66. Two districts are of equal population. Statistics show that the population of one increases at 18% while that of the other decreases by 12%. Compare the population of the two districts at the end of two years, the percentages of increase and decrease being on the population at the beginning of that year. (The answer must be in the form $1 : x$; x must be correct to three significant figures).

67. For cash down a motor car is offered by a company for Rs. 3,600. The alternative of an immediate payment of Rs. 750 and 6 monthly instalments of

Rs. 500 each, is also given. What rate per cent interest (to the nearest integer) is charged by the company?

68. (a) On one occasion a person lent Rs. a to his friend who had the money for b months. On another occasion the friend lent Rs. b to the former for c months and found that to cancel the obligation he should lend Rs. c more for x months. Express x in terms of a, b & c.

(b) I borrowod Rs. 750 from a friend and returned it at the end of 8 months. I afterwards obliged the friend with a loan of Rs. 450 which he returned 6 months hence. If again I lend him Rs. 300, for how many months is he entitled to keep the money?

69. A ball allowed to bounce vertically, rises to heights which gradually decrease in such a way that the height in any one bounce is 80% of the height in the preceding bounce. What is the height to which a ball will rise after the third bounce if the ball is dropped from a height of 6 ft.

Find also the total distance moved through, till the beginning of the fourth bounce.

70. The population of a certain district at the end of a certain year was 533,000. Next year the population increased by 10% and at the end of the third year it was found that the population had decreased owing to an epidemic by 12% of the strength at the beginning of that year. What was the population then? By how much per cent has it increased or decreased on the whole in the three years?

71. The Court of Wards takes charge of a Zemindari the successor being a minor. The deceased Zemindar was found to have incurred debts to the extent of Rs. 450,000, on an average rate of 9% compound interest. The average annual income from the state is Rs. 1,95,000. The court gives annually Rs. 24,000 to the minor for private expenses and the establishment expenditure comes to Rs. 1,000 per month. The balance is paid towards the interest and a portion of the Capital. Find in how may years the estate will become free and unencumbered.

72. Three brothers owned a jewel in common, having equal shares in it. The eldest kept the jewel and 1 year $10\frac{1}{2}$ months hence paid Rs. 526-4-0 to the next younger and Rs. 448-12-0 to the youngest seven months from the original date. What was the jewel worth and what rate per cent interest did he allow his brothers ?

73 A starts in trade with a capital of Rs, 6000. After 4 months B joins him with a capital of Rs. 2400 and after 3 months more C joins with a capital of Rs. 1800. At the end of 12 months the net profits are Rs. 918-8-0. What amount of profits does each get ?

74. Three persons A, B and C enter into partnership. A contributes Rs. 800, B Rs. 1500 and C Rs. 1100. A acts as manager for which he gets $\frac{1}{10}$ of the profits and the remainder is divided between all the three partners in proportion to their contributions. If the profits at the end of a year amount to Rs. 320, find the shares of A, B and C.

75. Three persons A, B and C contribute Rs. 3000, Rs. 3600 and Rs. 4200 respectively to the capital and open a shop. After 3 months B withdraws his capital on account of some misunderstanding with the other partners, who immediately replace the amount, each contributing a sum proportional to his original capital. If at the end of the year the profits amount to Rs. 432, find *B's* share of the profits.

76. Three persons invest sums of Rs. 4600, Rs. 3600 and Rs. 2400 respectively in a venture, on the agreement that the profits shall be divided in such a way that the *rate of interest* made by each shall be proportional to the sum invested. In 9 months the profits amount to Rs. 997. How much of this will each receive and what rate of interest per annum does each make ?

77. A starts a business with a capital of Rs. 1500 when B and C desire to join him with capitals of Rs. 2500 and Rs. 2000. A takes them as partners on the agreement that the profits be so divided that the *rate of interest* made by each shall be in the proportion of 5 : 3 : 4. If the profits at the end of the year amount to Rs. 920, what should each receive ?

78. A, B and C rent a meadow in common for Rs. +69-10-8 on the understanding that the rent each should pay shall be proportional to the use each has made, of the meadow. A puts 3 cows for 3 months and 50 sheep for 2 months. B puts 14 cows for 2 months and 10 sheep for 3 months. C puts 15 cows for 1 month and 16 sheep for 5 months. Supposing a cow to eat as much as 4 sheep what will be each man's share of the rent?

79. A person who is working as a partner in a joint speculation meets with a sudden loss and becomes a bankrupt. His assets consist of (1) 2 acres of lands worth Rs. 2400 each. (2) an undiscounted bill for Rs 1040 due 6 months hence at 8% per annum. (3) book debts to the amount of Rs. 1800 of which 60% consists of bad debts of which only 40 % is realisable. If he is able to pay his creditors only Rs. 0-12-0 in the rupee, find the amount of his liabilities. [Undiscounted bill—A bill for Rs. 104 due 6 months hence at 8 per cent is now worth only Rs. 100-0-0.]

80. A person who has a debt of Rs. 5000, loses nearly all his property in a litigation and becomes a bankrupt. His assets then consist of (1) A Life Insurance policy whose surrender value is Rs. 1200 (2) Furniture worth about Rs. 540 (3) A post office cash certificate for Rs. 500 due 2 years hence, interest allowed being 6 % per annum. Find how much in the rupee he can pay his creditors.

81. Two persons start a motor service, One invests Rs. 14,500 and the other 15,500, on the understanding that 60 per cent of the profits shall be treated as interest on the capital and the remainder divided equally between them. If at the end of one year one gets Rs. 180 more than the other, find :—(i) the total amount of the profits. (ii) What rate per cent interest each gets on the money he has invested.

82. A and B enter into partnership; A puts into the business Rs. 900 more than B who as active partner, is to receive a salary of Rs. 22 a month for management. At the end of a year, the gross profits computed at 15 % per annum on the capital invested are found to be Rs. 1485, from which, of course, B's salary is to be deducted.

Find the amount of capital subscribed by each partner and the rate of interest each receives on his investment.

83. A bankrupt's assets according to his account book, are just equal to his liabilities. On bad debts to the extent of Rs. 3000 he recovers only Rs. 0-10-8 in the rupee. The remainder, he receives in full. The expenses of bunkruptcy come to 5 per cent of the nominal assets. He is able to pay only Rs. 0-12-0 in the rupee. Find the amount of his liabilities.

84. A bankrupt has a house estimated at Rs. 3,900 and book-debts to the extent of Rs. 5,850. Had he realised these in full, his creditors would have received Rs. 0-13-0 in the rupee. The house was sold for 82·5 % of the estimated value and only 76·25% of the book-debts were realised. How much in the rupee could be paid as dividend?

85. Two persons invest Rs. 800 and Rs. 1,200 respectively in trade. One is to have 5% of the profits for managing the business and the remaining profits are to be divided in the ratio of the capitals contributed by each. At the end of the year one receives Rs. 20 more than the other. Find the amount of the profits and the rate of interest each receives.

86. The assets and liabilities of a declared insolvent are as follows:—

1 House, getting a monthly rent of Rs. 12: for which a half-yearly house-tax of Rs. 3-6 has to be paid. The purchase value being determined by the worth of money at 9 %.	A court decree for Rs. 10,873-8 as. including cost of suit, interest etc.
2. Apro-note in his favour drawn two years ago for a sum of Rs. 1,800 at 9% simple interest.	
3. Furniture valued at Rs. 4500, which if sold would fetch only 80% of the estimated value.	

Find how much in the rupee can be declared as dividend.

87. The assets and liabilities of an insolvent were as follow, on the 1st January 1918 :—

Assets—

(a) A promissory note for Rs. 400 at 7½% drawn on 15th September 1917.

(b) Hypothecation bond for Rs. 750 at 8 % drawn on 1st July 1916.

(c) House and furniture to the value of Rs. 5,000.

Liabilities—

(1) To A, Rs. 2,700 with interest at 10 % from 1st January 1917.

(2) To B, Rs. 4,000 with interest at 11 % from 1st July 1916.

(3) To C, Rs. 2,800 with interest at 9 % from 15th September 1917.

Find how much in the Rupee he can give ; only S. I. to be calculated.

88. The following three questions must be worked (i) by contracted methods and (ii) by referring to the ready reckoners-Intesest tables and (iii) by employing the third and tenth and tenth rule.

i. Find the interest on Rs. 247-12-6 from April 4th to July 9th, the rate of interest being 3½ %.

ii. A sum of Rs. 2500 is borrowed on April 15th and repaid on July 12th. How much must be paid if the rate of interest be 3⅛ %.

iii. If Interest has to be calculated on Rs. 26-4 as. at 12½ % from January first toFebruary 3rd, find the Interest.

N.B.—Interest is not charged for the day on which the loan is returned.

SECTION VIII. (*B*)

Averages.

1. The distance between Madras and Trichy is 199·5 miles. The boat mail leaves Trichy at 7-30 P.M. and reaches Madras at 6 A.M. What is its rate of running in miles per hour?

The student will divide the distance travelled *viz* 199·5 miles by the time $10\frac{1}{2}$ hrs. taken for the journey and then the rate of the train is $\left(\frac{199{\cdot}5 \times 2}{21}\right)$ miles per hr. = 19 miles per hour.

But we know that the train is not always running. The rate 19 miles, is true only when the train runs without stopping for a minute. Also the train must be uniformly running at that rate. We also know that the train moves very slowly either when it leaves a station or when it approaches a station. Therefore what does the rate '19 miles' mean? The train has sometimes run at more than 19 miles per hour, (25 miles even) and sometimes less than that rate; The rate 19 miles per hour is a mean of the different rates. This mean rate is called the *average rate*.

2. One day the student is cheery and the questions put by the teacher are of moderate difficulty and he scores 20 mks. Next day, he may be indisposed and may get 12 mks.; on the third day he gets 24 mks.; on the fourth day he gets 6 mks. on the fifth he studies hard and gets 38 mks.; the maximum being the same in all cass.

The student's ability cannot be judged by the 6 mks. that he gets on the 4th day.

The mean of the marks secured by him is a pretty sure indication of his worth or merit

The mean in this case is, $\frac{20+12+24+6+38}{5}$

$= \frac{100}{5} = 20.$

N B.—If the boy's mks. on a few more occasions be taken, the average will be a surer indication of his merit.

The average is therefore obtained by dividing the sum of the values on different occasions by the number of occasions.

The average of the 10 numbers :—7, 12, 8, 7, 12, 38, 24, 32, 18, 22 is the sum of the numbers divided by the number of the numbers. The average is $\frac{180}{10}=18$.

It should be noted that the average must be greater than the least number and less than the greatest number entering into the average.

3. The area of a district is 540 sq. miles and there are 18450 people.

How many on an average live in each sq. mile?

Evidently $\frac{1845}{54}=\frac{205}{6}=34\frac{1}{6}$.

N.B.—Famous towns are generally thickly populated. Even in these towns some parts are more thickly populated and it is very difficult to get houses on rent. Every town is in some parts thickly populated. The density of population is calculated by finding the number of persons per sq. mile.

4. The following points should be discussed with the boys :—

(i) The town proper is more thickly populated than the suburbs.

(ii) The greater the No. of persons per sq. mile, the more famous or the more important the town or district is.

(iii) The greater the average age of boys in a form, the greater the number of aged boys in the form. If the average is less, the greater the number of young boys and the less the number of big boys.

Qs. *Suggested*—

(i) What will be the average age of boys in form I?
(ii) ,, ,, ,, ,, VI?
(iii) ,, ,, the 5th class?

Actual figures may be had from the office and boys should work out the average.

(iv) If the school average in Ele. math. is 60 what does that show?
(v) ,, ,, ,, is 28 ,, ,, ?
(vi) ,, Presidency ,, ,, is 52 ,, ,, ?
(vii) ,, ,, ,, ,, is 24 ,, ,, ?
(viii) If the average daily temperature is 100·4 ,, ?
(ix) ,, ,, ,, ,, 92·8 ,, ?
(x) If the average score of runs of a cricketeer is 87·8 what does that show?
(xi) ,, ,, ,, ,, is 18·2 ,, ?
(xii) ,, ,, ,, ,, is 108·7 ,, ?

6. ***If the average of 'm' nos is 'n' their total is mn.***

If ,, total 'p' nos is 'q' their average is $\frac{p}{q}$

If the average of 7 nos. is 15 and if the average of 6 *of those numbers* is 13, find the seventh number.

The total of the 7 nos is 105
,, ,, 6 nos is 78
∴ the 7th no. must be $105 - 78 = 27$.

If the average of m nos. is n and of p of those numbers is q. the average of the remainder is $\frac{mn-pq}{m-p}$.

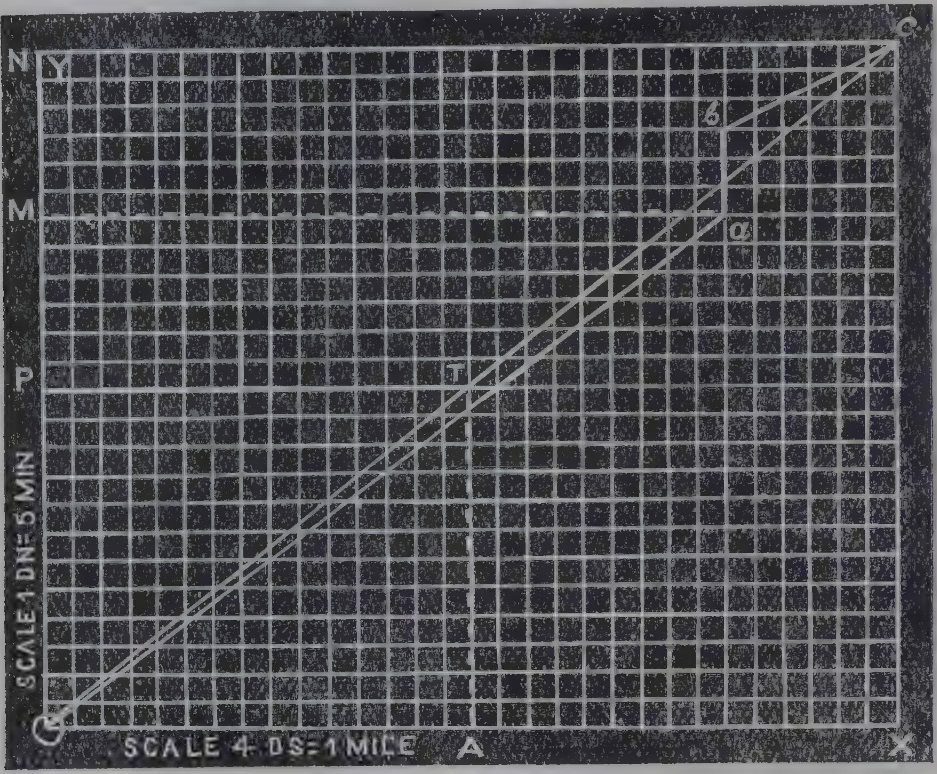
Method—The sum of m nos in mn.
,, ,, p of those is pq.
,, ,, the remaining $(m-p)$ is $(mn-pq)$

∴ the average of the $m-p$ nos. is $\frac{mn-pq}{m-p}$.

If the average of m nos. is n and the average of x other nos. is y, the average of the m + x nos is $\frac{mn+xy}{m+x}$.

N.B.—All these generalisations must be dilled in, by taking concrete examples. The practical utility of 'Averages' as giving a decently accurate idea or estimate of the value or the nature of things must be clearly brought home to the boys.

7. A man walks for 1½ hrs. at 4 miles an hour takes rest for 15 min. and then resuming his journey reaches his detination having run for 15 more min. at 6 miles an hour. Find the average rate graphically.



Oa graph of journey at 4 miles an hr. for 1½ hrs.
ab rest, no progress in distance made.
bc graph of journey at 6 miles an hr. for ¼ hr.
Total time taken *ON*.
,, distance travelled *OX*.
Hence average graph is *OC* which gives
PT i.e., *OA* for 1 hr. i.e., *OP*.
and *OA* = 15 divisions ; average rate being
3¾ miles per hour.

EXAMPLES XXIV.

1. A boy bought 32 marbles at 4 for 6 pies and 20 more at 3 for 6 pies. How much did he spend on the whole? How much had he paid on an average for 10 marbles?

2. The average age of 10 boys of a class is 12 years 6 months. of the remaining 20 other boys is 13 years. Find to the nearest tenth of a month the average age of the whole class.

3. The average age of 42 boys of a class is 13 years 4 months. Two boys whose ages are 12 yrs. 8 months and 14 years 2 months leave the school and one boy whose age is 14 is admitted. What will be the new average?

4. A piece of metal was weighed by ten boys and the results were 13·8 gr., 14·7 gr., 14·4 gr., 13·9 gr., 14·6 gr., 14·6 gr., 13·9 gr., 14·1 gr., 14 gr., 14·5 gr. The average of these was taken for the real weight.

How would this be altered if the average of the first five and the average of the last five be taken and added and divided by 2?

5. The average age of 30 boys of a class is 12 years 6 months. One more boy was admitted and the average rose to 12 yrs. 6·02 months. Find to the nearest tenth of a month, the age of the newly admitted boy.

6. A merchant bought goods for Rs. 3200; He sold ¼ at a gain of 10%; ⅙ of the remainder at a gain of 12½% and the rest at a gain of 6¼%. Find his average gain per cent.

7. A merchant wishing to clear out his old stock sold one lot of goods marked Rs. 1200 at a reduction of 10%: a
second lot of goods marked Rs. 1500 ,, of 20% and
the third lot marked Rs. 4000 ,, of 6¼%

What was the average reduction per cent?

8. (1) A man had run p miles in q hrs. and x miles in y hrs. Find his average rate of running.

(2) A train travelled a distance of 120 miles in 6 hrs. and then a distance of 45 miles in 4 hrs.: Find its average rate of running.

(3) A train travelled a distance of 20 mile at 20 miles an hour and then a distance of 45 miles a 11¼ miles per hr. Find its average rate of running.

(4) A cyclist goes 40 miles at 11 miles an hr and returns to the starting point at 7 miles an hr. Fin his average rate of cycling.

(5) A cyclist goes 40 miles and back. If th wind be blowing steadily at 2 miles an hour, all the tim and his rate of cycling in calm weather is 9 miles a hour, find his average rate of cycling for the journey?

(6) If the speed of an aeroplane is v miles pe hour, find its average speed from A to B and back, if the wind is blowing all the time from A to B at u miles a hour.

9. A district is divided into three taluks. A B & C A is a sq. miles and there are p persons to a sq. mile. B is b sq. miles and there are q persons to a sq. mile, C is c sq. miles and there are r persons to the sq. mile. Find the average population per sq. mile for the whole country.

10. In a certain competitive examination seven High Schools in a district sent in pupils to sit for the examination. The number of students presented by each school and the average for the school are given below. Find (1) the average for the district and (2) the names of the schools that secured more marks than the district average.

	No. of boys presented.	School average.
A	63	32
B	101	33
C	79	38
D	130	37·5
E	90	30·5
F	110	36·2
G	125	37

11. There are classes of 9, 12, 15, 19, 22, 24, 17 boys in a school. Their average ages are: 17 yr. 5 mths: 16 yr. 8 mths. 16 yr. 3 mths. 15 yr. 7 mths. 15 yr. 0 mths; 14 yr. 6 mths. and 14 year 3 mths: respectively. Find the average age of all the boys in the school.

Also find which class has an average above the average of the whole school.

12. A person bought 192 cocoanuts at 1 anna 3 pies each, 240 at 1 anna 2 ps. each and some more at 1 anna 7 ps. each and found that on the whole he had paid on an average Rs. 8-5-4 for each hundred. How many did he buy of the last sort?

13. (a) A train runs from a station P to another Q 24 miles distant, halting at an intermediate station R for 5 min. and at another S for 3 minutes. Its speed from P to R is 10 miles an hour, from R to S and S to Q, 12 miles an hour. Find its average speed including stoppages. (P to R = 8 miles: S to Q = 12 miles.)

(b) If on another occasion its average was 10 miles an hour because of the halt at S for a few more minutes, find for how many minutes it remained in that station, the other halt at R and the rates of running being the same as before.

14. (i) During this cricketing season a rising player has played 13 innings and his average is 96 runs. How many runs must he take in the 14th innings to raise the average to a century.

(ii) If he secured only a hundred runs what would be his average?

15. In a school a boys sat for the S. S. L. C. Examination; x of these had taken up Tamil under the C Group and these secured an average of $p\,\%$ in Ver. composition. The rest were Sanskrit students who secured $q\,\%$ in the subject. The average for the school was $c\,\%$. Find x in terms of a, p, q and c.

If $a = 78$, $p = 15$, $q = 38$, $c = 45{\cdot}5$ find x.

16. The average age of a class of 46 boys in September is 16. A boy whose age is 18 years 6 months in September leaves school at the end of the term. At the beginning of the next term (four months later) four new pupils are admitted whose ages are 13 years 11 months 14 years 3 months, 14 years 3 months and 14 years 11 months. Find the average age of the class at the beginning of the new term.

17. From a certain High School 63 boys were once sent up for the S. S. L. C. Examination. The school average in English at the Public Examination was 32 The boys that were eligible for a University Course in English secured 49·4 marks on an average, while the rest secured only an average of 28·3. Find the least number that should have failed.

SECTION IX.

Motion—Velocity.

1. *Rate of motion* is called *Velocity*: If equal spaces be travelled in equal times the velocity is called *uniform*.

If a person travels a distance of 18 miles in 1½ hrs. his velocity or rate of motion is 12 miles an hour.

N.B·—It must be noted however, thet it is not practically possible for a man to bike or walk or motor uniformly. The train again is slow when it starts and again reduces its speed when it is about to enter the station yard. Whenever, therefore we speak of uniform velocity, it should be borne in mind, that we speak of average velocity.

In general terms if s denote distance, v the velocity and t the time taken to travel that distance with that speed,

$$v = \frac{s}{t}$$

There are three letters involved in this equation; If any two of them be given the third can be determined.

2. A person motors a distance of 36 miles in 1½ hrs. and another bikes the distance in 4 hrs. Compare their speeds.

The motorist's speed is 24 miles per hour.
,, cyclist's ,, ,, 9 miles per hour.
∴ their speeds are as 24 : 9 or as 8 : 3.

If time taken be as 3 : 8, speeds are as 8 : 3 distance being same.
,, ,, 3 : 8, distances travelled are as 3 : 8 speed being same.
Speed be as 3 : 8, ,, ,, 3 : 8 time being same etc. etc.

3. Two persons start at the same time, from two places of A and B towards each other, the one from A walking at 3 miles an hour and the one from B riding at 8 miles an hr. If the distance between A and B be 33 miles find i. when and where they will meet.

ii. how far they will be apart in 1½ hrs. from the time of starting.

iii. when from the time of starting they will be apart 5½ miles.

SOLUTION—*Theoretical* :—

(i) In one hour they approach by 11 miles (*i.e.*, the distance between them will be 11 miles less than the distance between them when they started).
If they meet the distance between them is zero.

i.e., both together must have travelled the 33 miles.

∴ Time taken is 3 hrs......They will meet in 3 hrs. from the time of starting.

Again one of them will have travelled 9 miles and the other 24 miles.

i.e., they meet in 3 hrs. at a distance of 9 miles from A.

(ii) They will have approached each other by 16½ miles.

∴ the distance between them after 1½ hrs. will be (33 − 16½) miles = 16½ miles.

(iii) To be apart 5½ miles they must have together travelled 27½ miles which to do, they will require 2½ hrs.

Graphical :

Scale along X-axis is 1 dn. = 1 mile and along Y-axis is 1 dn. = 10 minutes.

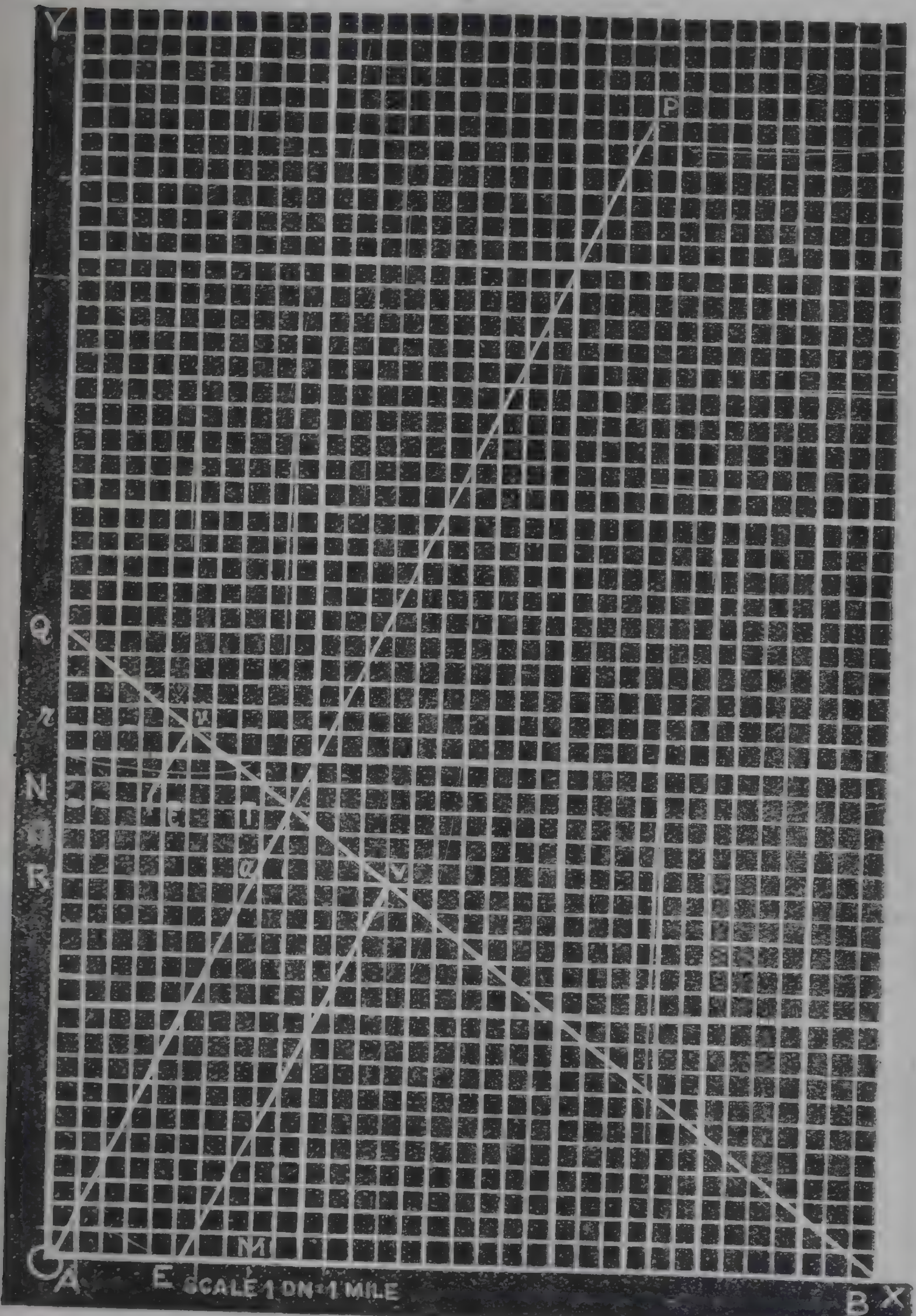
OP is the graph of the man walking at 3 miles an hour.

BQ „ „ riding at 8 miles an hour.

AB on the X axis represents 33 miles.

TM and TN the co-ordinates of T, give the time and place of meeting.

i.e., AM the place of meeting and AN time of meeting AE = 5½ miles; Through E draw a parallel EV to AP to meet BQ in V.



$Va = EA = 5\frac{1}{2}$ miles; Va produced meets OY in R; giving OR or AR—the time when they will be apart $5\frac{1}{2}$ miles.

Also vr gives another occasion when they will be apart $5\frac{1}{2}$ miles after meeting.

N.B.—(i) The student is advised to draw a big figure to the following scale.

Scale $\frac{2}{10}''$ or $\frac{1}{5}''$ along X-axis $= 1$ mile.

Scale $\frac{1}{10} = 5$ min. along Y-axis.

(ii) The student can also read off when each will reach his destination.

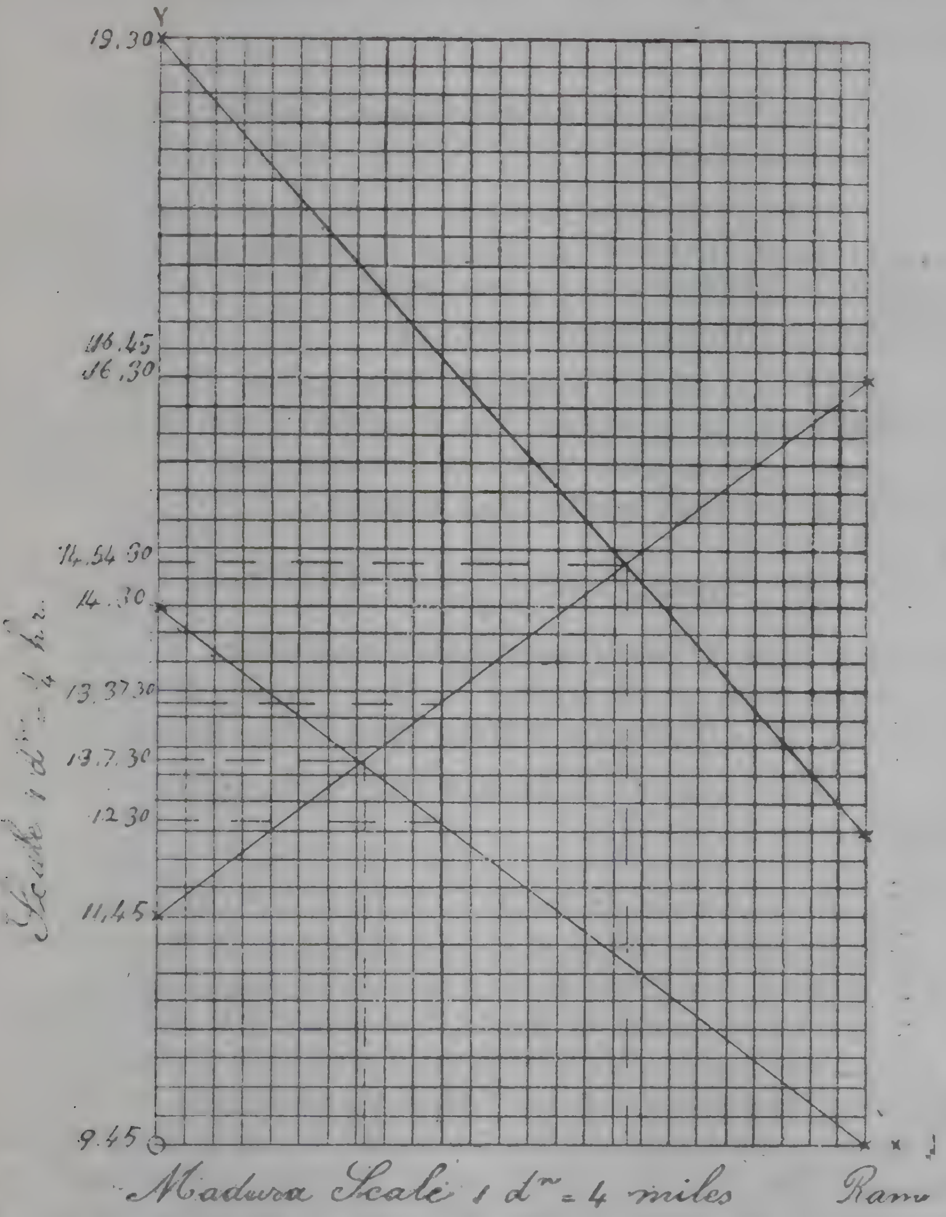
4. The following is a rough extract from a Railway Guide.

Trains from Madura.		
Distance from Madras in miles.	Stations.	Boat Mail to Ceylon.
348	Madura *dep.*	11-45
448	Ramesvaram *arr.*	16-30

Trains to Madura.			
Distance to Madras in miles.	Stations.	Boat Mail from Ceylon.	Rames-varam Express.
448	Ramesvaram *dep.*	9-45	12-30
348	Madura	14-30	19-30

[NOTE.—Time is calculated for 24 hours from midnight to midnight.]

Represent the motion of the three trains graphically. From the graph find when they are 40 miles from Madura, and when and at what distance from Madura the Boat Mail from Madura meets the other two trains. (S. S. L. C. 1918).



N.B.—As the trains run only between Madura and Rameswaram we need not fix on the graph sheet the position of Madars.

SOLUTION :—*See Figure.*

They are 40 miles from Madura.

i. at 13, 37′, 30″.
ii. at 12, 37′, 30″.
iii. at 16,45′.

No. 1 meets II and III. 29th mile from Madura at 13,7′ 30″; at the 66th mile from Madura at 14,54′ 30″ respectively.

5. The following points should be impressed in the minds of the pupils.

(1) If two persons walk in opposite directions towards each other; distance between at a particular moment ÷ sum of their rates gives the time of meeting after that particular moment.

(2) If they walk in the same direction; distance between at a particular moment ÷ the difference of the rates, gives the time of overtaking or of being together after that particular moment.

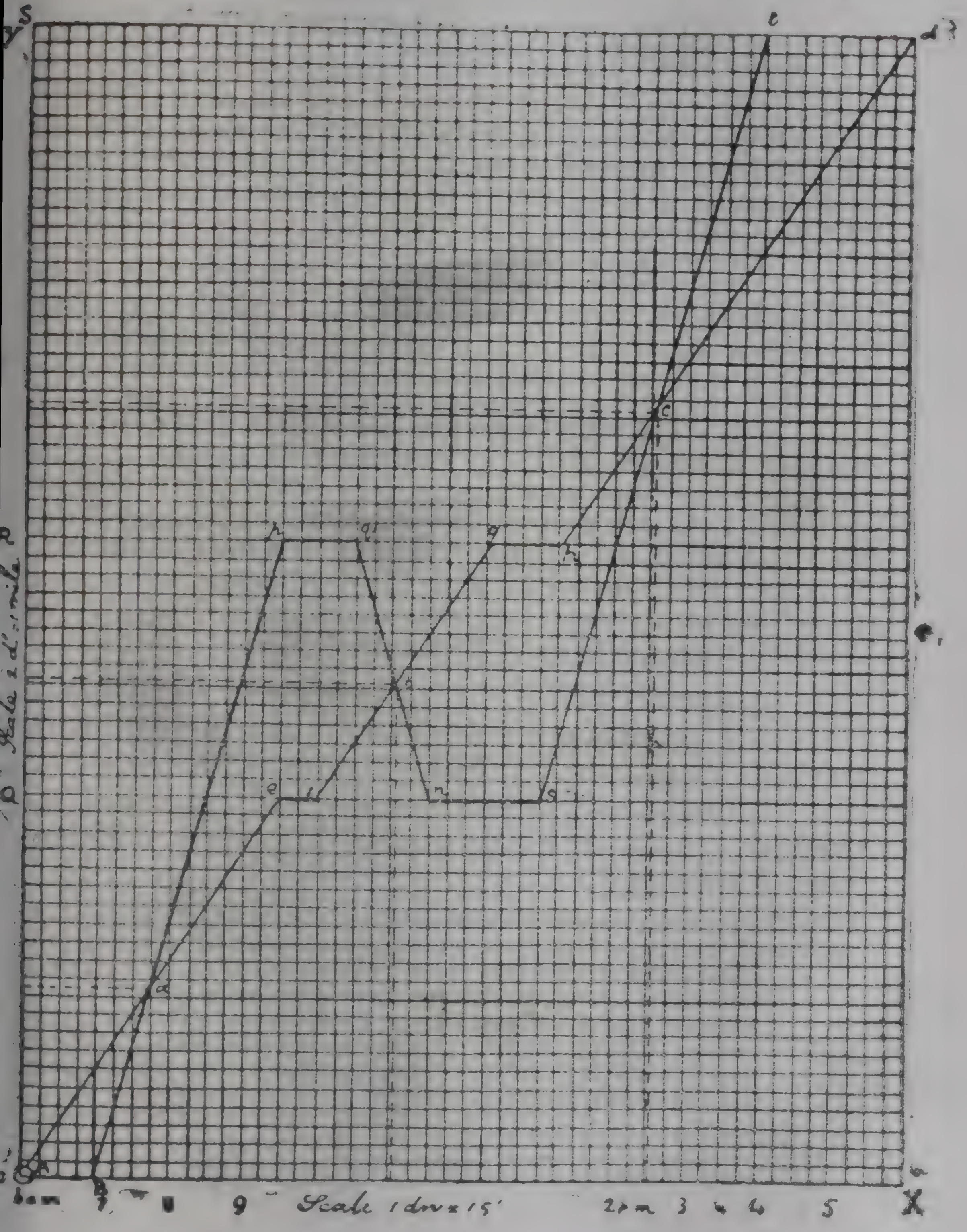
(3) Symbolically $\frac{D}{E}$ i.e. Distance between ÷ Effective rate, gives time of being together. The effective rate is the sum of the rates if they walk towards each other and the difference if they walk in the same direction.

The idea of 'Effective rate' should be thoroughly drilled in by examples.

6. A road runs from P to S passing through Q and R. PQ is 10½ miles and QR, 7 miles. A starts from P at 6 A.M. and walks to S at a uniform rate, halting at Q for half an hour and at R for one hour. B starts from P at 7 A.M. on his cycle, his rate of cycling being 7 miles an hour. He passes A at 7-45 A.M. and goes to R where he stops one hour. He returns to Q, where business detains him for an hour and a half. He then goes straight to S, which he reaches at 4 P.M. Find the distance from P to S, A's rate of walking, and the time when he reaches S.

Represent the above graphically and find from the graph when and where A and B will be together.

(S.S.L.C. 1920.)



i. Mark Q and R to scale along Y-axis. (We cannot draw the graph of A's journey to begin with, as data are wanting.)

ii. The graph of B's journey can be drawn. B*ap* in the figure.

iii. At 7-45 a.m. A and B are together; Therefore draw a vertical at a distance of 3 divisions from B. This ordinate meets B's graph at *a*. Therefore P*a* must be the graph of A's journey in as much as A had started at 6 a.m. and was with B at 7-45 a.m.

iv. Continuing A's graph, *et* shows halt at Q (time advances, no progress in distance) *tq* is parallel to P*e*, for A continues at the same rate, *gh* again shows halt at R and *hcd* is continuation of A's journey.

v. Taking B's graph, B*p* is the graph till he reaches R, *pq* rest at R. *qr* returning at the same rate to Q, so that the coordinates of *r* show that he has returned to Q and that while distance has become less, time has advanced. *rs* his rest at Q. *st* shows his graph. It is parallel to B*p* for he is at the destination at 4 p.m. travelling at his own rate. Therefore draw a parallel to X-axis through *t* to meet Y-axis at S;

Answer.—PS is the total distance = 31½ miles.

A's rate is given by the ordinate through B to meet A's graph, which is 6 ds. = 3 miles an hour.

Wherever the two graphs cut they will be together. Read from the graph. OX gives the time when A reaches S.

N.B.—This problem embodies a no. of Principles of graphical solution of questions on Time and Distance—viz (i) Finding rate graphically (ii) Finding times of being together (iii) Finding time of reaching destination (iv) Drawing graph of return journey (v) Representing rest graphically. Therefore the teacher will do well to explain each step clearly, eliciting step by step the answers from the boys. A big diagram must be drawn by them.

EXAMPLES XXV.

A

Motion in a straight line.

N.B.—Exercises marked with an asterisk will be solved *both* theoretically and graphically.

*1. Two persons start from the same place, at the same time and walk in the same direction. One of them walks at 3½ miles an hour and the other at 2½ miles an hour.

(a) What will be the distance between them at the end of 2½ hrs., 1¾ hrs., a hrs.?

(b) If the distance between them be 3 miles 4½ miles, p miles, how long have they walked?

***2.** (a) If they had walked in opposite directions what would be the distance between them at the end of 1 hour, 2½ hrs., m hrs.?

(b) If they walked in opposite directions how long would they have walked if the distance between them be 7 miles, 4½ miles, p miles?

***3.** (a) A person A, leaves P for Q and at the same time another B, leaves Q for P; The rate of A is 9 miles per hour and that of B is 11 miles per hour. What would be the distance between them at the end of 1 hour, if the distance between P and Q is 45 miles?

(b) What would be the distance between them at the end of 2 hours?

(c) What would be the distance between them at the end of 2½ hours?

N.B.—If there be no distance between them, they should meet.

(d) If P to Q be 60 miles and the rates of the persons be the same as before, when will they meet?

(e) If P to Q be x miles and the rates of the persons be a miles per hour and b miles per hour respectively, when will they meet?

4. A waks at x miles per hour and B walks a miles per hour less than A. If they travel towards each other starting at the same time from each terminus when will they meet, given that the distance between them originally was p miles?

*5. A is 5 miles ahead of B and bikes at 6 miles an hour. B bikes after him at 8 miles an hour. What will be the distance between them at the end of 1 hour? How many miles is this distance short of the original distance? What will be the distance between them at the end of 2 hours? When will they be together?

*6. A started from P for a place Q, 35 miles distant at 7 A.M. and biked at 10 miles an hour. At 8-15 A.M. B motored from P for Q at 16 miles an hour. (a) Where will A be, when B started? (b) What will be the distance between them at the end of an hour from the time B started? (c) What will be the distance between them at the end of x hours from the time B started? If this distance be equal to zero what does that mean? Hence find when B will overtake A; Also find where they will be, when they are together.

*7. A cat is a yards off and a dog sets in pursuit. The dog can run b yards a second and the cat c yards per second. When will the dog catch the cat supposing both of them run along a straight road? If $a=330$, $b=22$, $c=14\frac{2}{3}$, find in how many seconds the dog will overtake the cat.

*8. A thief had a start of 20 min. and ran away on a bike at 10 miles an hour. A police inspector sets in pursuit on a car at 25 miles an hour. When and where will the thief be caught?

9. Two persons start from the same place, at the same time and travel at the rate of 5 and 8 miles an hour respectively to a place 32 miles distant. Find:—

1. when they will be apart $\frac{1}{2}$ mile.
2. how many miles they will be apart at the end of 2 hrs. and 10 min. from the time of starting.
3. when each will reach his destination.

10. My office is a miles from my house. One day I find I should be there in t minutes (a) At what rate in miles per hour must I run to be there in time? (*b*) If having run at this rate half the distance, I am detained for *b* minutes on the road, at what rate must I run from that place to reach the office in time.

11. From the time a bus overtook me to its disappearance round a corner I took 30 steps and in 180 more steps I reached the corner itself. If my rate of walking was 2 miles per hour, what was the speed of the car?

***12**. A, B, C, D. E are five villages along a straight road. A postman has to go to these five villages to deliver letters and should be back at 8 P.M. in the office. One day he left the office at 2 P.M. From his office to A is 1 mile 2 fur. from A to B, 1 mile; From B to C. 2 miles 4 fur; from C to D. 4 fur. and from D to E, 6 furlongs. His rate of walking is 1 mile in 16 minutes. To distribute and deliver the letter etc. he took 32. 20, 16, 18 and 28 minutes at the stations respectively. Find graphically what his rate was while returning to office if he returned straight to his office without halting on the way and reached it just in time.

***13.** Two buses left a station for another 32 miles distant. One of them ran at 15 miles an hour. At the 20th mile the engine have trouble and having halted there for some time, it started again and was able to run only at $\frac{4}{5}$ of the original rate; while the other maintained a uniform speed of 12 miles an hour and both reached the destination at the same time. Find graphically for how many minutes the previous car was delayed on the way.

14. A prisoner of war, in attempting to escape to a friendly ship moored within sight of land, swims out at the rate of 3 miles an hour and is assisted by the tide which ebbs at the rate of 200 yards per hour. After he had been swimming for $\frac{3}{4}$ of an hour the tide turns and impedes him as much as it had previously assisted him. He also slackens his speed from 3 to $2\frac{1}{2}$ miles an hour and succeeds in reaching the vessel in another hour. How far was the vessel from the shore?

N.B.—Rate when assisted by the tide is 3 miles 200 yds. per hr. Rate when impeded by tide is ($2\frac{1}{2}$ miles − 200 yds.) = 2 miles 680 yds. per hour.

15. A train runs from A to C passing through an intermediate station B; A to C is p miles and the time taken is 't' hrs. The rate of the train is s miles per hour from A to B and k miles per hour from B to C. Find in terms of p, t, s, k, the distance between A and B.

16. The table given below shows the distances of a few important stations from Madras on the main line of the S.I.Ry., and the timings of the departure of trains. The left hand column shews 'Up train service' and the right hand column 'Down train service'. Time is calculated at 24 hrs. from midnight to midnight.

Distance from Madras Beach.	No. 3 R E.	No 261 P.		6 T. E.
177	16,41	13,50	Mayavaram	20,50
197	17,55	15,23	Kumbakonam	19,48
221	19,49	17,10	Tanjore	18,33
251	21,15	...	Trichinopoly	16,55

Find from a graph the times and places of the meeting of the trains.

*__17.__ On a certain day, the passenger motor left Pudukkottai at 7-20 A.M., and ran at the rate of 11 miles an hour. It halted at Keeranur (14 miles from Pudukkottai) for 10 minutes, and at Nallur (19½ miles from Pudukkottai) for half-an-hour, for repairs. The mail car left Trichy (33 miles from Pudukkottai) on the same day at 8-20 A.M., and stopping at Keeranur for 5 minutes reached Pudukkottai, running at the same rate as the other car. Read from a carefully and neatly drawn graph—

i. when and where the two cars met each other on the day;

ii. when the passenger car reached Trichy;

iii. when the mail car reached Pudukkottai.

18. A ship 78 miles from the shore springs a leak which admits $2\frac{1}{3}$ tons of water in $6\frac{1}{2}$ minutes. 68 tons would suffice to sink her, but the pumps can throw out 15 tons in an hour. Find the average rate of sailing that she may just reach the shore as she begins to sink.

***19.** A boy at Triplicane leaves his house at 8 A.M., for the Christian College and walks at the rate of 3 miles an hour. At 8-40 his brother leaves the house and bikes the distance at 6 miles an hour and both reach the College at the same time. How far is the College from their house and when do they reach it?

Again if the brother who goes on bike, after having travelled half the distance, returns to his house at the same rate, at whate rate should he bike again from his house to reach the College at 10?

***20.** A cyclist started from a place A to another station B, 32 miles distant. Having travelled for 2 hrs. at 8 miles an hour, he halted for $\frac{1}{2}$ hr. to do certain repairs in his machine and afterwards was able to bike only at 5 miles an hour. Just as he reached his destination his friend who had left A for the same place and who was biking uniformly at 8 miles an hour, also reached the place. When did the latter leave his destination? Find this time graphically.

***21.** A thief who had a start of 1 hr. 20 min. was pursued by an Inspector of police on horseback riding at the rate of 10 miles an hour and after a ride of 2 hrs. the Inspector caught the thief. Find :—

1. the rate at which the thief was running.
2. how many miles the two were apart at the end of 1 hr. 20 min. from the time the Inspector started.
3. when they were apart 1 mile.

***22.** At 6 A.M., one morning I left my house for a place 18 miles distant and travelled by a bullock cart which moved at the rate of 3 miles an hour. I forgot to take with me a document, and my father sent at 8-40 A.M. a friend on bike with the document and he overtook me at the 12th mile. He took rest there for a quarter of an hour and returned at 8 miles an hour. Find :—

i. the rate at which the friend followed me to hand over the document and the time when the document was handed over to me.

ii. the time when he would have reached my house back.

***23.** The No. 1 Ceylon Boat Mail may be taken to run at an average rate of 25 miles an hr. and the express at an average rate of 20 miles an hr. How many minutes will a traveller save by taking the Boat mail for a journey of (1) 125 miles. (2) 100 miles?

***24.** To go to a place 35 miles distant, a man takes a train having an average speed of 15 miles an hour, while another who starts 20 min. later motors at a certain rate and both reach the place at the same time. Find the rate of the motorist.

25. A field is in the form of a rt. angled triangle and the sides of the field are 480 yds. and 140 yds. Two men start together from one end of the hypotenuse, one of them walking along the hypotenuse and the other along a side. If they meet,

1. at the vertex of the rt. angled $\triangle$,
2. at the other end of the hypotenuse; find the rate of the one, if that of the other who walked along the hypotenuse be 220 feet per minute.

26. Two persons begin to walk from the corner of a square field a yds. long. One walks along the sides and the other along a diagonal. If the rate of the latter be b yds. per minute, find that of the other if they both reach the opposite corner at the same time.

***27.** A peon walks from A to B at the rate of 3 miles an hour, and after transacting some business which occupies him an hour, returns to A by the tramway at the rate of 5 miles an hour. He then finds he has been absent 2 hrs. 20 min. Find the distance from A to B.

28. A person in a railway carriage observes that there are 20 telegraph poles to the mile, at equal intervals apart, and that the train passes 13 of these intervals in 1 m. 37·5 s. Find the speed of the train in miles per hr.

*29. Two persons A and B set out together on a journey. They walked at the rate of 3 miles an hour; and after they had proceeded for three-quarters of a mile B returns walking at the same rate, to the place of starting. Here he was detained $\frac{3}{4}$ of an hr. Setting out again he overtook A, who had beèn walking all the time, at the end of $2\frac{1}{2}$ hrs. from the second time of starting. At what rate did he walk?

*30. A person A sets out from a place P to walk to a place Q. A quarter of an hour later, a second person B sets out from P to walk to Q, but after walking half a mile returns to P where he is detained 10 minutes. Again setting out from P, he reaches Q 5 min. after A. If A walks throughout, at tho rate of 3 miles an hour, and B at the rate of 4 miles an hour, find the distance between P and Q.

*31. Every evening a person at the railway station, waits for 16 minutes and then travels by train at 15 miles an hour and reaches home eight miles distant from the station, or walks a distance of two miles in 30 minutes, enters a bus which overtakes him and reaches home at the same time as he would by the train. Find graphically the rate of the bus.

32. A distance of 42 miles along a road from a station begins with a rise which continues for 12 miles. This is followed by an incline to a distance of 18 miles, from which place there is again a rise. Two men ride towards each other from either end. They have started at the same time. Find when and where they will meet if their rates be 5 miles an hour up and 6 miles an hour down the incline. Illustrate graphically.

*33. A man driving to a railway station at the rate of 8 miles an hour discovers when 12 miles from home that he has left one of his bags behind. He sends word back by a motor bicycle that happens to be passing, which travels at 20 miles an hour. The man continues his journey, and five minutes after the news is received, the bag is despatched from the house by motor car at 40 miles an hour. At what distance from the house does the car overtake the man? Illustrate graphically.

*34. A hare sees a dog 88 yards off and begins at once to run away. In half a minute the dog also sees her and sets in pursuit. If the dog runs at 15 miles an hour and the hare at 13½ miles an hour, how soon would the hare be overtaken? By choosing suitable axes, illustrate the problem graphically.

35. A hare makes nine leaps in the same time as a dog makes four; how many leaps will the dog have to make before catching the hare, supposing the hare to have a start of 16 yards on a straight road? A hare's leap is 2 ft. and a dog's 5 ft.

36. The distance between my house and the Branch School is nearly 2 furlongs, and the Local Mission School is about 30 yards farther off. I take $5'$ to walk from my house to the Branch School. One Saturday morning when the Mission School was dispersed at 10 A.M., I left my house at 10 A.M. and was detained on the way for a minute. I proceeded at my usual rate afterwards and met some children from the Mission School who were also walking at my own rate. Find from a carefully-drawn graph when and at what distance from the Mission School I met the children on the day.

*37. A and B are the termini of a railway 144 miles long. A train starts from B at 9 A.M. Another travelling *at the same rate*, starts from A at 10 A.M. and these meet at 11-30 A.M.

A third train leavs B at 10-20 A.M. and the train from A and this train meet at 12-32 P.M.

Find graphically:

(1) the rates of the three trains

(2) the time when each will reach its destination

(3) the distance one train has travelled in excess of the other when they meet.

38. A battleship B, with a speed of b miles per hour, starts in pursuit of an enemy vessel C, which is d miles ahead and whose speed is c miles per hour. If B's guns can hit at a distance of h miles, shew that B will be within hitting range in $(d-h) \div (b-c)$ hrs. Evaluate when $b=28{\cdot}6$, $c=27{\cdot}2$, $d=10$, $h=8{\cdot}4$

If in this numerical case, the firing began as soon as B was within hitting range and ceased when B was $5\frac{1}{2}$ miles from C, when C surrendered, how long did the action last?

· **39.** A thief steals a cycle and rides off on it at 9 a.m. at the rate of 16 miles per hour, followed 15 minutes later by the owner, who covers 15 miles an hour while in the saddle, but spends 5 minutes after every 10 miles, making enquiries. The thief at the end of every half-hour reduces his speed by 2 miles an hour. When and where will the thief be caught if both are riding along a straight road. Illustrate graphically.

***40.** A thief stole a bicycle and rode on it at the rate of 12 miles an hour, along a straight road. The owner discovered his loss and set out in pursuit on a car 40 minutes afterwards, at the rate of 20 miles an hour. He motored a distance of 10 miles where he stopped to make enquiries, and on starting, the car gave some trouble and on the whole he was delayed there for 50 minutes. Learning the thief had gone by the same road he went in pursuit, increasing the speed of the car to 26 miles an hour. Find graphically where and in how many hours from his starting the thief would have been caught.

41. A motorist and a cyclist start together. The motorist maintains a uniform speed of 33 yards to every 38 yards of the cyclist on the first sandy portion of the road and to every 30 yds. of the cyclist on the other portion. They however both reach the destination $5\frac{1}{2}$ miles distant at the same time. What length of the road was sandy?

***42.** A man walks to a place at the rate of $4\frac{1}{2}$ miles an hour. At 8 miles from his destination he meets B, and turns back with him (walking at B's rate) for a mile; if A is half-an-hour late at his destination, what is B's rate and at what rate should A have walked after parting with B so as to arrive at the proper time?

***43.** A hunter goes along a straight path in a forest, with his dog by his side. The dog sees a hare 220 yds, in front and sets in pursuit running 1760 ft. per minute. The hare also begins to run along the st. path just at the same time at 1320 ft. per min. The dog catches the hare and returns with it to the hunter who has been walking all the time at 3 miles an hour. When and where will the hunter meet the dog ?

44. Assuming that sound travels at the rate of 1100 ft. per second, and that a man walking towards a gun fired once a minute hears the fourteenth report $12\frac{27}{29}$ min. after hearing the first, find the rate in miles per hr. at which the man is walking.

45. A steamer A is in distress and stationary and fires a gun which is heard on another steamer B coming direct towards her. A fires another gun 10 min. after the first and this is heard on B, 9 min. 48 sec. after the first. At this moment B is 4 miles from A. How soon will the two steamers be alongside of each other ? Sound travels at the rate of 1130 ft. per second. Find also to the nearest tenth of a mile the rate of the 2nd steamer coming to the rescue.

46. Eleven guns of salute were once fired in honor of the visit of a prince ; the time between successive reports being 2 min., a motorist going towards the gun heard the last report 19 min. 30 sec. after hearing the first. Just then he was 3 miles from the gun. When will he be alongside of the gun ? Sound travels at the rate of 1144 ft. per second.

B

Races, Handicaps.

*__47.__ In the time A can run a hundred yds., B can run only 78 yds. How many yds. will B be behind in (1) a hundred yds. race, (2) a furlong race, (3) a half-mile race ?

*__48.__ A can beat B by 20 yds. in a furlong race and C by 30 yds. Who is the faster runner, B or C ? By how many yds. can B beat C in (1) a furlong race (2) in a half-mile race ?

*__49.__ (a) If A can beat B by 25 yds. in a quarter mile race, how many yds. start must be given to run a dead heat ?

(b) How many yds. start had B if they ran a dead heat in a mile race?

50. (a) In a mile race A can beat B by x yds. and C by y yds. If B be the faster runner than C, which is greater x or y?

(b) By how many yds. can,

i. A beat B in a half-mile race,
ii. A beat C in a half-mile race,
iii. B beat C in a half-mile race,
iv. A beat B in a 800 yds. race?

51. In a half mile race, if A gives B 10 yds. start A wins by 10½ sec.; but if A gives B 15 sec. start, B wins by 22 yds. Find the time that each takes to run the ½ mile.

52. Three men run a mile race; the first beats the third by 80 yds. and the second beats the last by 12 secs. The pace of the first is to the pace of the second as 45 : 44. In what time to the nearest tenth of a minute does each run the mile?

53. In a half-mile race A gives B a start of 20 yds. and C a start of 30 yds.; and beats the former by 10 yds. and the latter by 20 yds. Find, from a carefully-drawn graph, at what distances from the winning post A overtook B and C respectively.

***54.** In a quarter-mile race A can beat B by 80 yards and C by 90 yards. By how many yards can B beat C in the same race?

Supposing their rates to be uniform wherever they run and whatever the length of the race-course be, find by how many yards A can beat B or C, and B can beat C in a furlong race? With suitable scale illustrate the problem graphically.

C

Motion in a Circle.

Ex. 1.—A motors at 60 miles an hour and B goes by a jutka running at 5 miles an hour. Both travel in the same direction.

(*a*) If they start from the same place, when will they be (i) 15 miles apart, (ii) 30 miles apart, (iii) 45 miles apart?

(*b*) If B is 10 miles ahead of A, when will they be (i) together, (ii) 15 miles apart, (iii) 45 miles apart, (iv) 30 miles apart, supposing both of them started at 2 P.M.?

Ex. 2—When will the hands of a clock be

(*a*) together, at rt. angles and in a st. line between 12 and 1 o'clock?

(*b*) together, at rt. angles and in a st. line between 2 and 3 P.M.?

N.B.—Note the relation between this question and the previous one.

Ex. 3.—When will the hands of a clock be (i) together, (ii) 10 min. apart, (iii) at rt. angles, (iv) in a st. line between,

(*a*) 5 and 6 o'clock.
(*b*) 11 and 12 o'clock.

Ex. 4.—(A) N.B.—When the min. hand makes 12 rounds the hr. hand makes 1.

i. e., the min. hand gains 11 rounds in 12 rounds.
1 round in $\frac{12}{11}$ rounds.
2 rounds in $\frac{12}{11} \times 2$ rounds.
x rounds in $\frac{12}{11} \times x$ rounds.

When $x = 11$, the min. hand gains 11 rounds in 12 rounds i.e. the hr. hand and the min. hand are together after 12 rounds.

(B)—Two persons whose rates of running are as 13 : 10 run round a circular course. At how many points are they together if they start from the same place?

the faster gains 3 rounds in 13 rounds.
1 round in $\frac{13}{3}$ rounds.
2 rounds in $\frac{13}{3} \times 2$ rounds.
3 rounds in $\frac{13}{3} \times 3$ rounds.
x rounds in $\frac{13}{3} \times x$ rounds.

Hence they will be together when the faster has run $4\frac{1}{3}$ rounds, $8\frac{2}{3}$ rounds and next 13 rounds, when, they both will be together at the starting point.

Hence if rates of two people running in the same direction round O^{r}. course be as $a : b$ $(a > b)$ they will be together $(a - b)$ times at the last of which they will be together at the starting point, and if they continue, the same thing will be repeated. It should be noted however that $\frac{a}{b}$ must be in its lowest terms.

(C)—If they walk in opposite direction with rates as 13 : 10, then

When the faster goes $\frac{13}{23}$ of the round the slower goes $\frac{10}{23}$ of the round—1st time of meeting.

When the faster goes $\frac{26}{23}$ of the round the slower goes $\frac{20}{23}$ of the round—2nd time of meeting.

When the faster goes $\frac{39}{23}$ of the round ,, ,, $\frac{30}{23}$ of the round—3rd time of meeting.

When the faster goes $\frac{13x}{23}$ of the round ,, ,, $\frac{10x}{23}$ of the round—xth time of meeting.

If $x=23$; they meet again at the starting point.

Hence $a+b$ denotes the no. of meetings, the last of which is being together at the starting point.

56. A mounted serjeant, a corporal and a soldier have been posted on duty and they have to go round a fort 8 miles in circumference. The serjeant rides on horseback at 7 miles an hour, the soldier walks at 3 miles an hour and the corporal bikes at 5 miles an hour. They start from the same place, and while the corporal and the soldier go clockwise the serjeant goes counter-clockwise. Supposing they began the patrol at 8 p.m.

find 1. the time and place of their meetings.
2. the time when each pair will be together at the starting point.
3. the time when all of them would be at the starting point.

D

Upstream—Downstream—etc.

57. A man cycles at 12 miles an hour. If the wind blow steadily in the direction he is travelling, at 4 miles an hour, how many miles would he have gone in (a) 2 hrs. (b) 4 hrs., (c) x hrs., (d) $a+b$ hrs. ?

58. If the cyclist travels against the wind on his, return journey, how many miles would he have travelled in (a) 3 hrs., (b) 2½ hrs,, (c) 1 hr. 40 min., (d) x hrs., (e) a hrs. b min. ?

59. Wind is blowing steadily from P to Q at 4 miles an hr. How long will a cyclist have to bike from P to Q and back if he can go 12 miles an hr. in calm weather, P to Q being 18 miles.

60. A stream is running at 3 miles an hr. If a boat which can be rowed 5 miles an hr. in still water goes 6 miles upstream and returns with the current to the starting point, what time will have elapsed before it will be again at the starting point ?

61. An aeroplane flying at a miles an hour in still weather goes b miles with the wind blowing at c miles per hour. If t denote the time taken, establish a relation between a, b, c, and t.

62. A stream runs at the rate of 'a' miles per hour. If a man who can row 'b' miles an hour upstream goes 'c' miles downstream and returns to the starting point in 'x' hours, establish a relation between x, a, b and c.

If the rate of the stream be 3 miles an hour, that of the man in still water 5 miles an hour, find using the above relation the time he will take to row 6 miles downstream and back again to the starting point.

63. A man starts to row on the still water of a canal and completes a mile in ¼ of an hour; he then reaches a river, and entering it rows with the current 1⅓ miles in the next quarter of an hour. How long will he take to return to the starting point, if he rows back with the same strength as on his outward course?

64. The width of the Cauvery at a certain point is 7 fur. The boatmen can row 5 miles an hour in still water. One day I find it takes them 45 min. to reach the exactly opposite point on the other shore. Find the rate at which the river is running.

E

Trains and their lengths

65. (1) A train is 132 yds. long and runs at 15 miles an hour. How long will it take to clear a telegraph post?

Note. Distance run = its own length.

(2) How long will it take to clear a platform 242 ft. long?

Distance run = its own length + length of the platform.

(3) How long will it take to clear a man walking in the same direction at 4 miles an hour?

Distance run = its own length and rate being, (its own rate — the man's rate.)

(4) How long will it take to clear another train running in the same direction at 10 miles an hour; the length of the latter being 220 yds.

Distance run = its own length + the length of the other and the rate = its own rate − the rate of the other.

(5) How long will it take to clear a man walking in the opposite direction at 5 miles an hour?

Distance run = its own length and rate (being its own rate + the man's rate.)

(6) How long will it take to clear the train running in the opposite direction; vide 4 *spura* for particulars of the other train.

Distance run = its own length + the length of the other. Rate being the sum of their rates.

N.B.—All the six cases might be included in one formula,

$$T = \frac{L_1 + L_2}{E}$$

where T denotes time of crossing, L_1 & L_2 lengths and E the effective rate.

The teacher is requested to apply this formula in the solution of questions of the type. The applicability to all cases must be clearly explained.

66. The Coleroon Railway bridge is nearly 6 furlongs. How long will it take the Boat mail to clear it, if the speed be 10 miles an hr. and the length of the Boat-mail be 110 yds.

67. A tunnel is 3 miles long. In how many minutes will a train 132 yds. long running at 15 miles an hour clear the tunnel?

68. The Cauvery bridge is 2 fur. long. A column of infantry is marching at 3 miles an hour. If the file be 100 ft., when will it clear the bridge?

69. Two trains measuring 330 feet and 264 feet respectively, run on parallel lines of rail. When travelling in opposite directions they are observed to pass each other in 9 seconds, but when they are running in the same direction at the same rates as before, the faster train passes the other in $27\frac{1}{2}$ seconds. Find the speeds of the two trains in miles per hour.

70. A train l yards long, running at m miles an hour clears a platform in n seconds. If the length of the platform in yards be x, establish a relation between x, l, m and n.

A person standing on a Railway platform 264 yds. long noticed that a train passed the platform in 20 seconds and himself in 8 sec. Using the above relation find the length of the train and its rate per hour.

F

Work in specified time.

71. (1) A pipe can fill a tub in x min. In how many minutes will the tub be half-full?

(2) A pipe can fill a cistern in 15 min. How many such cisterns will it fill in an hour, in 2 hrs., in x hrs.?

(3) A pipe can fill a cistern in 15 min., another can empty it in 20 min.

[In an hour the supply pipe alone would have filled 4 such cisterns and the other would have emptied 3 such cisterns if full and if both the pipes are open the cistern will be full at the end of an hour.] How much of the cistern will be full at the end of half an hour; 15 min.?

72. A vessel sprang a leak and when the hands began to pump out water, it had only 6 ft. more to rise to sink the vessel. Of 18 that formed the crew 12 worked for 8 minutes at the pumps and the water gained 3 ft. against them; then 14 pumped for 6 minutes and the water rose 2 ft. more. All hands took to the pumps till the ship floundered. From the time they began to work at the pumps, find how long a time elapsed.

73. A certain pipe can fill a cistern in 'x' hrs. and another in 'y' hrs., while a third pipe can empty it in 'z' hrs. If the three pipes are turned on at 12 noon when the cistern is empty, find how much of the cistern would have been filled by 3 P.M.

If $x=3$, $y=4$ and $z=2$, find when the cistern will be half-full.

74. A, B and C are three pipes connected with a reservoir; A and B are supply pipes and C is a waste-pipe. A is turned on at 6 A.M., B at 8 A.M., but C was found to be on also, at 8 A.M. and was turned off. Find graphically when the cistern will be full if A and B can each fill the cistern in 4 hours and 6 hours respectively while C can empty it in 5 hours.

75. A cistern whose capacity is 4309 gallons, is to be filled with water by a pipe which conveys 23 gals. 1 qt. per minute. On account of a leakage the cistern is only just filled in 3⅓ hours. What is the average amount of leakage per hour?

(b) How many gallons will there be in the cistern at the end of 15 min. from the start?

76. A cistern can be filled by 3 pipes in 30, 40 and 60 min. respectively and emptied by an escape pipe in half-an-hour. The three taps are turned on at noon, but the escape pipe is at the same time accidentally left open and not closed for quarter of an hour. At what time will the cistern be full?

77. A cistern, the cubic contents of which are 360 cub. ft., has two pipes which can empty it in 3 and 4 hrs. respectively. It has also a third pipe with an orifice of 1 sq. ft. through which water flows into the cistern at the rate of 1 yard per minute. If all the three pipes be opened together when the cistern is full, in what time will it be emptied?

N.B.—Find the volume of water brought in by the third pipe.

G

Miscellaneous.

78. Of two watches, one loses 5 seconds and the other gains 4 seconds in 12 hours; they are both right at noon on a certain day. When will one be 5 min. before the other and what time will each then show?

79. Two watches one of which gained at the rate of 1 m. 54·6 sec. and the other lost at the rate of 1 m. 55·8 sec. daily were set correctly at noon on the first Jan. 1896. When did the watches next indicate the same time and what time did each then indicate?

80. A well is fed by a spring which flows continuously and uniformly into it. When there are 10,000 cub. ft. of water in the well, 7 men can empty it in 20 days; and when there are 15,000 cub. ft. of water in the well, 5 men can empty it in 15 days. How many cub. ft. of water flow in one day?

81. A person finds, it takes him 5 times as long to row up a river as to row down it. If the rate of the stream be 6 miles an hour, find how many miles per hr. the man can row on still water.

82. In a certain meadow there is a crop of 525 stone of grass, which grows uniformly. If 11 oxen turned in would consume all the grass in 48 days, but 6 oxen would require 98 days, what weight of grass has been growing per day ?

83. Two clocks A and B, whose rates are uniform, at noon yesterday indicated 11 hrs. 55 min. A.M. and 0 hrs. 2 min. P.M. respectively ; A indicated correct time at 9 P.M. yesterday and B at 6 A.M. this morning. When did A and B last agree and what time did they then indicate ?

84. On a river, B is intermediate to and equidistant from A and C ; a boat can go from A and B and back, in 5 hrs., 15 min. and from A to C in 7 hrs. What time would it take to go from C to A ?

85. Two clocks, one of which gains 1′ 15″ per day and the other loses 45″ per day, were set right at noon on Jan. 6th 1904. When will they indicate the same time and what time will they then show ?

86. Of two watches, one loses 5 seconds and the other gains 4 seconds in 12 hours ; they are both set right at noon on a certain day ; when will one be 5 minutes before the other, and what time will each then show ?

87. 300 coolies are set to build a tank-bund. In 14 weeks they have done $\frac{7}{10}$ of the work, when rain stops the work for 4 weeks and washes away $\frac{2}{7}$ of what they had done. At the end of that time the work is resumed with only 250 coolies. In what time from the commencement will the work be finished ?

SECTION X (A).

Ratio, Proportion and Mixtures.

1. How do we generally compare two magnitudes of the same kind? Either by subtracting the less from the greater or by saying how many times one is contained in the other. The first process is comparison by subtraction and the second, comparison by division.

We take a concrete example:—

A boy A has 10 marbles while B has 3 marbles. We may say that (i) A has 7 marbles more than B, (ii) B has 7 marbles less than A, (iii) A has $\frac{10}{3}$ times what B has, (iv) B has $\frac{3}{10}$ of what A has.

i and ii are comparisons by subtraction and iii and iv are comparisons by division.

The name '*ratio*' *is given to the fraction which expresses the relation between two magnitudes of the same kind.*

Hence the fraction $\frac{3}{10}$ may denote,

(i) 3 times the unit—the unit being $\frac{1}{10}$.

(ii) the tenth part of 3.

(iii) the ratio of 3 to 10; this is written thus 3 : 10.

N.B.—For purposes of arithmetical calculation we write as $\frac{3}{10}$ and for purposes of comparison we write as 3 : 10. It is also told that in $3 \div 10$, the bar – in the sign $\div$ has dropped and that $3 \div 10$ is expressed as 3 : 10.

Again the ratio $\frac{3}{10}$, being a fraction, has all the properties of fractions and can therefore be exhibited graphically.

2. $\frac{3}{10} = \frac{15}{50}$ *i.e.*, 3 : 10 :: 15 : 50. ... ... Proportion, (Equality of two ratios).

Note that in a proportion the product of the extremes is equal to the product of the means *i.e.*, $3 \times 50 = 10 \times 15$.

$\frac{a}{b} = \frac{c}{d}$; We know by cross-multiplication that $ad = bc$.

If expressed in the form of proportion $a:b::c:d$, the product of the extremes which is ad is equal to the product of the means which is bc.

3. We solve a few examples below :—

(a) What fraction of Rs. 8-8 as. is Re. 1-4 as. ?

Re. 1-4 as. = 20 as. ; Rs. 8-8 as. = 136 as. The fraction required is $\frac{20}{136}$ *or* $\frac{5}{34}$, *i.e.*, the two amounts are in the ratio 34 : 5.

(b) The areas of two estates are as 5 : 3. The bigger is 8 acres in extent. Find the area of the smaller.

$8 : x :: 5 : 3$; $\therefore 24 = 5x$ or $x = 4\cdot8$ acres.

(c) Divide Rs. 85 between A and B in the ratio 8 : 9.

If the total be $17x$, A must get Rs. $8x$ and B Rs. $9x$.

Hence A gets Rs. 40 and B Rs. 45.

(d) The weight of a cub. ft. of water is 1000 oz. and the weight of 50 cub. feet of teak is a ton.

The wt. of one c. ft. of teak is $\frac{20\times4\times28\times16}{50}$ oz.

$$=716\cdot8 \text{ oz. } \left\{ \text{Note rough work} \left| \begin{array}{c} 28\times16 = \\ \times \\ 160 \quad \ldots \end{array} \right| \begin{array}{r} 448 \\ \\ 71680 \end{array} \right\}$$

$\therefore$ wt. of 1 c. ft. of water : wt. of 1 c. ft. of teak

as $1000 : 716\cdot8$

or as $1 : \cdot7168$

i.e., as $1 : \cdot72$ (correct to two decimal places.)

We say that the Specific gravity of teak is 0·72

N.B.—It will be a very good exercise if the students be made to compare the weights ot a substance and its equal volume of water (got, of course, by displacement). The idea of *Specific gravity* should clearly be grasped by the students.

(e) A merchant mixes inferior coffee worth Re. 1-7 as. a viss with a superior sort worth Rs. 2-12 as. per viss, in the ratio 2 : 5. (i) What is the cost price of a viss of the mixture? (ii) How much per cent will he gain by selling each viss at Rs. 2-10-9 ps.?

Solution.—

(i) Cost of 2 viss of inferior coffee is Rs. 2-14 as.
,, 5 ,, of superior coffee is Rs. 13-12 as.
∴ Cost 7 ,, of mixture is Rs. 16-10 as.
∴ Cost 1 ,, of ,, is Rs. 2-6 as.

(ii) Gain on Rs. 2-6 as. is 4 as. 9 ps. *i.e.*, 12½ %.

(f) A merchant mixes inferior coffee worth Re. 1-7 as. a viss with a superior variety worth Rs. 2-12 as. a viss and by selling each viss of the mixture at Rs. 2-10-9 ps. a viss gains 12½ %. Find the ratio of the mixture.

Solution.—

The actual cost of a viss is $\frac{8}{9}$ (Rs.2-10-9) = 8(4 as. − 9ps.)
= Rs. 2-6 as.

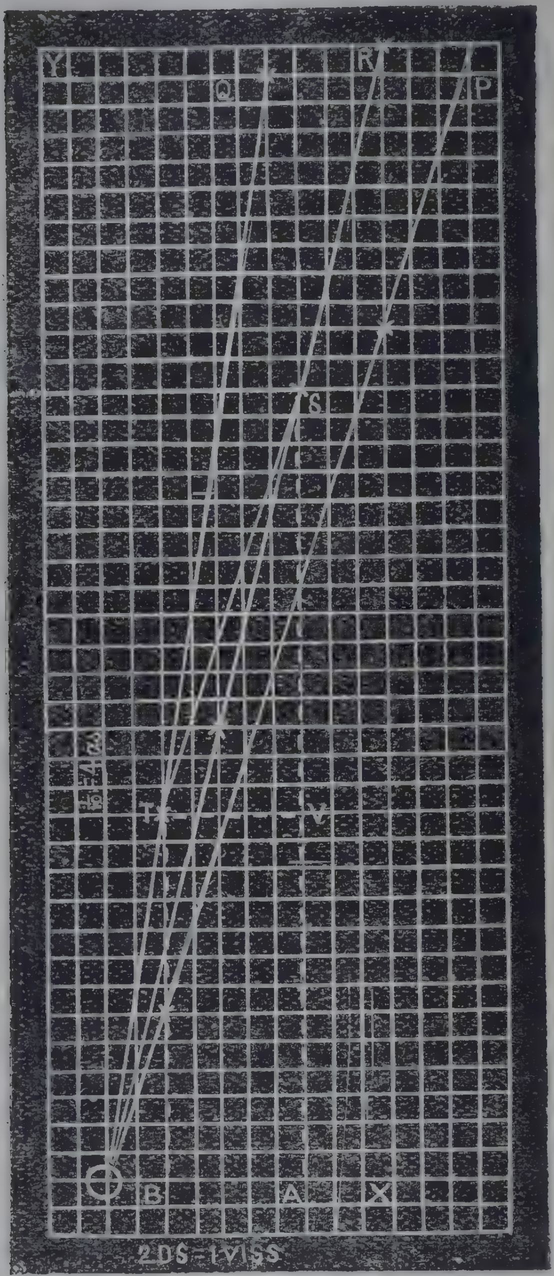
If the whole had consisted of the inferior sort he will have gained 15 as. on each viss sold at Rs. 2-6 as.

If the whole had consisted of the superior sort he will have lost 6 as. on each viss sold at Rs. 2-6 as.

But by selling at Rs. 2-6 as. a viss he will neither gain nor lose. Therefore he must have taken 6 viss of the inferior sort and 15 viss of the superior sort.

The ratio of the mixture is 2 : 5.

(g) If a certain variety worth Rs. a per viss be mixed with another variety of the same commodity worth Rs. b per viss and if the mixture be sold at Rs. c per viss, the ratio of the mixture is (a being greater than b and consequently than c) $c-b : a-c$.



(h) A merchant mixes inferior coffee worth Re. 1-8 as. a viss with a superior variety worth Rs. 3-4 as. a viss and by selling the mixture at Rs. 2 a viss, does neither gain nor lose. Find the ratio of the mixture graphically.

Explanation.—

OP, OR and OQ are graphs of prices of Coffee at Re. 1-8 as., Rs. 2 and Rs. 3-4 as. per viss respectively.

OA represents 3 ½ viss;

Price of 3 ½ viss at Rs. 2 per viss is given by AS.

Through S draw ST parallel to PO to meet OQ in T.

Drop the ordinate TB.

TB is price of 1 viss at Re. 1-8 as. a viss.

SV is price of 2 ½ viss at Rs. 3-4 as. a viss.

and TB + SV is price of (1 + 2 ½) viss at Rs. 2 a viss.

Therefore ratio of mixture is OB : BA *i.e.*, 2 : 5.

Note.—If through S, a parallel ST′ is drawn to OQ to meet OP in T′ and if the ordinate dropped from T′ cuts x-axis in B′ the ratio of the superior to the inferior will be given *i.e.*, 5 : 2.

Example.—Pure gingelly oil worth Rs. 1-6 as. a measure is mixed with groundnut oil worth 14 as. a measure and the mixture is sold for Re. 1–1 a.–6 p. a measure and the merchant neither gains nor loses. Find how much of the ground-nut oil he should mix for every 7 measures of the gingelly oil.

Solution.—If the whole had consisted of pure oil, for every measure sold he would have lost 4½ as.

If the whole had consisted of groundnut oil for every measure sold, he would have gained 3½ as.

∴ for 3½ measures of the former he should take 4½ of the latter.

For 7 measures of the former he should take 9 of the latter. We say that they must be mixed in the ratio of 7 : 9.

EXAMPLES XXVI.

Oral

1. Two partners have invested Rs. 3000 and Rs. 4500 respectively in a business. In what ratio must the profits be divided? If a profit of Rs. 1500 is to be divided amongst them, how much will each get?

2. A st. line AB is divided in the ratio 11 : 9 and the length of the shorter segment is 0·72 cms. What is the length of the whole line?

3. If 6 furlongs of gravel road have to be repaired and there are two batches of coolies of 21 and 12 men respectively, what length of road should each batch work at?

4. Find the ratio of the speeds of a train running at 45 miles an hour and that of a race horse covering a distance of 6 fur. in 1 min. 30 secs.

5. A diamond jewel was purchased by three marwaries for Rs. 7500. The gold in the jewel was equal to the cost of a single diamond and there were 14 diamonds of equal value in the jewel. If they paid Rs. 2500, Rs. 3500 and Rs. 1500 respectively, find how many diamonds each had, if the gold was taken by the last marwari.

6. Express in its simplest form the ratio between
   (1) the values of a 4-anna piece and a three-pie coin.
   (2) a side and the diagonal of a square.
   (3) a side and altitude of an isosceles rt. $\angle$d$\triangle$.
   (4) ,, ,, ,, of an equilateral $\triangle$.
   (5) the circumference and the radius of a circle.

7. An English mile : An Irish mile : : 11 : 14. How many yards are there in an Irish mile?

8. A bankrupt has assets to the value of Rs. 13800 and his assets are to his liabilities as 3 : 4. Find the value of his liabilities.

9. The price of sugar rose in the ratio 5 : 7. If it was 7½ as. at the cheaper rate what was it at the dearer?

10. Sovereign gold consists of 22 parts of pure gold to 2 parts of alloy. Pure gold is said to be 24 carats fine and sovereign gold 22 carats fine. What is the degree of fineness of gold which contains 9 parts of gold and 3 parts of alloy?

11. If 2 sovereigns be melted with 2 sovereign weight of inferior gold 17 carats fine, what is the degree of fineness of the resulting gold?

12. Two merchants A & B speculated with Rs. 1800 and Rs. 4200 respectively. A gained $11\frac{1}{9}$% of his investment and B lost $16\frac{2}{3}$% of his capital. Find the ratio of their capitals afterwards.

13. A person invested Rs. 4800 in a venture and increased his capital in the ratio 6 : 7. (i) How much did he gain? (ii) What was the percentage gain?

14. Two lengths are in the ratio 17 : 15. The sum of their lengths is 3 fur. 140 yds. Find the shorter.

15. The ratio of the fares between a Christmas season return ticket and an ordinary ticket to and fro is 3 : 4. The price of the Christmas season ticket being Rs. 7-8 as. find the price of an ordinary return ticket.

16. The rate of income-tax is 5 pies per rupee. Find the ratio of the net income after paying the tax to the gross income.

17. The population of a district is 432000. The males are to the females in the ratio 4 to 5. Find the strength of the female population.

18. The Boatmail leaves Trichy at 7-30 P.M., and reaches Egmore 6 A. M., next day. If the speed of the Boatmail : the speed of the Express as 5 : 4 when will the Express, which leaves Trichy at 12 Noon to-day, reach Egmore?

19. Ghee worth Rs. 3-4 as. a measure is mixed with ghee worth Rs. 4-8 as. a measure in the ratio 3 : 2. If the mixture be sold at Rs. 4 a measure, find the gain per cent.

EXAMPLES XXVII.

(*Written.*)

1. A rice-dealer buys 3 cartloads of paddy at Rs. 22-8 a pothi and 5 more cartloads at Rs. 23-8 a pothi. At what price per bag of 50 measures must he sell (1) so as neither to gain nor lose; (2) to gain Rs. 45 by the transaction? A cartload is 3 pothis, a pothi = 150 measures, a bag contains 50 measures.

2. If x viss of ghee at Rs. 2-4 as. a viss be mixed with 12 viss of ghee worth Rs. 3-2 as. a viss and the mixture sold at Rs. 3 a viss the merchant (1) neither gains nor loses, (2) gains 12½ per cent. Find in each case how much of the inferior sort was taken.

***3.** In what ratio should an oil monger mix cocoanut oil worth Re. 1-4 as. a measure with an inferior variety worth Re. 1-2 as. a measure that he may sell the mixture at Re. 1-3-6 a measure.

4. Gold costs £ 3-17-10½ per oz. and silver 5 s. 6 d.; in what proportion must these metals be mixed that a lb of the alloy may be worth £ 32-5 s.?

5. A grocer mixes 4 kinds of tea which costs him 5*s.*, 4*s.*, 3*s.*, 2*s.* per ℔ respectively in the proportions 2, 3, 4, 7 respectively. Find at what rate he must sell the mixture so as to gain 25 p. c. on the whole.

*6. A merchant mixes coffee worth Rs. 2-13-6 a viss with a superior variety costing Rs. 3-1-6 and retails the mixture at Rs. 2-14 as. a viss. Find the ratio of the varieties in the mixture.

(*a*) If the mixture be sold for Rs. 3–3–9 and the merchant gain 12½ per cent, in what ratio were they mixed?

(*b*) If again there be 64 viss of the mixture how many viss were taken of each sort?

(*c*) If 12 viss of the superior sort be taken, how many viss of the inferior must have been taken to form the mixture?

7. A milkseller starts from her house with x measures of pure milk. In the first house she sells a measures and fills the vessel with as much water. In the 2nd house she sells b measures of the adulterated milk and puts in b measures of water. In the 3rd house she sells c measures of the mixture and fills the vessel with an equal quantity of water. Find the quantity of milk in the mixture.

Ans. $\dfrac{(x-a)\,(x-b)\,(x-c)}{x^2}$;

N. B.—(1) If $a_1\ a_2\ a_3\ a_4 \ldots\ldots\ldots a_n$ denote the quantities sold on each occasion, if the same quantity of water be added then and there, the quantity of milk after the nth operation will be $\dfrac{(x-a_1)(x-a_2)(x-a_3)\ldots(x-a_n)}{x^{n-1}}$.

(2) Note the special case when $a_1=a_2=a_3=a_4\ldots=a_n$.

8. A milkmaid carrying 500 seers of milk to a king had 3 guards to pass. The sentinels at the 1st guard removed 80 seers from her vessel and supplied the place with an equal quantity of water. The sentinels in the 2nd guard removed 100 seers and poured water instead. Those in the third guard removed 120 seers and filled the place with water. How much milk actually reached the king?

9. A person had 6 oz. of medicine. He drank $1\frac{1}{2}$ oz. and filled the bottle with an equal quantity of water. Next time he took 2 oz. and substituted 2 oz. of water. On a third occasion he took 3 oz. of the mixture and replaced it with water. How much of the pure medicine did he drink altogether?

10. Forty maunds of tamarind, if sold for the price it was bought, would give the merchant $1\frac{1}{19}$% gain, as the quantity originally bought was first soaked in water to make it 40 mds. How much water must he add to his wet tamarind to gain on the whole $1\frac{1}{2}$ %?

11. Pure gold is said to be 24 carats fine. Gold 22 carats fine contains 22 parts pure gold and 2 parts of alloy or inferior metal. If 12 oz. of gold 21 carats fine and 18 oz. of gold 16 carats fine be fused together, what will be the fineness of the resulting gold?

12. A goldsmith has 16 oz. of gold 22 carats fine. How much pure gold must he put, that if both be melted together, the resulting gold may be 23 carats fine?

13. An alloy contains 10 parts of silver and 4 of copper; Another contains 6 parts of silver and 2 parts of copper. If these be melted together the former weighing 21 oz. and the latter 32 oz., what will be ratio of copper to silver?

14. A cask contains 16 gallons of spirit. Two gallons are drawn off and the cask filled up with water. Two gallons are again drawn off and the cask filled up as before. This is done a third time. Compare the quantities of spirit and water remaining in the cask.

15. Two partners start a Motor Bus service. One had contributed Rs. 5,000 and the other Rs. 3,500.

(a) In what ratio should the profits be divided?

(b) If the share of the former be Rs. 430; what is the total amount of profits?

16. A gentleman found from his accounts that his average monthly expenditure on necessaries of life in a certain year was to his expenditure on luxuries as $a : b$ Next year the price of necessaries rose by p per cent and the prices of luxuries fell by q per cent, so that his expenditure increased by r per cent.

If $p=35, q=20$ and $r=10$ find the ratio $a:b$.

17. A rod x ft. long is cut into two parts in the ratio of $a:b$. Each part is again cut into two parts bearing the same ratio. Find in terms of x, a and b the lengths of the four parts. Verify your results by adding together the lengths of the four parts. If $x=4\cdot9$, $a=4$ and $b=3$ find in inches the length of each part.

18. A pint of milk weighs a lb. a pint of water b ℔. and a pint of a mixture of the two c ℔. What fraction (by volume) of the mixture is milk?

If $b=1\cdot25$, $a=1\cdot288$, $c=1\cdot276$; what fraction is milk?

19. A pint of water weighs a pounds and a pint of milk b pounds. There is a mixture of the two and this measuring a pint weighs c pounds. Find the proportion of water to milk in the mixture. If $a=1\cdot125$, $b=1\cdot164$, $c=1\cdot16$ find the ratio of water to milk.

20. (a) If x viss of oil at Rs. a per viss be mixed with y viss at Rs. b per viss and the average cost of 1 viss of the mixture be Rs. c per viss, shew that $\frac{x}{y}=\frac{c-b}{a-c}$.

(b) What weight of gold 22 carats fine must be melted with 4½ sovereigns' weight of gold 18 carats fine, that the fused gold may be (i) 19 carats fine (ii) 21 carats fine?

21. I wanted to make a gold cup of 25 sovereigns' weight. I had two watch chains each of five sovereigns' weight, but the gold was only 18 carats fine. If the balance that still was wanting in weight to make up the 25 sovereigns was given in sovereigns, find the degree of fineness of the gold of which the cup was made.

22. An old jewel was melted and the resulting gold weighed 14 *vrahanadais*, 18 carats fine. I had with me 10 sovereigns. These had to be melted together with some pure gold such that the degree of fineness of the resulting gold may be that of the sovereign which is 22 carats fine. How many *vrahanadais* of pure gold must I put. Given that £ 1 weighs 2¼ *vrahanadais*.

23. A goldsmith had with him 44 *vrahanadais* of gold 14 carats fine. He bought some gold 20 carats fine. He melted these with 8 sovereigns and the resulting gold was 18 carats fine. How much did he pay for the gold he bought if a sovereign weight of gold of the fineness he purchased is worth Rs. 13-8 as. Sovereign gold is 22 carats fine and a sovereign weighs $2\frac{1}{4}$ *vrahanadais.*

24. Six coins of equal weight made of gold and silver mixed, are melted together and recast. In one of them the gold and silver are in the ratio of 2 : 1, in two others in the ratio 3 : 5 and the rest in the ratio of 7 : 5. In what ratio will the gold and silver be mixed in the resulting fused solid?

25. The number of men, women and children employed in a tank bund is as 1 : 2 : 3; When the number of men employed is 50, the total earnings of the men, women and children is Rs. 1181-4; If the wages of a man, woman and child be 4 as., 2 as. 6 ps., and 1 a. 6 ps. per day respectively, find how much is earned on the whole by each man, woman and child.

26. A crown, made of an alloy of copper and gold weighs 16·5 oz. while the water it displaces weighs $1\frac{1}{2}$ oz. How much copper does it contain? (S. G. of gold is 19·3 and that of copper 8·96.)

27. From a vessel containing 'a' gallons of spirits 'b' gallons are taken and replaced by water. From the mixture 'c' gallons are taken and replaced by water. If the vessel now contains 'x' gallons of pure spirits, establish a relation between x, a, b, and c.

A vessel is filled with a liquid 3 parts of which are water and 5 parts syrup. How much of the mixture should be drawn off and replaced with water so that the mixture may be half water and half syrup.

28. An alloy of gold and copper weighs 60 grammes and by the water which it displaces its volume is found to be 5 cub. cms. If a cub. cm. of gold weighs 19·3 grammes and a cub. cm. of copper 8·9 grammes, determine the weight of pure gold in the alloy and the degree of fineness of the alloy.

29. (a) A man buys 25 ollocks of milk at 1 a. 6 p. per ollock and sells it at 1 a. 3 p. an ollock, making a profit of 5 annas; how many ollocks of water did he add to the milk?

(b) If he had added half this quantity of water and sold each ollock at the price for which he bought it, find as shortly as you can, the profit % he would have made.

Proportionate division.

30. The shares of three men in a business are as 3 : 5 : 7. If the profits at the end of the year amount to Rs. 348-12 as. how much should each receive? If a further profit comes later and the share of the biggest shareholder of this additional profit be Rs. 2-7 as.; find the total gross profits.

31. To invite Jagadguru Swamigal a sum of Rs. 810, had to be raised by subscription among landowners. A had 87 acres, B 12, C 30, D 18, E 4, F 13, G 3½, H 7½, K 12, L 7, M 38 and N 35. Find how much each should subscribe.

32. During the recent Cauvery floods, the people of a certain village strengthened the bund to avert a breach. This cost them Rs. 840. The Government paid half the cost. The balance had to be borne by the people and it was agreed that it should be divided amongst the villagers in the ratio of the value of the property owned by each. If a mirasdar whose property was estimated at Rs. 7500, paid Rs. 9-13-6, what is the rateable value of the property of the village?

33. A sum of Rs. 128-4 as. was collected last year to celebrate a festival in a temple and it was spent as follows:—

	Rs.	As.	Ps.
Abishekam	18	12	0
Piper	15	0	0
Flowers	20	10	0
Lighting charges	20	0	0
Prasadams	35	6	0
Miscellaneous	18	8	0

If this year, a sum of only Rs. 106–14, is collected, find the proportionate amount that can be spent on each item referred to.

34. In a distribution of alms, each child had 1 a. 4 p. each woman thrice as much, and each man four times as much. There were twice as many children as women, and thrice as many children as men, and the whole sum distributed was Rs. 23. How many were there in all?

35. Three farmers take the contract for grazing cattle in a meadow. The years' rent is Rs. 145-8 as. One of them puts 20 sheep for 7½ months on an average. Another puts 32 sheep for 10 months and the third puts in 28 sheep for only 4 months. How much should each pay towards the rent?

36. The expenses of collecting the Government Revenue is 10% of the gross receipts. If, for every net rupee that is collected the Army costs 4 as. 6 ps. Public Works 2 as. 3 ps., Education 1 anna, the interest on the Public Debt 6 as., and the civil establishment 1 an. 3 ps., find the gross receipts and the expenses of Education, if 3⅜ lacs goes annually into the Savings Bank.

37. The maximum number of marks in an examination was 400 and it was distributed in the proportion of 2 : 1 : 1 : 0·5 : 0·5, among the subjects, English, Mathematics, Vernacular Composition, History and Elementary Science. A boy secured 240 marks in all. In English he secured as many marks as he did in Mathematics and Vernacular Composition taken together, and 80% in each of the other two subjects. If he had obtained 50% in Elementary Mathematics, find how much per cent. of the maximum for the subject he obtained in Vernacular Composition.

38. A ship's company takes a prize of £ 1000 which is to be divided amongst them in proportion to their pay and to the time of service. The officers, four in number have 40 shillings each a month, and the midshipmen 12 in number have 30 shillings each a month and they have all served 6 months; the sailors 110 in number have each 22 shillings a month and have served 3 months. Find each man's share.

39. Three merchants A, B and C engage in speculation with a total capital of Rs. 35,000; A's Share : B's : C's = 1 : 3 : 3; They agree to deposit at once $\frac{1}{5}$ of their respective contributions to the capital, and an equal amount at the end of the month and the remainder at the expiration of another month. After paying the second instalment B dies and A and C make up the remainder, A paying $\frac{7}{9}$ and C $\frac{2}{9}$; The profit at the end of the year when the business is wound up is Rs. 7,420; How much does each receive and how much should be paid to B's heirs including the capital he had put in?

40. The staff and menials of a High School consist of, 1 Head-master who is paid Rs. 130: 10 Licentiates in teaching, 10 Intermediate trained teachers, 3 Matric trained teachers and 2 attenders, 4 peons, 2 gardeners and 3 clerks. The pay of each matric-trained teacher is Rs. 40, of an attender Rs. 25, of a peon Rs. 15, of a gardener Rs. 10 and that of a clerk Rs. 30. If from fee collections, grants etc., the school has an average monthly income of Rs. 2000 and the pay of an L.T. is to that of an intermediate grade trained teacher as 17 : 10 and a sum of Rs. 180 per month is spent on repairs to building, taxes, stationery etc., find how much each L. T. and each secondary grade trained teacher is paid if the school is self-supporting.

41. A school is given by the government a special grant of Rs. 1500, which is to be divided among the teaching staff according to the pay of the masters and the service they have put in the school. The staff has a headmaster of 14 years of service drawing a pay of Rs. 215, 3 senior L. T's drawing Rs. 100 each who have each put in a service of 12 years, 2 junior L. T's drawing Rs. 85 each who have each put in a service of 9 years. 2 other L. T's drawing Rs. 80 each who have put in a service of 8 years, 4 secondary grade trained masters drawing Rs. 45 each who have put in a service of 6 years each and four masters of the lowest classes drawing Rs. 25 each and who have put in a service of 15 years. How should it be divided? Find to the nearest pie each master's share.

42. Three persons *A*, *B* and *C* started on a pilgrimage for Tirupati. *B* was to have all the money with him and it was agreed that all expenses other than those incurred privately shall be divided equally among them.

A gave Rs. 173-8 as., C gave Rs. 100 and B brought Rs. 98-6 as. A bought a slab for grinding sandal for Rs. 1—12 as. and an effigy of God Venkateswara for Rs. 2—4; B bought a like effigy of God :while C bought 3 effigies of the God of a superior make each costing Rs. 4—12 and it was noted afterwards that one of the currency notes which A took for a 50-Rupee note turned out to be a ten-rupee note. On reaching home they had a balance of Rs. 41—3 as. How should the money be divided? If A had received the balance due to him how did B and C stand?

43. The estimated cost of a college day celebration was Rs. 240. The students in the Intermediate classes proposed to raise it among themselves by subscribing at 12 as. each. The students in the High School classes however joind them, whereupon a fresh rate was proposed, the amount paid by each college class student was to the subscription paid by each pupil of the High School classes as 8 : 5 so that, what each college class student would have paid as originally proposed, without the High School students, was reduced by 33⅓ per cent. Find the number of boys in the college classes and in the High school department.

44. Two farmers own adjoining lands of 450 *Kulis* and 300 *Kulis* respectively. They unite their farms, taking at the same time a new partner who pays Rs. 750, on the understanding that a third of the land will in future belong to each. How should the Rs. 750 be divided among the original owners?

SECTION X (B).

Similar Figures.

1. ABC is a triangle; AB = 7·8 cms; BC = 3 cms. AC = 7·2 cms.

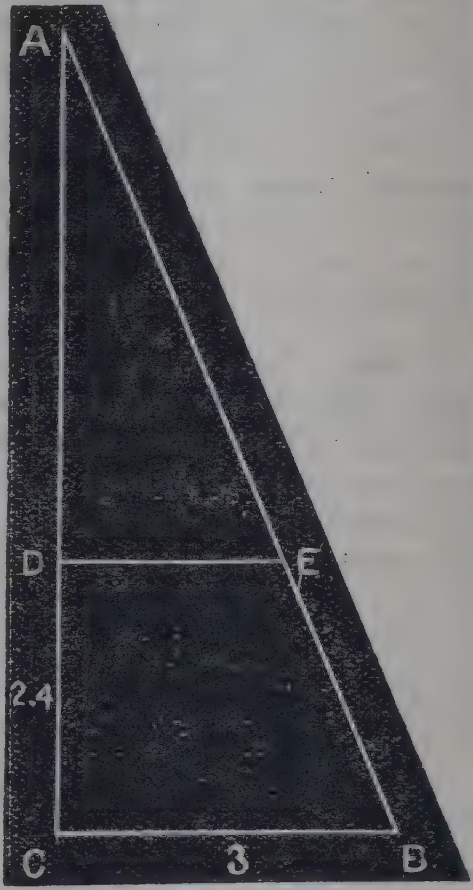
D is a point in AC, such that CD = 2·4 cms.

DE is drawn parallel to CB.

The following points must be verified by the students.

(i) △s; ADE and ACB are equiangular.

Hence if through a point in one side of a △, a parallel be drawn to the second side, to meet the third side, the new triangle thus formed is equiangular to the original △



(ii) AD = AC − CD = (7·2 − 2·4) cms. = 4·8 cms.

By measurement AE = 5.2 cms. and BE = 2·6 cms.

We find that AD : DC :: AE : EB.

Also AC : AD = AB : AE = CB : DE for the ratios are 7·2 : 4·8 = 7·8 : 5·2 = 3 : 2.

2. (a) Describe a tiangle with the following data.

c = 3 cms., **A** = 60°, B = 45°; Find by careful measurement a and b.

(b) Again describe another triangle whose base angles are 90° and 45° as before, but the base is 4 cms. Find by careful measurement a and b. Fill in the following table by taking the measurements required, as carefully as possible.

Table I.

	(1)	(2)
Side a	—	—
Side b	—	—
alt. on side a	—	—
alt. on side b	—	—
Area of △	—	—

Table II.

Ratio of side a to side a in (1) & (2)
,, ,, b to side b in (1) & (2)
Ratio of area (1) to area (2)

N.B—1. Note that the △s are equiangular.
2. The corresponding sides are in the same ratio.
3. The areas are as the squares on the corresponding sides.

3. ***(i) Such triangles in which the ratios, of corresponding sides are equal are said to be similar. Such △s are always equiangular.***

Hence.—equiangular △s are similar.

(ii) Two rectilineal figures are similar, if they are equiangular and have the ratios of all pairs of corresponding sides equal.

Note.—Two maps both of the same district, but drawn to different scales must be necessarily similar.

Two photographs of the same person, taken at the same time (but on different sizes of plates) must also be similar.

(iii) Exercise.—Name some geometrical figures which are always similar.

4. (a) Take a square whose side is a''; Its area is a^2 sq. in.
Take another whose side is b''; ,, ,, b^2 sq. in.
The areas are as $a^2 : b^2$.

(b) The three sides of a *rt.* angled △ are 3. 4 and 5 in..(i)
and ,, ,, another *rt* angled △ similar to the first are 2·4, 3·2 and 4 in...(ii)

The area of (i) is 6 sq. in. and the area of (ii) is 3·84 sq. in.

∵ the area of (i): the area of (ii) = 6 : 3·84 = 1 : ·64 = 25 : 16 *i. e.* as $5^2 : 4^2$ *i. e*, as squares of corresponding sides.

Hence—***areas of similar figures are as squares on corresponding sides.***

(c) Each side of a cube is 6 in. and each side of another cube is 8 in.

Their volumes are to one-another as the cubes on the corresponding lengths.

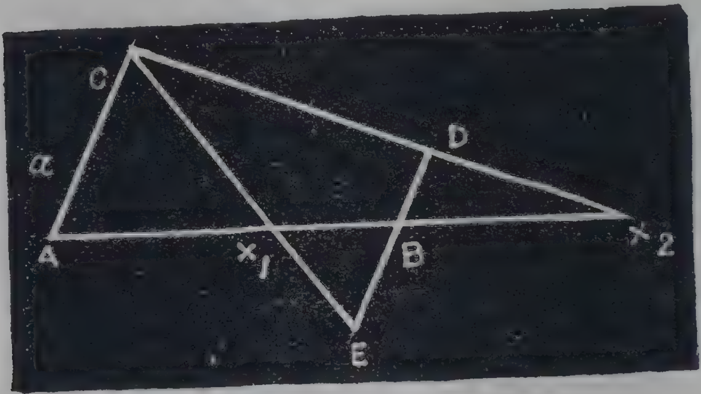
i. e., ***Volumes of similar solids are to one another as cubes on corresponding lengths.***

5. The following is a constrn. for dividing a given st. line in a given ratio, internally and externally.

Let AB be a given st. line; It is required to be divided internally and externally in the ratio of $a : b$.

At A draw AC = a cms. and at B draw BD and BE each b cms. long; DBE must be paralled to AC.

N.B.—Generally AC and DBE are drawn perpendicular to AB, using set squares.



AB is divided by CE internally, and by CD produced externally, at x_1, x_2 respectively, in the given ratio.

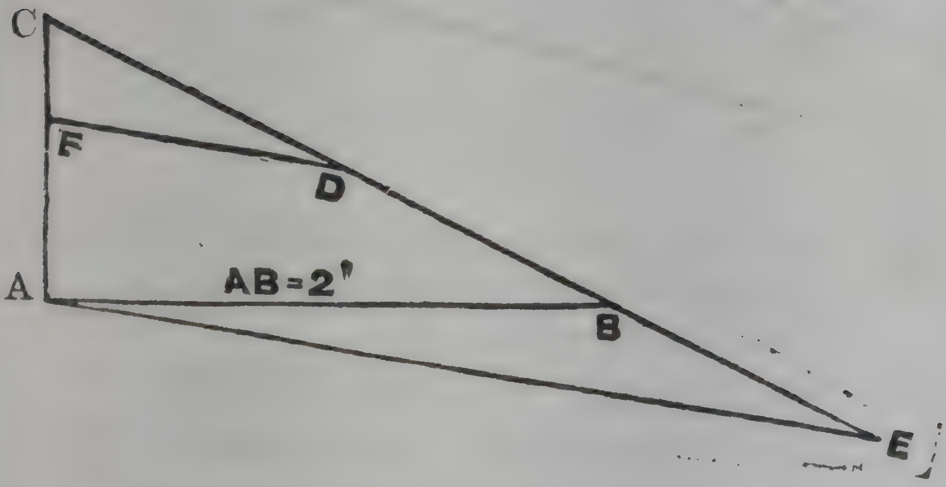
Ex. Supply Proof. Apply Property of ∥ lines and similar △s.

6. (*a*) Find graphically the value of $\frac{\sqrt{5}-1}{\sqrt{5}+1}$

$$\text{Let } x = \frac{\sqrt{5}-1}{\sqrt{5}+1}$$

Then $1 : x = (\sqrt{5}+1) : (\sqrt{5}-1)$

Find $\sqrt{5}+1$ and $\sqrt{5}-1$ graphically: This is done as follows:—$CE = \sqrt{5}+1$ and $CD = \sqrt{5}-1$,
In the figure, join AE and through D draw DF parallel to AE. CA is of unit length.



Then $CD : CE = CF : CA$

i.e., $\sqrt{5}-1 : \sqrt{5}+1 = CF : 1$; $\therefore CF = \frac{\sqrt{5}-1}{\sqrt{5}+1}$

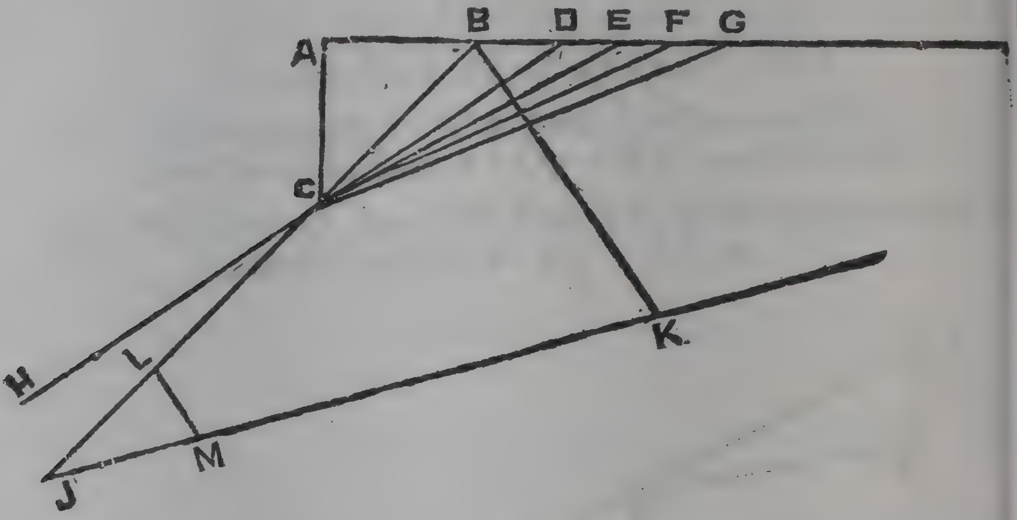
The student is now able to represent any surd graphically and fractions whose numerators and denominators are surds can also be represented graphically.

7. (*b*) *Example.* Find graphically,

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{2}+\sqrt{6}}$$

$\sqrt{5}$ can be found graphically; also $\sqrt{3}$, $\sqrt{2}$ and $\sqrt{6}$. $\sqrt{5}+\sqrt{3}$ can be found graphically by producing the line representing $\sqrt{5}$ and by attaching $\sqrt{3}$ to it by means of the dividers. Similarly $\sqrt{2}+\sqrt{6}$.

Produce D C backwards, put one point of the compasses at C and measuring CF cut off CH = CF. $\therefore DH=\sqrt{5}+\sqrt{3}$. Similarly $BJ=\sqrt{2}+\sqrt{6}$.



Draw JK = DH. Join BK, Cut off JL = 1unit = CA.
Through L draw LM parallel to BK.
Then JL: JB = JM : JK.
But $JB=\sqrt{2}+\sqrt{6}$, and $JK=\sqrt{5}+\sqrt{3}$; and $JL=1$; $\therefore\ 1:(\sqrt{2}+\sqrt{6})=x:(\sqrt{5}+\sqrt{3})$ or $x=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{2}+\sqrt{6}}$

N. B.—We have done all the constructions in one figure.

8. We solve a few examples on maps and map drawing.

The students' attention is drawn to Articles 3 and 4 of this section.

The fraction whose numerator is unity, representing the scale of a map, is called the Representativ Fraction, If a map is drawn to a scale of 1 in = 50 yds the Representative Fraction or briefly the R. F. of th map is $\frac{1}{50\times3\times12}=\frac{1}{1800}$.

If again the R. F. of a map is $\frac{1}{2400}$, the scale c the map is 1 in. = 2400 in. or 200 ft. or 1 ft. = 2400 ft. c 800 yds. The length of the original aud the equivalen length on the map must be reduced to the same denomination; i.e. the numerator 1 unit length represents on the map, as many of the same unit lengths as the denominator indicates.

Ex. 1.—The area of a district is 18000 sq. miles. A map of the District is drawn to a scale of 1″ = 100 miles (*a*) Find the area on the map occupied by the district? (*b*) To the same scale, how much area will another district of 4800 sq. miles occupy?

Solution.—(*a*) The district and its representation on the map are similar.

Areas of similar figures are as squares of corresponding sides.

$$\therefore \frac{\text{Area on map}}{\text{Area of Dt.}} = \frac{1^2}{(100 \times 1760 \times 36)^2}$$

$\therefore$ Area on a map =

$$\frac{1}{1000000 \times 176 \times 176 \times 36 \times 36} \times 18000 \times 1760 \times 1760 \times 9 \times 144 \text{ sq. in.}$$

$$= 1{\cdot}8 \text{ sq. in.}$$

(*b*) From the previous result an area of 4800 sq. miles must occupy $\frac{48}{180} \times 1{\cdot}8$ sq. in. $= {\cdot}48$ sq. in.

The work can be much simplified by adopting the unitary method.

Ex. 2.—A may of a country has been drawn to a scale of 1 in. = 200 miles and the area of the map is $4{\cdot}32$ sq. in. If it is reduced that the area may be 3 sq. in. what is the scale of this new map?

$$\frac{\text{Area of reduced map}}{\text{Area of original map}} = \frac{3}{4.32} = \frac{1}{1.44}$$

$$\therefore \frac{\text{Scale of reduced map}}{\text{Scale of original map}} = \frac{1}{1.2}$$

$\therefore$ Scale of reduced map $= \frac{1}{1.2}$ times the R. F. of the original.

i.e., 1 in. represents $1{\cdot}2$ times 200 miles = 240 miles.

Ex. 3.—There are two maps of the same estate. One is drawn to a scale of 1 in. = 250 yds. and the other 1 in. = 200 yds. If the area of the estate on the former be 1·2 sq. in. what is its area on the second map?

$$\frac{\text{Area of the estate on the former map}}{\text{Area of the estate on the second map}} = \frac{(\text{Scale of former})^2}{(\text{Scale of latter})^2}$$

$$= \frac{(\frac{4}{5})^2}{(1)^2}$$

$$= \frac{16}{25}$$

∴ Area of estate on the second map = $\frac{25}{16}$ times area of the estate on the first map.

$$= \frac{25}{16} \times 1{\cdot}2 \text{ sq. in.} = \frac{25 \times {\cdot}3}{4} \text{ sq. in.} = \frac{7{\cdot}5}{4} \text{ sq. in.} = 1{\cdot}875 \text{ sq. in.}$$

EXAMPLES XXVIII.

1. Describe a triangle ABC, given that $a = 4\frac{1}{2}$ cms. $b = 5$ cms. and $c = 7$ cms. Through D, 1½ cms. from A along AB draw DE ∥ to BC to meet AC in E.

Calculate the following lengths :—DE, AE, EC.

2. The sides of a triangle are 5, 12 and 13 cms. The shortest side of another equiangular triangle is 2½ cms. Find as quickly as you can (1) the lengths of the other sides (2) the area of the second triangle as deduced from that of the first. Check the results by measurement.

3. The shadow of a man 5 ft. 4 in. high, due to a lamp is 9 ft. 4 in. long. The man himself is 12 ft. from the foot of the lamp. How high is the lamp? Determine this both by calculation and measurement, drawing a figure to scale.

4. (a) The slope of a hill is 1 in 30. How far up must one go to be at a vertical height of 150 feet?

(b) A rod fixed vertically in the ground is 8 ft. long and its shadow is 10½ feet long. What would at the same time be the length of the shadow of a man 5 ft.

high, of a tree 20 feet high, of a tower 60 feet high and of a child 3 feet high.

5. (a) Two friends walk one along each road from the junction of two roads which meet at an angle of 90° Their rates are as 4 : 3; If the faster has walked 150 yards find geometrically how far, the other would have walked along the other road.

(b) If at a time, the distance between them be 120 yards how far would each have walked?

6. (a) Compare.

i. the areas of two circles whose radii are a and b inches respectively.

ii. the areas of two equilateral triangles whose sides are p and q inches respectively.

iii. ,, perimeters are x and y inches ,,

iv. ,, squares whose sides are p and q inches.

v. ,, perimeters are x and y ,,

vi. ,, similar rt. < d triangles whose hypotenuses are a and b inches.

vii. the sides of two regular hexagons whose areas are p and q square inches.

viii. the areas of two ,, ,, perimeters are p and q inches.

(b) The radius of the base of a cylinder is x in and its height is h inches.

,, ,, another cylinder is $\frac{x}{2}$ in.

and its height is $\frac{h}{2}$ in.

Compare. i. the areas of their curved surfaces.
ii. their volumes.

7. A marble statue is 5 ft. 6 in. high. A statuette which is an exact copy of the statue, is 9 in. high and is half the weight it would have been in marble. The statuette weighs ½ ℔. What is the weight of the statue to the nearest half-cwt?

8. The height of a cone is h in. and the radius of the base is s in.

another cone is $\frac{h}{3}$ „ $\frac{s}{3}$ in.

Compare (1) the areas of their curved surfaces
(2) their volumes.

9. If a stone whose volume is a cub. in. be dropped in each cylinder in Q. 8, each being half full of water, compare the heights to which the water would rise.

10. (a) Describe a circle with 1·5 in. radius. Inscribe (i) a regular hexagon, (ii) a regular octagon.

(b) At the corner of each figure draw tangents to the circle. Compare the areas of the two hexagons, of the two octagons so formed.

11. Thales used the following method to find the heights of the Egyptian Pyramids. He took a stick of known length and found the length of the shadow. He then measured the length of the shadow of the Pyramid and calculated its height? Describe the principle employed by him.

A stick 4 feet long casts a shadow 5·8 feet long. If a shadow cast by a tower at the same time is 104·4 feet, find the height of the tower.

12. AOB is the diameter of a circle whose centre is O and radius $= x''$. OD is another perpendicular radius; P is a point in OD such that $\text{OP} = y''$. Shew that the triangles APO, ACB (C being on the circumference in AP produced) are similar. Hence find in terms of x and y the lengths of AC and CB. Also find the area the triangle ACB.

13. I want to find the height of a tower. I know the compound wall is 80 yards from the feet of the tower and each turret on either side of the gate in the compound wall is 7½ ft. high. From a distance of 50 ft. from a turret, I find the top of the tower in a line with the top of the turret. Can I find the height of the tower? If so what will it be?

14. If a cord 55 yards long be so stretched from the top of a pole 44 yards high, to a nail driven in the ground that it might just touch the top end of another pole distant 30 yards from the nail, find by careful drawing and by calculation, the height of the longer pole over the shorter.

15. The width of a river is ascertained thus :—Two poles one 8 ft. and the other 6 ft. are taken to the bank. The shorter is fixed vertically on one bank. Receding at right angles to the straight bank in a line with the post fixed, the longer one is fixed at a distance of 100 ft. from the shorter. Looking from the top of the longer pole this top, the top of the shorter and the point just opposite in the other bank are in a line. Find from a carefully drawn figure, the width of the river.

Also calculate the width and check the measurement.

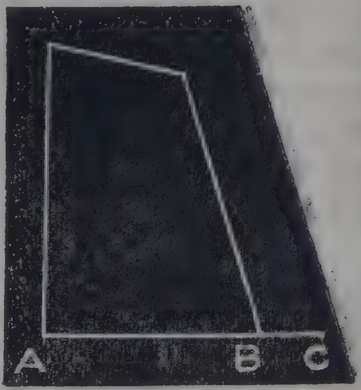
16. Standing at a certain distance from a straight road, I can see two lantern posts along the further side. The posts are, I know, 121 yards apart. I saw a cyclist riding straight along the road between me and the posts, in a line with the first lantern post and 30 seconds later I saw him in a line with the other; I walked straight to the first lantern post and having walked a distance of 80 feet, crossed the cycle-track. Walking 8 ft. more in the same line I reached the lantern post itself. At what rate was he cycling ?

17. Two roads meet at an angle of 60 degrees and from the point of meeting two cyclists A & B start, one along each road, A travelling at 8 miles and B at 5 miles an hour. Find, geometrically how far A will be from the starting point when B has travelled a distance of 8 miles and also where B will be, when A has travelled a distance of 10 miles.

18. In the painting of a scenery a house 45 ft. high is of height 2·4 in. What would be the heights in the picture of :—

(i) A man standing in front, of the house whose height is 5 ft. 4 in.

(ii) A tree, by the side of the house, 40 ft. high ?



19. Prick off the given quadrilateral. Then enlarge the figure so that the side AB shall have the length AC.

What is the ratio of AC to AB? What is the ratio of the area of the enlarged figure to that of the given one?

20. The boiler of a steam engine holds 12,000 gallons of water. A model of the steam engine is made and the boiler in the model holds 12 pints. What is the length of the connecting rod of the steam engine if its length in the model is 6·75 in.?

21. Two similar ships A and B are loaded similarly. B is twice the length of A. The wetted area of A is 12000 square feet, and its displacement 1500 tons. State the wetted area and displacement of B.

22. (a) Draw a straight line AB whose length is 1·4 inches.

(b) Find geometrically a length which is $\sqrt{5}$ times AB.

(c) Hence find another length which is $\dfrac{\sqrt{5}}{\sqrt{3}}$ times AB.

23. Find geometrically the values of—

(i) $\dfrac{\sqrt{5}}{\sqrt{3}}$ (ii) $\dfrac{\sqrt{5}}{2\sqrt{3}}$ (iii) $\dfrac{\sqrt{2}+1}{\sqrt{2}-1}$: check your results by values obtained (correct to one decimal place) by calculation.

24. The number of acres in the county of Suffolk is 961,000. What is the area which represents this county on a map of England which is drawn to the scale of 1″ to 100 miles? Give your answer to the nearest hundredth of a square inch and draw a square of this area on any convenient scale.

25. The map of Australia is printed on a sheet of paper 9·8″ by 7·2″. The scale of the map is given as 1 inch to 500 miles. The area of the island is known to be 2974581 sq. miles. Find, correct to two significant figures, what fraction of the area of the sheet is occupied by the map.

26. A plan is to be enlarged so that a plot of ground which occupied 1·5 sq. in. in the original shall occupy at least 1 sq. foot in the enlarged plan. If the original measured 6 in. by 4 in. what must the least dimensions of the enlarged plan be, if the enlarged plan is of the same shape as the original?

27. The scale of a map is 1 : 1,000,000. (a) Find in miles, furlongs and yards the distance between two towns which in the map are 2·88 inches apart. (b) If the map occupies about 0·75 of a page 14″ by 10″ find the area of the country it represents.

28. A railway rises at a gradient of 1 in 12. How many feet does it rise in a mile? If a train runs up the line at a speed of 20 miles an hour, how many inches does it rise in a second?

29. A map is drawn to a scale of 1 inch = 500 miles. If a country occupies 70% of a sheet 14 inches by 12 inches what is its actual area in sq. miles?

30. In a map, a piece of land is enclosed by three roads 11, 12, 13 inches long. The map itself is 2 ft. by 2½ ft. and represents 2 acres. Find the area of the land enclosed by the roads.

31. The sides of a rectangle are reduced as per scale 1 ft. = 10 inches. By what percentage is the area of the rectangle diminished?

32. On a map drawn to a scale of $\frac{1}{2500}$ the sides of a rectangular field are 4·7 and 3·2 cms. Find the area of the field in sq. metres.

33. i. The scale of a map is 1 : 1,000,000. How many miles is this to an inch?

ii. If the map of a country drawn to the above scale occupies three-fourths of a sheet 2½ ft. by 1½ ft., find the area of the country to the nearest square mile.

34. (1) The scale of a map is 1 in 5,000,000. Find the scale in miles per inch to the hundredth of a mile.

(2) If on a map drawn to the above scale a country is represented by a quadrilateral whose diagonal AC is 2·5 inches, AB = 2″, AD = 1·5″ and angles B and D are right angles, find the area of the country.

SECTION XI.

The Circle.

The Circumference and Area of the Circle.

Chords and Arcs of Circles and Angles in a Circle.

1. (i) Draw two or three circles of different radii and find with the help of a thread, the circumference in each case.

(ii) Place an ink dot on the circumference of a rupee coin and roll it along a sheet of paper. Measure the distance between two consecutive dots made by the wet ink on the paper. This distance is, of course, the circumference of the rupee.

(iii) Roll a strip of paper round a cylinder and prick a hole with a pin at a place where the ends of the strip overlap. The circumference of the cylinder is the distance between the holes.

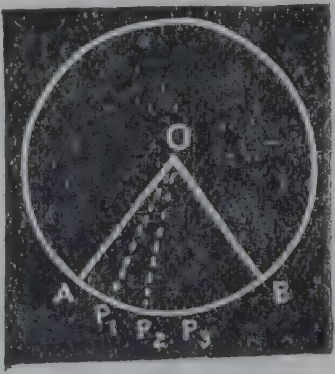
(iv) Divide in each of the above cases, the length of the circumference by the diameter.

In all the above cases the result i.e. the ratio of the circumference to the diameter, is constant, being very nearly equal to $3\frac{1}{7}$.

Hence we have the formula :– $C = \pi d$ **where** C **denotes the circumference and** d **the diameter.** π is a number i.e. the ratio of the circumference to the diameter. The ratio is generally taken as $3\frac{1}{7}$. A closer approximation will be $3{\cdot}14159$. Sometimes the value of π is also given as $\frac{355}{113}$.

The figure formed by two radii and the arc intercepted by them is called a *Sector*.

2. Draw any circle with centre O and radius r. Let OAB be any sector. Let P_1, P_2, P_3 etc. be points in the arc AB, very near each other so that arc $P_1 P_2$ almost coincides with chord $P_1 P_2$ and area of sector $OP_1 P_2$ is almost equal to the area of the triangle $OP_1 P_2$. Then the area of the sector $OP_1 P_2 = \frac{1}{2}r\ P_1 P_2$. Similarly if arc AB be divided into a number of, say, n equal parts each of which is very small, the area of the sector is n times the area of OP_1P_2. Hence *the area of the sector is* $\frac{1}{2}r \times P_1 P_2 \times n = \frac{1}{2}r \times l$ for $AB = n \times P_1 P_2$.



(i) The area of the sector is, half the length of the bounding arc multiplied by the radius.

(ii) If the whole circumference be divided into p parts each of which is very small, the student can easily deduce the area of the circle, $\frac{1}{2}$ of the $O^{ce} \times r$; but the semi-circumference $= \pi r$; $\therefore$ **the area of a circle is** πr^2 **where r denotes the radius.**

3. (i) *The length of an arc is that fraction of the whole circumference of the circle as the angle subtended by the arc at the centre is of 360°.*

(ii) The area of a sector is that fraction of the area of a circle as the angle contained by the radii is of 360°.

In general terms. (i) $l = \frac{n}{360} \times 2\pi r$ where l denotes the length of the arc, n the no. of degrees at the centre subtended by the arc and r the radius.

(ii) $S = \frac{n}{360} \times \pi r^2$ where S denotes the area of the sector.

Ex. From the formula $S = \frac{n}{360} \times \pi r^2$; deduce the formula $S = \frac{1}{2}lr$.

Solution :— $$S = \frac{n}{360} \times \pi r^2$$

$$\therefore \ 2\,S = \frac{n}{360} \times 2\pi r^2$$

i.e. $2\,S = \frac{n}{360} \times 2\pi r \times r = lr$ from relation (i) in the previous paragraph $\therefore$ $S = \frac{1}{2}lr$.

4. (a) It is very easy for the student now, to describe regular polygons in circles.

(i) Divide the four rt. angles at the centre into five equal parts with the protractor and describe a regular pentagon in the circle.

(ii) Divide the four rt. angles at the centre into 6 equal parts and describe a regular hexagon in the circle. Note that the side of the hexagon = the radius.

(iii) Divide the four rt. angles at the centre into eight equal parts and describe a regular octagon in the circle.

(iv) Divide the four rt. angles at the centre into twelve equal parts and describe a regular dodecagon (a regular polygon of 12 sides) in the circle.

(b) In each of the above cases by taking necessary measurements, find the area of the polygon and find in each case, the difference between the area of the circle and the area of the polygon.

In each case the area of the polygon is

(a side × ½ the altitude of a △ × no. of triangles)
= (a side × ½ the altitude of a △ × no. of sides)
= (Perimeter × ½ the altitude of a △).

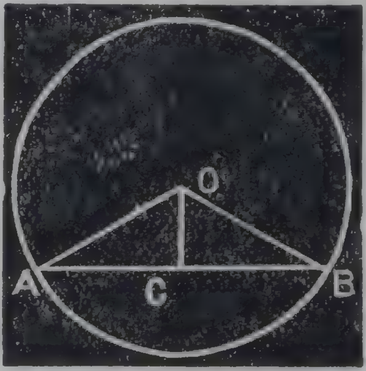
As the number of sides increases the difference between the area of the circle and the polygon becomes smaller.

When the number of sides is very, very great the sides of the polygon almost coincide with the circumference and the area of the polygon is almost equal to the area of the circle and the altitude of a △ is very nearly the radius itself.

So that we deduce the formula for the area of the

$$\text{circle} = \text{circumference} \times \tfrac{1}{2}\text{ radius.}$$
$$= 2\pi r \times \tfrac{1}{2} r = \pi r^2.$$

5. (A) *The chord of a circle.*—Take any two points A and B on the circumference of a circle. The st. line AB is called a chord. Join AB. Also take O the centre of the circle. Join OA, OB. From O, drop a perpendicular OC to AB; compare △s ACO and BCO; We have ∠OAC = ∠OBC for AO = BO and ∠ACO = ∠BCO being both right ∠s ∴ the △s ACO and BCO are congruent. ∴ i. e. ∠AOC = ∠BOC and AC = BC.



i. *The perpendicular from the centre to a chord bisects the chord.*

Also, the student can likewise prove that :—

ii. *The st. line joining the mid-point of a chord to the centre, is perpendicular to the chord.*

iii. *The perpendicular bisector of a chord passes through the centre.*

(B) (i) The chord AB is 'a' inches long. The radius of the circle is r in. long. Find the distance AB of the chord from the centre.

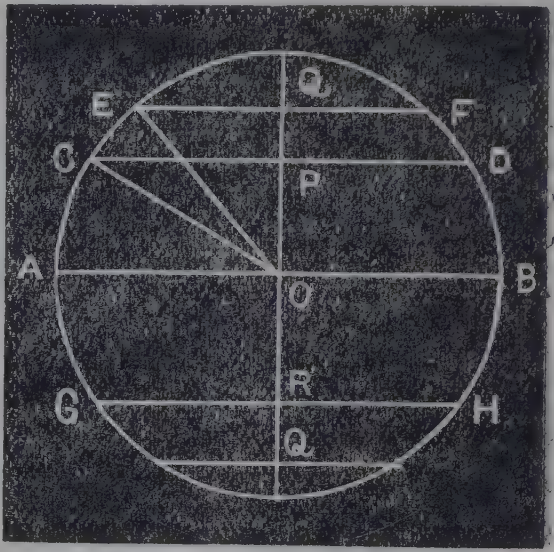
$$\left.\begin{aligned}\text{In the figure } AC &= a \text{ in.}\\ OA &= r \text{ in.}\end{aligned}\right\} \begin{aligned}&\therefore\ OC^2 = (r^2 - a^2) \text{ in.}\\ &\therefore\ OC = \sqrt{r^2 - a^2} \text{ in.}\end{aligned}$$

ii. In the same circle place another chord = AB. Find its distance from the centre. Deduce the fact that—

Equal chords are equidistant from the centre and the chords which are equidistant from the centre are equal.

(C) Two chords are parallel to each other and are placed in a circle whose radius is 6 cms. The length of the longer chord is 4·8 cms. and that of the shorter is

3·6 cms. Find the distance between them. How will you geometrically place the two chords in the circle ?



The distance of EF = (3·6 in.) from O is $\sqrt{6^2 - 1{\cdot}8^2}$ cms.

CD = (4·8 in.) from O is $\sqrt{6^2 - 2{\cdot}4^2}$ cms.

i.e. $OQ = \sqrt{(6 + 1{\cdot}8)(6 - 1{\cdot}8)}$ cms. $= \sqrt{7{\cdot}8 \times 4{\cdot}2}$ cms.

$= 6\sqrt{1{\cdot}3 \times {\cdot}7}$ cms. $= 6\sqrt{{\cdot}91}$ cms.

$OP = \sqrt{(6 + 2{\cdot}4)(6 - 2{\cdot}4)}$ cms. $= \sqrt{8{\cdot}4 \times 3{\cdot}6}$ cms.

$= 12\sqrt{{\cdot}7 \times {\cdot}3} = 12\sqrt{{\cdot}21}$ cms.

$\therefore PQ = OQ - OP = 6\sqrt{91} - 12\sqrt{{\cdot}21}$ cms.
$= {\cdot}6\sqrt{91} - 1{\cdot}2\sqrt{21}$ cm.
$= {\cdot}6 \times 9{\cdot}54 - 1{\cdot}2 \times 4{\cdot}58$ cm.
$= 5{\cdot}724 - 5{\cdot}496$ cm.
$= 5{\cdot}72 - 5{\cdot}5$ cm.
$= {\cdot}22$ cms.
=

```
            9 54
       91 | 953
       81 |
 185 |1000
     | 925
1903 |  7500
        5709
1906  | 179 100
            458
```

The figure is only a rough figure drawn for the purpose of calculation only.

```
     21 | 458
     16 |
 85 |500
    |425
908 | 7500
    | 7264
916 |23600
```

The student should note that the distance may also be

OP + OR = 11·22 cms. for, the chord at a distance = OP, on the other side of AB must be equal to CD. In the figure the chord GRH = CD for, OR is cut off = OP.

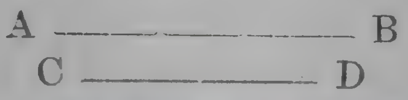
To place two parallel chords of given lengths in a circle.

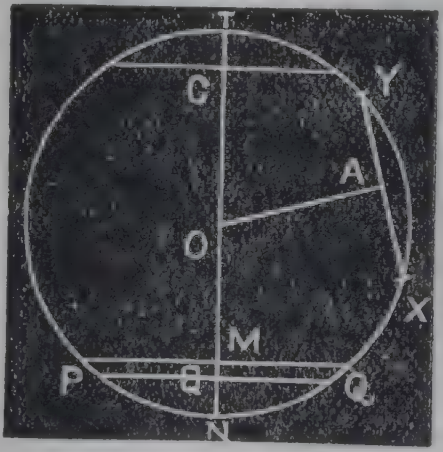
One of the chords is easily placed.

The other chord must be parallel to this and must have a given length. Place the second chord anywhere in the circle. Drop a perpendicular on this chord from the centre. Take the compasses and measure the distance...............(1)

Draw the perpendicular bisector of the first chord. Produce it to meet the circumference on either side. From the centre cut off distances on either side of the perpendicular bisector equal to the distance (1) we have already taken. Chords drawn parallel to the first chord through these two points will be equal to the second chord.

Let O be the centre of the circle given and AB & CD—two lengths. It is required to place in the circle two chords parallel to one another and equal respectively to AB and CD.





Take the length BA. Take P in the O^{ce} and cut off PQ = AB. From O the centre draw OM perp^{r}. to PQ to meet the O^{ce} at points N and T.

Place another chord XY = CD anywhere in the ⊙; OA is its distance from the centre. Cut off OB and OC in TN.

Draw $\|^{s}$ to PQ through B and C. These two chords will be the chords required.

What will happen when a given length is greater than the diameter?

(D) The following points must be noted.

(i) *The nearer the chord to the centre the longer it is.*

(ii) *The farther the chord from the centre the shorter it is.*

(iii) *The greatest chord of a circle is the diameter.*

(E) The perpendicular bisector of a chord passes through the centre. Hence

(i) *Find the centre of a circle, a portion of the circumference i.e. an arc being given.*

(ii) *Which is the same thing as describing a circle through three given points.*

(iii) *Which again is the same thing as describing a circle round a triangle i.e. drawing the circum-circle of a △.*

(F) (i) The length of the chord AB of an arc ACB is given $= 2a$. The perpendicular bisector of the chord meets the ⊙ in C. CD is called the height of the segment. If this height is also given (say) h, find the radius of the circle.

Suppose the length of the radius is r. CD produced backwards must go through the centre. Suppose O is the centre.

In the △ ODA, ∠ ODA is a rt. ∠

$$OD = r - h.$$

$$OA = r.$$

$$\therefore \quad \text{we have } r^2 = a^2 + (r-h)^2$$

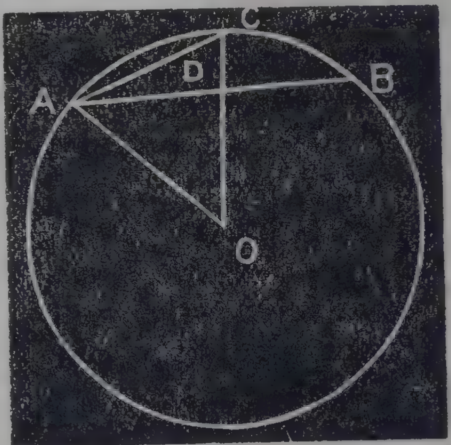
$$\text{i.e. } r^2 - (r-h)^2 = a^2$$

$$\text{i.e. } h(2r-h) = a^2$$

$$\text{i.e. } 2r - h = \frac{a^2}{h}$$

$$\text{i.e. } 2r = \frac{a^2}{h} + h$$

$$r = \tfrac{1}{2}\left(\frac{a^2+h^2}{h}\right)$$



(ii) Given radius r and height of segment h. Find the length of the chord.

$$r^2 - (r-h)^2 = a^2$$

$$\therefore \; a = \sqrt{h(2r-h)}$$

(iii) Given a and r find h i.e., given the length of the chord and the radius, find height.

$$(r^2 - a^2) = (r - h)^2$$

$$\therefore r - h = \sqrt{r^2 - a^2}$$

$$h = r - \sqrt{r^2 - a^2}$$

(iv) (a) Given length of chord and the height, find chord of half the arc cut off by the chord. From (iii) supra $h = r - \sqrt{r^2 - a^2}$

$$\therefore \text{AC} = \sqrt{a^2 + \{r - \sqrt{r^2 - a^2}\}^2}$$

$$= \sqrt{a^2 + r + r^2 - a^2 - 2r\sqrt{r^2 - a^2}}$$

$$= \sqrt{2r^2 - 2r\sqrt{r^2 - a^2}}$$

$$= \sqrt{2r(r - \sqrt{r^2 - a^2})}$$

6. **To the teacher.**—We have so far detailed at some length formal proofs of propositions and algebraical relations between chords, radii and arcs etc. The other properties of circles need not be dwelt upon so elaborately. However the following geometrical truths must be known to the students and we therefore give below some notes.

Work again Ex. 32. P. 45 and deduce therefrom that

(a) *The angle subtended by an arc of a circle at the centre, is double the angle subtended by the same arc at the o^{ce}.*

hence

(b) i. *Angles in the same segment of a circle are equal, and*

ii. *the angle in a semi-circle is a rt. angle,*

and from this

Give the construction for drawing a tangent to a circle (a) at a given point on the O^{ce}, and (b) from a given point outside—(See Qs. 68 and 69 in examples that follow).

Applying knowledge of congruence of Δ^s *prove that the two tangents are equal and that the st. line joining* *the given point to the centre of the circle, bisects the angle between the tangents and is the perpendicular bisector of chord of contact of tangents.*

(ii) Other geometrical truths that might be deduced are that

i. the opposite angles of a quadrilateral inscribed in a circle are together equal to two right angles.

ii. if a chord be drawn from the point of contact of a tangent, the angle which this chord makes with the tangent is equal to the angle in the alternate segment.

Hence give construction to draw a segment of a circle to contain an angle equal to a given angle.

(iii) (*a*) If the sum of the radii of two circles be equal to the distance between their centres, the two circles touch one another externally.

(*b*) If the difference be equal to the distance, the $\odot^s$ touch one another internally.

What must be the relation between the sum of the radii and the distance between the centres that two circles may cut one another?

(*c*) Circles that have the same centre are said to be *concentric.*

Concrete exercises have been proposed in the examples that follow, that many of the geometrical truths enumerated above may be derived by the students. The construction for drawing tangents to a circle is also given as an exercise, step after step, indicated.

The definition of a tangent must be deduced from the fact, that the tangent is the limiting position of a secant.

Hints. Draw any st. line to cut a circle at P and Q. Place a ruler along this secant and roll it away from the centre. P and Q approach ; call them P_1 and P_2. If again we roll P_1 and P_2 approach each other still further and at last if we go on rolling, there must be a position where P_1 and P_2 coincide.

7. The Regular Polygons.—The Hexagon and the Octagon.

(a) The student will revise the working of questions 37 to 39 in Examples VI. He will then easily find out how to draw a regular hexagon (i) in a circle, (ii) on a given base.

(b) If tangents to the circle be drawn at the angular points of an inscribed hexagon, another hexagon will be formed.

The area of a hexagon is six times the area of the equilateral triangle on the radius and the student knows that the radius = a side of the hexagon.

The area of the interior hexagon : The area of the circumscribed hexagon = 3 : 4.

(c) To construct a regular octagon on a given base :—

Let AB be the given base. Bisect AB at C and draw CD perpendicular to AB.

Make CD = CA or CB.

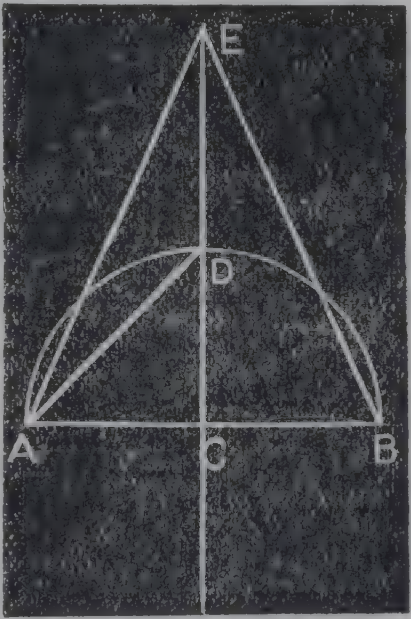
Join DA and DB.

Produce CD to E such that DE = DA or DB.

Join EA and EB.

E is the centre of the circle round the regular octagon whose side is AB.

Draw a circle and step off lengths equal to AB along the O^ce^. Join the points in order.



Proof—$\angle$ADB is a rt. angle why?
$\angle$AEB is 45° why?

Area of the octagon.—Let $AB = 2a$. Then $AC = a$ and $CD = a$; $\therefore AD = a\sqrt{2}$; $\therefore DE = a\sqrt{2}$;

$$\therefore EC = CD + DE = a(\sqrt{2}+1)$$

$$\therefore \text{Area of } \triangle AEB = a^2(\sqrt{2}+1)$$

Hence area of the octagon is $8a^2(\sqrt{2}+1)$ where $2a$ is the length of the side of the octagon.

(d) If the octagon be inscribed in a circle of radius r, a side of the octagon will be $r\sqrt{2-\sqrt{2}}$ i.e. 0·7654 times the radius.

EXAMPLES XXIX.

N. B.—Questions marked with an asterisk must be done in Form IV.

*1. (a) Find the circumference of circles that have radii of 3″, 4″, 3½″, 7″, 10½ cms., 2′ and 2½′.

(b) Find the radii of circles that have a circumference of 11″, 20″, 33 cms., 38½ cms., 7′ 4″ and 5½ yds.

*2. What is the circumference of a wheel whose radius is 2 ft. 4 inches. How many times will it turn round in travelling over a mile? If another wheel makes 20 more revolutions in travelling over the same distance, will its radius be greater or less and by how much?

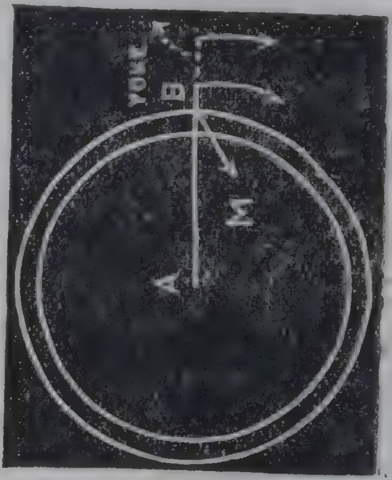
*3. The circumference of the earth is 40,000000 metres and the length of a metre is 39·3708 inches. Find the diameter of the earth in miles if $\pi = 3{\cdot}1416$.

*4. The hindwheel of a certain bicycle revolves x times for one revolution of the pedal. If the diameter of the wheel be d ft and n the number of revolutions of the pedal, express D (the distance travelled by the wheel) in terms of x, n and d. Find the distance if $x = 3\frac{1}{2}$, $d = 4\frac{1}{2}$ and $n = 500$.

*5. The diameters of the forewheel and hindwheel of a carriage are 2′ 11″ and 4′ 8″ respectively. Two nails, one on the outside of each wheel touch the ground together when the carriage starts. After what interval of time will the nails touch the ground together once again if the carriage goes at 3⅔ miles per hour and how often will this happen before the carriage goes 1½ miles?

6. The bund round a circular tank has a width of 14 ft. all round at the top and 5 times round the tank along the inner edge is an exact mile. If a man walking at the

rate of 3 miles an hour goes round the tank along the outer edge, in how many minutes will he complete the round?



*7. A stone roller M, moving along a circular roller pit is attached to a log AB 14 ft. long fixed to a peg A at the centre of the pit. Mortar is ground in the pit when the stone is rolled along it, by a pair of oxen yoked together by a pole $3\frac{1}{2}$ ft. in length. If the ox near the pit keeps always to the extremity B of the log and makes 240 complete rounds in an hour, find the speed of the other ox in miles per hour.

*8. (a) Find the areas of circles of radii 1″, 5″, 7″, $2\frac{1}{2}$″ yds. and $3\frac{1}{2}$ yds. taking π to be equal to $3\frac{1}{7}$.

(b) Find the area of a sector whose bounding radii are 5″ long and contain an angle of 40°.

(c) If the length of the arc AB of a sector OAB be 3″ and the radius OA of the circle be 5″ find the area of the sector.

(d) An arc of length 11″ subtends an angle of 50° at the centre. Find the area of the sector. What will be the radius and the ⊙ce of the circle of which the arc forms a part?

9. The chord forming the base of a segment of a circle is 2″ in length and subtends an angle of 60° at the centre. Find the area of the smaller segment cut off by the chord to the nearest length of a sq. in. If the chord subtends (1) a rt. angle, (2) an angle of 120° at the centre, find to the nearest tenth of a sq. in. the area of the minor segment formed.

*10. (a) What is the radius of a circle whose area is equal to 6·16 sq. inches?

(b) If the area of a circle is equal to that of a square of side 22 inches, find its diameter.

(c) Find to the nearest inch, the radius of a circle whose area is equal to—

i. the sum of the areas of 2 circles of radius 9″ and 3¾″.

ii. the difference in area of 2 circles of radius 10″ and 6″.

iii. the area of an equilateral triangle of side 3″.

iv. the area of a regular hexagon of side 2·2″.

v. the area of a regular octagon of side 2″.

*11. A man has two circular plots of ground of diameter 36′ and 15′. If he gives them in exchange to a square plot of ground whose area is equal to the sum of the areas of the 2 circular plots, find the cost of fencing the square plot at 7 pies per yard. If instead of the square plot he chose a circular plot how much will he have to spend in fencing it at the same rate?

*12. (a) In the midst of a circular table of diameter $2x$ ft. is spread a circular carpet of radius y ft. leaving a border all round. Find the area of the border.

(b) If the radius of the carpet be a ft. and the width of the border be b inches, find the area of the border.

*13. (a) Find the area of a circular plot of ground of diameter 70 yds.

(b) If there is a pathway round the field having a width of 3½ yds. find the cost of gravelling it at 6 annas per sq. yd.

(c) Find in question 6 the cost of levelling the circular track on the bund at 4 ps. per sq. yd.

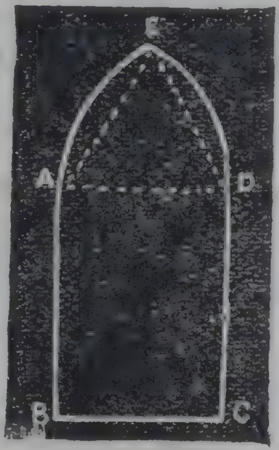
*14. The semi-circular benches 1½ ft. in breadth are placed round a circular table 12 ft. in diameter, so that there is a space of 1 foot, all round the table, for the persons sitting on the benches to move about. How many persons can seat themselves round the benches if each person requires a sitting space $5\frac{1}{7}$ sq. ft. and a vacant space of 1½ sq. ft. on either side?

*15. A stone roller of diameter 2 ft. gets reduced in weight by 25% after some years of use. Supposing that it has worn out uniformly all round, find by how much its diameter has decreased. If its diameter gets reduced to 1½ ft., what fraction of its original weight will it have?

*16. ABCD is a square of side 2″. With the mid-points of the sides as centres and with radii ·7″ semi-circles are described all round, outside the square. Find the area of the figure formed.

*17. If the fig. represents a lawn drawn to a scale of 1″ to 5 yds., find i. the cost of fencing the lawn at 3 annas a yard ii. the cost of turfing it at 1 a. 3 ps. per square foot.

18. ABCDE represents the section of gothic archway leading to a lecture hall. If AB = 6 ft, AE = ED = DA = 4 ft, find (1) the area of the archway and (2) the cost of planks required to fit it up with doors at 12 as. per sq. foot.



*19. The total area of the surface of the Earth is about 197 millions of square miles. Of this area the water covers 145 million square miles. The area of the Arctic ocean is 5½ million square miles, of the Atlantic 35 million square miles, of the Pacific 68 million square miles, of the Indian 25 million square miles.

(a) Find correct to one decimal place what percentage of the whole area, that of each of the oceans and that of the land mass is.

(b) Draw a circle with any convenient radius to represent the surface of the globe and draw sectors therein whose areas are proportional to the areas of the land mass and the oceans. The name of the place which each sector represents, should neatly be entered in that sector.

*20. The main divisions of the Races of the world and the estimated number of each are given in the table below.

Division.	Estimated No. in millions.
Mongolian	610
Caucasian	725
Negro	210
Semitic	100
Malayan	104·5
Red Indian etc.	30

(*a*) If the total population be represented by 360 what number correct to one decimal place would show the number in each race respectively.

(*b*) If a circle of 3 cms. radius represent the area of the earth, divide the circle into sectors whose areas are proportionate to the strength of the races. Enter the name of the race occupying the area, the sector represents.

***21.** Two plots of ground, one a square and the other a circle, each contain 73,686 sq. ft. Find the difference in their perimeters.

22. (a) Draw a circle whose radius is 2 in.
(b) Place a chord AB in the circle, of length 0·8″
(c) Place another chord CD = 0·8″ parallel to AB
(d) Place a third chord EF = 1·2″ ,, ,,
Shew that there are two positions for chord EF.

23. A sphere of diameter 3 in. rests on a hollow cylinder placed on a table. The diameter of the cylinder is 2·8 in. Find the height of the centre of the sphere above the table. Height of the cylinder is 8 inches.

24. Two parallel chords AB, XY, 1·8″ and 2·4″ respectively are placed in a circle whose diameter is 3·1 in. Find the area of the trapezium ABXY.

25. Describe a circle and let its radius be *a*.″ Draw a radius OP, O being the centre. Take a point M in OP at a distance of *b* in. from O. Through M draw MNR perpendicular OP, and meeting the ⊙ at N and R. Find in terms of *a* and *b* the length of NR.

26. (a) If the radius of a circle be *p* in. and the length of a chord be 2*q*. in. find the heights of the two segments into which the chord divides the circle.

(b) Draw a circle of radius 3·6 cms. and place in it a chord 4·2 cms. in length. Find its distance from the centre. Also find the heights of the major and minor segments. Verify your results by calculation.

27. The ends of a diameter of a sphere as centres, two circular caps are cut off, to make a paper weight. If the diameter be 3 in. and the radius of each circle at the ends 1 in. find the distance between the centres of the circles *i.e.*, find the height of the weight.

(a) If the greater diameter be $2a''$ and that of each of the smaller be $2b''$ find the distance between their centres.

28. AB is a chord of a circle 3·2 cms. long. If its distance from the centre O of the circle be 1·2 cms., draw the circle and find its radius.

(b) The chord AB of a circle is 3·4 cms. long and the height of the arc AB = 1 cm. Draw the arc in position and find the radius of the circle of which the arc forms a part.

29. (a) Draw a circle of diameter 3 inches and place in it two parallel chords of length $2''$ each. Measure the distance between their midpoints.

(b) In a circle of radius $1·6''$ draw 2 parallel chords $2·4''$ and $2''$ in length, such that the centre lies between them. Measure the distance between the chords.

(c) If the chords be on the same side of the centre, what is the distance between them?

30. (a) Draw a circle whose radius is $(\sqrt{5}-1)$ inches.

(b) Describe an equilateral triangle in it. Measure the length of a side.

31. Out of a circular disc of metal 35 equal circular holes are punched; the weight of the metal thus punched out is to the weight of the perforated disc as 45 : 67; compare the diameters of the disc and of the holes.

32. The range of a gun is 3½ miles. It is stationed at a place 1½ miles from a straight road. What length of the road can it command? If a fur. more of the road has to be guarded, find from a diagram, the least distance the gun should be brought nearer to the road.

33. A circular race track 48 ft. broad is made such that twice the distance round the course midway between the edges of the track is a mile and a half. Find :—(1) the radius of the inner edge of the track and (2) the cost of turfing the track at 3 ps. per sq. foot.

34. The points (—2, 8), (6, 12) and (8, 6) are the points of contact of the inscribed circle of a triangle. Draw the triangle and find its area by any method.

35. (a) A piece of wire a inches long is bent in the form of a square with a circle round it. Find the length of a side of the square.

(b) If the same piece could be bent in the form of a circle with a square whose sides touch the circle find the length of a side of the square. Evaluate these results when $a = 24$.

36. (a) The sides of a triangular field are 78 yds. 74 yds. and 69 yds. Draw the plan of the field to any convenient scale.

(b) A cow is tethered to a stake by means of a rope such that when the rope is fully stretched the cow can just touch each of the sides of the above field at one point only. Find the position of the stake, the length of the rope, and calculate the area of the portion of the field the cow can graze through.

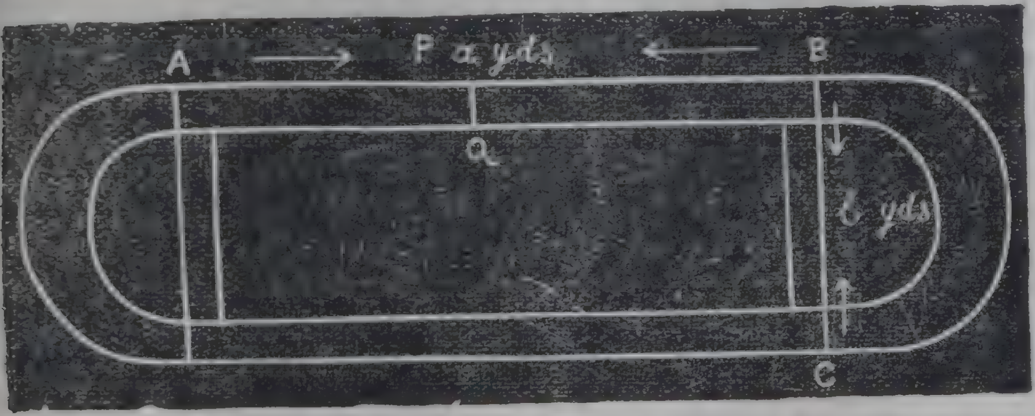
37. A rectangular field of 5 acres, 200 yds. long, is planted with trees in rows perpendicular to the length one yard from row to row and one yd. from tree to tree in the same row. If a width of a yard all round the field remain unplanted, find the number of trees.

If the same number of trees were planted along the circumference of a circle at the same distance apart, what would be the diameter of the circle ?

38. A circle of 123 ft. radius is divided into three parts by concentric circles. Find the radii of these circles so that the three parts may be of equal area.

39. The chord of an arc of a circle is 1·8″ and the height of the arc $\frac{2}{5}''$. Draw the arc and find the length of the radius of the circle of which the arc forms a part.

40. (a) The accompanying diagram represents a race-course. The curved portions are semicircular. Find the distances (1) along the inner edge of the track and (2) along the outer edge.



(b) Two competitors starting abreast at P & Q run, one keeping to the outer edge, and the other to the inner edge. Find where P will be when Q reaches the starting point, their rates of running being 5 and 4 miles an hour respectively, given that $a=340$ and $b=180$, the width of the track being 30 yds.

41. (a) A chord of a circle subtends an angle of 60 degrees at the centre. If the chord is 10 cms. long find the areas of the two segments into which the chord divides the circle.

(b) From a point on the circumference of a circle two chords are drawn at right angles and their lengths are 1·2″ and 1·6″. Find the area of the circle and draw the circle with the chords.

42. A barn is a square of 40 ft. side. At one corner a cow is tied by a rope 60 ft. long. Making all possible calculations and taking necessary measurements, find the area the cow can graze through.

43. (a) Describe a circle whose radius is 2·5 inches. Place in it a chord AB of length 4 inches. Draw the perpendicular diameter COED, C and D points on the circumference, O centre of the circle, E being on the chord. On CE and ED describe semicircles one on each side of the line COED. What fraction of the original circle is the figures bounded by the semi-circumferences CAD, DE and EC?

(b) What peculiarities do you observe regarding the three circles and the chord AB?

44. In the centre of a square-walled enclosure of 12 ft. side, there is a stake to which a bear is tied by means of a rope 14 ft. long. There are narrow openings in the middle of two of the opposite walls through which the bear can come out. Ignoring the width of the opening, find (i) the area through which the bear can freely move and (ii) the length of a fence at a distance of 2 ft. from the farthest limits the bear can reach. Also draw a neat plan to scale.

45. The area of an equilateral triangle is 1,7320·5 square feet. About each angular point as centre, a circle is described with radius equal to half the length of a side of the triangle. Find the area of the space included between the three circles. ($\pi = 3{\cdot}1416$ and $\sqrt{3} = 1{\cdot}73205$.)

46. A field is in the form of an equilateral triangle of side 220 yards. A cow is tethered at a corner to a stake by means of a rope, such that it can graze only through half the area of the field, Find to the nearest inch the length of the rope.

47. A circular disc of cardboard one foot in diameter is divided into six equal sectors by straight lines through the centre and in each sector a circle is inscribed touching the bounding radii and also the arc at its middle point. If the six circles be cut and removed, find the area of the cardboard remaining.

48. A circular plot of ground is such that the times (taken by two men walking at the same rate of 3 miles per hour from one point on the circumference to the opposite point, one along a diameter and the other along a semicircumference, differ by 2 minutes. Find the area of the circular plot of ground.

49. A square field is of side 50 ft. A cow is tethered to a stake at a corner by a rope 25 ft. long. Next day it is tethered at the next corner by the same rope and the third day at the third and the fourth corner on the fourth day. If it grazes throughout the area it is possible for it to graze, find the area of the portion impossible for the cow to reach.

Draw a neat plan to scale and shade tde portion the cow could not reach.

50. A towel 6 ft. in length hangs under gravity over a roller 2·8 inches radius. One end of the towel touches the floor and the other end is 1 ft. 6 in. above the floor. Find the height above the floor of the centre of the roller.

51. Two points A and B are 18·6 cms. apart. Two equal circles whose radii are 10·4 cms. are described with centres A and B to cut AB in C and D and each other in E. EN is perpendicular to AB. Find EC, ED, EN.

52. Two equal circles with centres P and Q and radius PQ and QP are drawn to cut each other at A and B. If $PQ = r$ shew that:—

i. Arc AQB subtends an angle of 120° at P.
ii. $AB = r\sqrt{3}$.
iii. area of $\triangle PAB = \dfrac{r^2\sqrt{3}}{4}$

Also find the area common to both circles in terms of r.

53. Draw a circle whoses radius is an inch. Place a chord one inch long in the circle. On this chord and on side remote from the major segment, describe an isosceles triangle whose height is 4 inches. Find the area enclosed by the major segment and the sides of the isosceles triangle.

54. A process-server had to go to four places B, C, D, and E from A; B is 10 miles due east of A; C, D are such that CA, DB, CD are all equal, and E is 3 miles from C, and angle CDE is a rt. angle. Find the distance of the shortest route he had to go through and draw a figure to scale.

55. The side of a square is 12 in; find the area included between the square and the circumscribed circle.

56. An equilateral triangle inscribed in a circle has an area equal to $144\sqrt{3}$; What is the area of the circle?

57. A window is in the form of a rectangle 2 feet wide, surmounted by a circular arc (less than a semircle) of radius 3 feet. The height of the window from the base to the highest point of the circular arc is 4 feet. Determine the height of the rectangle.

58. The length intercepted on a certain straight line by a circle of radius 3″ is 5″. What will be the length intercepted on the same straight line by a concentric circle of radius 5″?

59. Find the ratio of the areas of two circles inscribed in equilateral △s, if the perimeter of one triangle is four times that of the other.

60. An enclosure is bounded by two sides of an equilateral triangle and a semicircle whose diameter is the third side of the triangle. If the side of the triangle is 24 ft. what is the area of the enclosure?

61. (a) Draw AB, 3″ long. On AB as diameter describe a circle. Take a point K in AB ·5″ from the centre O. Draw KC at right angles to AB to cut the semicircle in C. Show that $CK^2 = AK \cdot KB$.

(b) Show how the above principle can be made use of, in finding the square root of a given number geometrically.

(c) Obtain geometrically the square roots of 7, 17 and 23 correct to the first decimal place.

62. (a) The diameter of a circle is 2·5.″ Draw the circle and insert in it a chord AB of length 2.″ Bisect the minor arc at C and join AC. If the diameter through C cuts AB at K and the circumference again at D, show by measurement and by calculation that $AC^2 = CK \cdot CD$. Hence deduce the formula (Chord of half the arc)^2 = diameter × height of the arc.

(b) Show how the above formula gives us another method of finding the square root of a given number geometrically. Do question **61.** (c) according to this method.

(c) Measure with the help of a thread or with dividers the length of the arc ACB and show that the length obtained is approximately equal to $\frac{8AC - AB}{3}$ (the formula given by Hughens.)

63. A boy while playing on a trapezium sweeps a horizontal distance of 11·25 ft. during a swing. If the vertical rise of the boy during a swing is observed to be

3·25 ft. find geometrically or otherwise the angle traced out by the boy during the swing.

64. A small canal 12 ft. broad is to be spanned by a bridge with a circular arch resting on two piers each 8 feet high. If the greatest height of the bridge be 12 ft., represent in a diagram the section of the arch, and find the radius of the circle of which the arc forms a part.

65, An arch like the minor arc of a circle 8 ft. in diameter is built over two pillars 6 ft. apart. Find by means of a diagram drawn to scale the height of the arch.

66. Find by calculation the chord of half the arc, if the length of the arc is 7 cms. and the chord of the arc is 6 cms. Then describe the circle and find its radius. Find also the height of the arc. Apply. Q. **62** (c).

67. A strong flexible bamboo stick 7 ft. in length is bent in the form of a bow and a string 6 ft. long is tied to its ends. If the length of the arrow used should be at least 2 inches greater than the height of the arc, draw an outline of the bow and find the least length of the arrow required.

68. (a) Draw a circle with centre O and radius 2 cms. From any point A on the circumference draw a st. line AM at right angles to the radius OA. What is AM in relation to the circle?

(b) Show how a tangent can be drawn from any point on the circumference of a circle. A is a point on the circumference of a circle with centre O and radius 2·4 cms. If the length of the tangent AM from A be 1·8 cms, find OM.

(c) O is the centre of a circle of radius 4 cms. A & B are two points on the circumference 2½ cms. apart. Draw tangents through A and B and measure the distance between the centre of the circle and the point of intersection of the tangents.

69. (a) Describe a circle with centre A and radius 2·6 cms. Take any point B, 4 cms. from A and on AB as diameter draw a circle cutting the first circle in C and D. Show that BC and BD are tangents to the first circle.

(b) Hence deduce a method for drawing a tangent to a circle from any point outside it.

70. Draw AB, 5 cms. long. With A as centre and 3 cms. as radius describe a circle. Draw tangents from B to this circle and show that they are equal. Measure the angle between the tangents and the distance between their points of contact with the circle.

71. A circular picture frame of diameter 1 ft. has a groove all round. A string with its two ends tied together is passed round the frame and the frame is then hung from a nail on the wall by the string. If the least distance between the nail and the frame be 2 feet, find (1) the angle which the two parts of the string make at the nail. (2) the length of the string that is not in contact with the frame. (3) the length of the full string.

72. From a point without a circle two tangents are drawn making an angle of 60°. If the length of each tangent is 15 in. find the diameter of the circle. Draw a figure to scale.

73. Draw a circle of radius 3·3″. Place a chord AB in it 2·5 inches long. Take points C, D. E etc., in the major arc and join them to A and B. Measure angles ACB, ADB, AEB etc. What do you observe? What relationship do these angles bear to the central angle AOB? (Vide Q. 32 Examples VI.) Draw through A a tangent AM to the circle. How is this angle BAM related to the angles ACB etc.,

74. (*a*) Let AB be 2 inches long and let AM drawn below it make an angle of 45° with it. Where will the centres of circles passing through A and B lie? What is the locus of the centres of the circles having AM as their tangent, the point of contact being A? Find hence the centre of the circle passing through A and B and touching AM at A, Show that the angle in the major segment =45°.

(*b*) Hence deduce a method for constructing a segment on a given base, containing a given angle.

(*c*) Describe segments containing angles of 60°, 80°, 120° and 135° on bases equal to 3·5 cms., 4 cms., 1·8″ and 1″ respectively. Find in each case the height of the segment and the radius of the circle of which the segment forms a part.

75. (a) The base of a triangle is 3·4 cms. and its vertical angle 60°. Where will its vertex be?

(b) If one of its other 2 sides be 3 cms., draw the triangle and measure its other side.

(c) Construct a triangle ABC given,

1. $c = 2{\cdot}2$ cms. $b = 2{\cdot}5$ cms. $C = 45^\circ$.
2. $c = 1{\cdot}8''$, altitude on $c = 1{\cdot}2''$, $C = 70^\circ$.
3. $c = 3$ cms., median on $AB = 2{\cdot}4$ cms. $C = 55^\circ$, and measure the remaining parts.

76. (a) A, B, C are three points such that $AB = 2''$, $BC = 2{\cdot}4''$ and $ABC = 60^\circ$. Find a point equidistant from A, B and C and hence describe a circle passing through them. (The circum-circle).

(b) In the above question, find a point O within the fig. ABC equidistant from sides AB, BC and CA and hence describe a circle touching the sides of the triangle ABC.

(c) Describe a triangle ABC with $AB = 2''$, $BC = 3''$ and $CA = 2{\cdot}5''$. Circumscribe a circle about and inscribe a circle within the triangle and compare their radii.

77. (a) Fold a circular sheet of paper along a diameter AB. Fold the paper again so that A and B coincide with the centre. Show that the points marked by the creases on the circumference form the angular points of a regular hexagon.

(b) Show how a circular sheet of paper can be folded so that the points marked by the creases on the circumference may form the angular points of (1) an equilateral triangle. (2) a square. (3) a regular octagon.

78. Two boys worked out the area of a circle of a given radius; one used the approximation $\frac{22}{7}$ for π and the other $\frac{355}{113}$ and gave their answers correct to the nearest square inch. The answers differed by 1 square inch and both had worked out correctly however. What was the radius of the circle whose area they had been asked to compute?

79. (a) Describe a circle of 3 cms. radius. Draw AB and produce it to D such that the tangent from D to the circle may be $3\sqrt{3}$ cms. Do the whole construction and shew that two such tangents can be drawn. AB is a diameter.

(b) If DP and DQ be the tangents each of $3\sqrt{3}$ cms. find correct to the nearest centimetre, by calculation, the area of the figure formed by the tangents and the arc PQ.

80. A hollow cylinder of height 8 inches and internal diameter 6 inches is placed on a table. A sphere of diameter 10 inches is placed to rest on the cylinder so that part of the sphere is within the cylinder. Find at what height the centre of the sphere is vertically above the table.

81. If the engine-wheel make 'r' revolutions per minute when the engine is moving at the rate of 'm' miles per hour and if 'd' be the diameter of the engine-wheel, in feet establish a relation between r, m and d.

82. Two circles of diameters 3·4 and 2·0 inches cut each other and the common chord is 1·6 inches in length Describe the circles and shew that the distances between their centres is either 2·1 or 0·9 inches.

83. A watch chain has 60 circular links joined in the ordinary way. The outside diameter of each link is $\frac{5}{16}$ of an inch ; the inside diameter $\frac{5}{32}$ of an inch. What is the length of the chain ?

84. By drawing correct figures compare.

(i) The height of a hexagon and of an octagon on equal bases.

(ii) The bases of a hexagon and of an octagon of equal heights.

Check your results by calculation.

N.B.—In drawing the figures, the constructions must all be done geometrically.

85. Show how to inscribe a regular octagon in a square. If the side of the square be 2·4″, what will be the area of the circumscribed circle of the octagon inscribed? Draw the figure.

86. Take figure of Q. 32 in Examples VI. Join AB. The magnitude of $\angle$ ACB is $(3x+y)^\circ$, that of $\angle$AOB is $(7x-y)^\circ$ and that of $\angle$OAB, $\left(\frac{x}{6}+12y\right)^\circ$. Find the magnitude in degrees of the angles.

87. Take any four points A, B, C and D on the O^{ce} of a circle in order, and join them. ABCD is a quadrilateral. $\angle$A contains $(6x+y)^\circ$; $\angle$B, $\left(\frac{x}{3}+13y\right)^\circ$; $\angle$C, $(9x-y)^\circ$; $\angle$D, $\left(8x+\frac{y}{3}\right)^\circ$. Find the angles of the quadrilateral.

SECTION XII.

Solids.

The Prism, the Cylinder and the Cone.

Introductory:—

The articles in this section are more for the teacher than for the student, who, however, will have a clear grasp of the subject if he carefully studies what we have detailed below. Whether it is practically possible for teachers to experimentally teach the subject on the lines described and that in the limited time at their disposal, we are not to answer, though we should tell the teacher that lessons on solids must be given at least to boys in Form II or III, if not to the younger in the lower classes. If a little of each portion be done in each of the lower forms, the whole subject will have been gone through in the three years of the high school life of the student.

We are quite averse to giving ready formulae; but an author has to satisfy so many of diverse ideas and notions and therefore, we have effected a compromise in having given the mensuration formulae earlier in Examples III.

1. **A Word to Masters.**—Even from the lower forms the children must have had considerable practice in card board modelling. What is meant by surface or a line (the two-dimensional and the one dimensional) is not as easily understood by children as the idea of bulk or of volume The child or for the matter of that, every one of us, sees, observes and handles only three-dimensional things. We propose below a series of examples on paper cutting etc.. and hope that, before the topic of solids is taken up, the student will be asked to go through the exercises. He will not then blindly rely on the formula for the volume of a cone or for the curved surface of a cylinder.

2. **The Laboratory for the teaching of solids.**

(i) A few cement square bricks.

(ii) A gross of match boxes.

(iii) A few deal wood boxes—made of planks of uniform thickness—a few with lids and a few without lids. A few boxes made of 2″ planks are necessary.

(iv) Chips of wood of varying thicknesses—(i) all being rectangular. (ii) blocks of wood on rectangular bases and on polygonal bases. Prisms.

(v) Circular pieces of wood—each ½″ thick; about 50 of the same diameter and thickness and a few others of different sizes.

(vi) Cardboard—ruled and unruled. Coloured or tinted if possible.

(vii) One or two good sets of Mathematics Instruments, for blackboard work and also for work on paper. The latter for teachers' use in the class-room. The boys must have a look each day, at the ideal instruments the teacher is handling.

(viii) Boxes made of galvanised tin—cylindrical, rectangular ......Two or three in each variety.

(ix) Ounce glasses of varying capacities.

(x) Glass Jars graduated—cub in and cub cm. About a dozen each to hold not less than a pint.

(xi) Lead weights of known volume, and a few irregular solids of known volume.

(xii) Cheap balances—about a dozen. (Excellent balances can be made locally with good twisted rope and scale pans of tin or iron.)

(xiii) Models of solids.

EXAMPLES XXX.

Oral and Practical.

1. The teacher shows a box without the lid :—

How many copies of your Modern Elementary Mathematics will the box hold? (Let the copies in the class be collected and boys find out by the actual doing of the thing.)

2. Let the teacher then bring a big book, like the Encyclopaedia and ask the boys how many such books the box will hold.

Elicit "The bigger the book, the less the number of copies the box will hold."

"The bigger book occupies more space."

"The capacity of the box is such that it can hold so many copies of Mod. Ele. Math. and only a *less* no. of copies of the Encyclopaedia."

The idea of *Volume* in thus neatly elicited.

3. Arrange 4 match boxes end to end. Form four such rows and close them up. Place another such layer above this. How many boxes are there? In each layer there are 16 boxes. There are two such layers. The number of boxes must therefore be 32.

4. Each layer contains 5 along the length and 4 along the breadth and there are three layers. How many boxes are there?

5. If each layer contains 6 along the breadth and 8 along the length, how many will there be?

6. If each layer contains 6 along the breadth and 8 along the length and if there are 4 such layers one above the other, how many will there be on the whole?

7. I ordered for 80 copies of a book. I tore open the parcel along two sides and found 4 along the breadth and 5 along the height. The parcel was rectangular. How many must there be along the length?

8. I ordered a few thousand bricks. I was not able to count them as cart-loads arrived. I therefore asked the bandyman to arrange them into equal rectangular blocks. I found that there were 5 blocks. Each block had 15 bricks along the length and 12 bricks along the breadth. The height was that of 10 bricks placed one above the other. How many bricks were there?

9. *N.B.*—(*i*) ***Generalise, and elicit the formula for the volume of a prism*** *viz.* ***Area of base × height.***

ii. Take a number of blank cards all of the same shape and size. Cut off a rt. angled triangle from one corner, so that the sides of the △ cut off coincide with two adjacent sides of the card. Place this card above another, rule off and take away the rt. angled triangle Similarly cut off equal rt. augled triangles from about 100 cards.

What kind of rectilineal figure is a card thus cut off?

What is its area?

Place all the cards, one above the other, so that the equal sides coincide, and make it into a *nice* packet.

Measure the height.

What will be the volume of the packet?

How many sides has the packet including the top and the base?

What kind of figure is each face?

iii. Similarly put the rt. angled △s cut off, one above the other, so that the sides containing the rt. angle coincide.

Again note that the bottom and the top are rt. angled △s and that the faces are rectangles. What will be the volume of the solid formed?

Any solid which is capable of being sheared into a number of equal and congruent plane figures is called a prism.

The students must be shown some models and must be asked to reason out if that particular one is a prism. Which, in each case, must be the base?

(i) The room itself (ii) a cube (iii) a loaf of bread (iv) a book (v) The gable roof (vi) A block of cylindrical wood (vii) The cylindrical wood from which a plank has been sheared off. etc. etc.

A diagram must in each case be drawn to represent the base.

10. *Note.*—(*a*) We do not like the idea of telling the students what a cube is, a rectangular solid is and then what a prism is. Let the boys handle the concrete things, and intuitively they will find out that the volume of a prism is area of the base multiplied by the height or length. They handle the solids, observe them, find the volume in so many cases and then obtain the rule. This rule they will afterwards apply to all prisms an idea of which they will have, by simply reading the descriptions given in the questions.

(*b*) The students must make models with cardboard and they will soon learn how to find out the surfaces of solids.

(*c*) The students will be asked to derive the formulae and the teacher will draw their attention to the formulae given in Examples III if necessary.

11. (*a*) The students must be given cardboards and asked to

i. Make a rectangular—open tray
ii. (*a*) „ „ closed box.
  (*b*) Let them be shown a box made of thick planks and then asked to find the external dimensions and the internal dimensions.
iii. „ a cube.
iv. „ a cylinder open at both sides.
v. „ „ at one end and closed at the other.
vi. „ „ closed at both ends.

(*b*) The boys must be taken to a carpenter's shop or the services of a carpenter may be requisitioned. The carpenter will

i. drill a big cylindrical hole, in a rectangular solid, perpendicular to the base
ii. „ „ „ cylinder „ „ „
iii. saw out a plank from a cylindrical piece of wood.

(We have ourselves taken to the class room, the soft interior of the plantain tree வாழைத்தண்டு) and have successfully given the lesson on cylinders.

12. (i) Practical work—Cut out of a circular strip of filter paper of radius 3 in. a sector containing an angle of 90°. Fold the sector in such a way that the radii coincide and the bounding arc becomes a circle. What shape has it assumed?

The arc of the original sector is now the circumference of the base.

What will be the radius of the base?

The original radius is now a slant side—what is the height of the solid?

Note that the curved surface is equal to the area of the sector itself.

(iii) Take two vessels, one cylindrical and the other conical· of the same height and also having the diameters of the bases equal. (A few sets of different sizes may be made of tin.) Find how many times water should be poured from the conical vessel into the cylinder to make it full.

(The glass funnel in the laboratory and cylindrical graduated jars might be used for this purpose.)

iii. Practically demonstrate that a right circular cone is generated by the rotation of a rt.-angled triangle about one of the sides. The side about which the △ revolves is the axis of the cone. The end of the axis or top of the axis (away from the rt. angle) is called the vertex. The slant side is the hypotenuse.

(iv) Take the tin mould which our females have at home to prepare பருப்புத் தேங்காய். Ask the tinker to get ready another but cylindrical of the same height and of the same base. Place the tin-mould inside the cylinder, so that the bases coincide (vertex-topmost point). Pour water into the remaining space until the water is flush with the surface. Divide the height of the cylinder into 3 equal parts. [This you can do by taking a strip of paper $\frac{1}{4}''$ wide and of the same height as the cylinder and folding it into 3 equal parts. (Have you seen how doctors paste the strip of paper thus cut, on to the side of a bottle?) Gently take away the mould. Note that the water stands to $\frac{2}{3}$ of the height of the cylinder. What therefore is the volume of the cone?

EXAMPLES XXXI.

N.B.—Questions marked with an asterisk relate to rectangular prisms and must be done in Form IV.

Written.

*1. (a) Take a few bricks and arrange them thus:—Place 4 bricks end to end leaving no space between adjacent bricks. Form four such rows and press them together leaving no space between rows. Arrange 4 such layers. How many bricks do you want?

(b) Take 4 inch-cubes and put them side by side in a row. Place 3 such rows so that the twelve cubes form a layer. Arrange 5 such layers one above the other and say what kind of solid is formed. Measure its length, breadth and height. What is their product equal to? If V gives the volume of solid and l, b, h, its three dimensions, establish a relation between V, l, b, h.

***2.** (a) Show that the volume V of a rectangular solid can also be expressed as Ah where A gives the base area and h the height.

(b) In the two formulæ $V = lbh$ and $V = Ah$ express each measurement in terms of the rest.

(c) i. If $l=5''$ $b=3''$, and $h=4''$ find V
ii. If $b=5''$, $h=6''$ and $V=360$ c. in. find l
iii. If $A=10$ s. ft. and $V=11{\cdot}25$ c. ft. find h
iv. If $l=2b$, $h=5''$ and $V=1690$ c. in. find b.

***3.** A tennis court 80 ft. by 40 ft. is to be covered with gravel to a depth of 3 inches. Find to the nearest cwt., the weight of gravel required, given that 1 c. ft. weighs 110 lbs.

***4.** Find the volume of a log of wood 20 ft. long, 15 inches broad, and 12 inches thick. Find how many planks 5 ft. by 12 in. by 1 in. can be sawn from the log if during sawing 10% of the wood gets wasted.

***5.** A tin 10 inches by 10 inches by 16 inches is four-fifths full of ghee and is purchased for Rs. 59-5-0. If the ghee is sold by means of a measure of capacity 112 c. inches at Rs. 3-8-0 per measure, find the profit made in the transaction.

***6.** Find the cost of making a table from the following data :—

Wood required :—

4 legs each	3″ by 3″ by 2½′	at Rs.	6	per c. ft.
2 frames each	3″ by 1″ by 5′	,,	6	,,
2 frames each	3″ by 1″ by 2¼′	,,	6	,,
2 planks each	5′ by 14″ by 1″	,,	7	,,
Labour charges ...	...	,,	4	
Nails, Screws etc., Varnishing.		,,	2	

***7.** The earth dug out from a rectangular field 80 ft. long, 60 ft. broad, is just enough to raise the level of another field 50 ft., by 40 ft., by 6 inches. Find to what depth the earth has been dug out from the first field.

*8. Taking the dimensions of a brick to be 8″ × 4″ × 2½″, find the number required to build the walls above the floor of a room 28 ft. long, 18 ft. broad and 15 ft. high, the walls being 2 ft. thick and the room being provided with 2 doorways each 8′ × 3¼′ and 4 windows each 4′ × 2½′ and making no allowance for the space taken by the mortar. Find the cost of the bricks at Rs. 5-8-0.

*9. The foundation for the walls round a rectangular room 24 ft. by 16 ft. is 2½ ft. wide and 3 ft. deep. If the earth dug out is spread evenly over the floor to raise its level, find by how much will the level of the room be raised.

*10. Given that 1 oz. of gold is worth £ 4-5 sh. and that 1 c. in. of gold weighs 10 ozs.; find the average thickness of a gold leaf 40 sq. feet of which can be made from a sovereign.

*11. If a sheet of lead 10·89 metres long and 20 decimetres wide weighs 1678·32 kilograms and 1 c. centimetre of lead weighs 11·4 grams, what is the thickness of the sheet to the nearest tenth of a millimetre?

*12. A book of gold leaf contains 25 leaves, 3⅓″ square and $\frac{1}{282000}$ of an inch thick. If 1 c. in. of gold weighs ·697 lbs. and 1 lb. of pure gold is worth £61-19 s., find the value of the gold in the book to the nearest penny.

13. A rectangular stone trough is 3 ft. 9 in. long, 2 ft. 7 in. broad and 1 ft. 2 in. deep externally and weighs 874 lbs. when empty. The interior surface is irregular, but it is found by trial that the trough holds 38 gallons. Find to one place of decimal how many times the stone is as heavy as water bulk for bulk; (1 c. ft. of water weighs 62·3 lbs. and 1 gallon of water weighs 10 lbs.)

*14. An inlet to a tank 80 yds. long, 50 yds. broad and 5 yds. deep has a rectangular cross-section 1 ft. by ½ ft. and water flows through it at 3 ft. per second. If the tank be empty when water begins to flow, find when it will be half full? If the tank has also an outlet having the area of its cross-section equal to ⅜ sq. ft. and if water just begins to flow out at the same rate through it when it is three-fourths full, find when the depth of water in the tank will be 4 ft.

***15.** From the corners of a rectangular card-board x'' by y'', 4 equal squares of side a'' are cut off. The four projecting rectangles that result are folded up to make the sides and ends of a box. Express in terms of x, y and a the capacity of the box. Find the capacity if $x = 21$, $y = 3a = 16\frac{1}{2}$.

If in the above question the sides and ends are squares what will be the shape of the box and what will be its capacity?

***16.** An open box made of wood 1″ thick weighs 46 lbs. and its external dimensions are 26″ by 20″ by 16″. What would be the weight of a closed box of the same wood and of the same internal dimensions, the planks used being ½″ thick.

***17.** The internal dimensions of a closed box are x'' by y'' by z'', and the box is made of wood a'' thick. Find (1) the capacity of the box; (2) the No. of bocks a'' by b'' by c'' that can be packed inside; (3) the maximum length of a ruler that can be placed in it; (4) the volume of wood in the box; (5) the cost of painting it with colour inside at m annas per sq. foot. (6) the cost of varnishing it outside at n pies per sq. ft.

18. A prism of height h'' stands on a base in the form of (1) an equilateral $\triangle$ of side a'', (2) a regular hexagon of side b'', (3) a regular octagon of side c''. Find in each case the volume of the prism and the total surface area. Give numerical answers if $a = 4$, $b = 3$, $c = 2$ and $h = 8$.

19. (a) Describe a trapezium having its parallel sides equal to 2·5″ and 7″ and the slant sides 1·2″, each and find its area.

(b) If this represent the cross-section of a hip-bath tub 2 ft. long to a scale of 1 in. to 1 ft. find (1) the capacity of the tub, (2) the area of the zinc plate required to make the tub.

20. The channel which conveys the water required for a certain city is 8 ft. broad at the bottom and 12 ft. at the top and is 2 ft. deep. If the average rate of flow is 1 mile per hour, find to the nearest hundredth of a gallon the number of gallons supplied per day to each inhabitant, given that the population of the city is 540,000 and that a gallon contains 277·274 c. inches.

21. (a) Construct a pentagon ABCDE given $AB = 2{\cdot}4''$; $AE = ED = DC = CB = 1{\cdot}5''$; $\angle A = \angle B = 90^\circ$ and find its area.

(b) If this represents the front elevation of a school hall 40 ft. long drawn to a scale of 1 in. to 10 ft., find the number of students that it can accommodate allowing 127 c. ft. of air space to each student.

22. Take a rectangular strip of paper and bend it so that the two lengthwise sides coincide and the breadthwise sides become circles? What is the shape of the solid formed? If the radius of its circular ends be r inches and the height be h'', what is the volume V, of the solid? What is its curved surface equal to?

23. In the above question if

i. $r = 2$, $h = 8$, find V and S.
ii. $r = 6$, $V = 1760$, find h.
iii. $h = 3\frac{1}{2}$, $V = 539$, find r.

24. What is the volume of a solid cylinder 5 cms. high standing on a circular base 4 cms. in diameter? Find to the nearest tenth of a centimetre, the length of the edge of the cube which has the same total surface as the cylinder.

25. Equal quantities of water are poured into 2 cylindrical vessels of different diameters, and it is seen that the depth of the water in the one $1\frac{87}{169}$ times that of the water in the other. Compare the radii of the bases.

26. The rain from a flat roof falls into a gutter, the section of which is a semi-circle of diameter 5 in. The area of the roof is 800 s. ft. If the pipe from the gutter is blocked, how many hundredths of an inch of rain must fall before the gutter which is 45 ft. long overflows, supposing that all the rain which falls goes into the gutter.

27. An observatory building consists of a cylindrical base of 12 ft. radius surmounted by a hemisphere of the same radius. If the height of the cylindrical portion is 13 ft., find the cost of covering the outside with waterproof material at 4 sh. 6 d. per sq. yd. (The surface of a sphere of radius $r'' = 4\pi r^2$ sq. in.)

28. The dumb-bells which athletes use consists of a cylinder in the middle, with 2 spheres at the ends. If the diameter of the cylindrical part be 1 in. and that of the sphere be 2 inches, find the weight of a pair of dumb-bells, given that it is made of iron whose specific gravity is 7·6 and that 1 c. ft. of water weighs 62½ lbs. (Volume of a sphere $= \frac{4}{3}\pi r^3$ where r is its radius.) Height of the cylindrical portion = 4″.

29. To prevent the accumulation of dust on polished and varnished pillars, while the construction of a house is in progress, stiff rope is wound closely round the pillars. A cylindrical pillar 5 ft. 4 in. high required 192 turns and 300 feet of rope. Find the diameter of the rope and that of the pillar.

30. In order to find the bore of a piece of thermometer tubing, mercury is let into it and it is found that 0·134 gram of mercury occupies a length of 40 cm. of the tube. What is the mean diameter of the bore? (1 c. cm. of mercury weighs 13·6 gms. approximately.)

31. A coil of wire has 72 turns; the average diameter of the turns is 5 in., and the thickness of the wire is $\frac{1}{10}$ inch. Find (1) the length of the wire, (2) the volume of the wire in the coil.

32. Faraday declared that an ounce of gold can be drawn into a wire stretching round the earth. Given that the circumference of the earth is 4×10^7 metres and 1½ c. c. of gold weigh an ounce, find the diameter of the cross-section of the wire in millimetres correct to 2 significant figures [$\pi = 3·14159$].

33. The length of a cast iron pipe with flat ends is 20 ft. and the diameter (internal) of each end is 5½″. Find its weight in lbs. correct to the nearest tenth assuming the thickness of the pipe to be ½ inch (1 c. inch of cast iron weighs 26 lb.)

34. Two pipes, one of lead and the other of tin, are resply. 49 and 61·6 in. long. They have the same internal diameter of 1″ and the external diameter of the lead pipe is 1·2″. If lead is 11 times and tin 7 times as heavy as water what must be the external diameter of the tin pipe in order that the pipes may be of the same weight?

35. A cylindrical wooden tub is made of wood a'' thick. If the external diameter of the tub be $2x$ inches, find (1) the volume of water the tub can hold, (2) the volume of wood in the tub. Height of the tub $= h$ inches.

36. A pedestal for a statue consists of 3 circular slabs. The top slab 1 yd. in diameter rests symmetrically on the next which is 1¼ yds. broad. This rests on a slab 1½ yds. broad which rests on the ground. These slabs form a circular staircase, each step being 6″ high. What will it cost to gild the pedestal at Rs. 3-12-0 per sq. foot, omitting the upper surface of the uppermost layer.

37. A human being generates 1 cub. ft. of carbonic acid gas every hour. If fresh air contains 3 parts of this gas in 10,000 and air becomes unfit to breathe when it contains 15 parts in 10,000, find how long it will be before the air in a room 100 ft. by 50 ft. by 24 ft., will be unfit to breathe, if the room contains 150 people and there is no ventilation?

38. A hall with a flat terrace is to be built to accommodate 384 pupils. The site is 32 ft. broad. If each pupil requires 10 sq. ft. of floor and 120 cub. ft. of space, what must be its least length and height?

39. The reservoir from which a certain city draws its water supply has a surface area of 2½ square miles. If the city has a population of 450,000, and if the average daily supply is at the rate of 20 gallons to each inhabitant, find to the nearest inch, what must be the average depth of the reservoir so that, when full, it may contain a year's supply? (1 gallon $= 277{\cdot}274$ cu. in., a year $= 365$ days.)

40. The length, breadth and height of a rectangular room are l, b and h yards respectively. Find formulæ for (i) V, the volume of the room, (ii) A, the total area of walls, ceiling and floor, (iii) D, the diagonal of the room. Evaluate these when $l = 8$, $b = 6$ and $h = 4$.

If v, a and d denote the volume, area and diagonal of a similar room, each dimension of which is half the corresponding dimension of the larger room, state the relations between v and V, between a and A and between d and D.

41. Given that 1 oz. of gold is worth £ 4-5 s. and that 1 cub. inch of gold weighs 10 oz. find the average thickness of gold leaf, 40 sq. ft. of which can be made from a sovereign. Give the thickness, correct to 3 significant figures.

42. On a base 4·2 cms. long describe a square. On the top of this square describe an isosceles triangle whose height is equal to the height of the square. If this represents to a scale of 1 cm. = 1 foot, the section of a tower by a plane right through the vertex dividing the whole into two symmetrical portions, find the cost of white-washing the outside of the whole structure at 2 as. a square yard.

43. A rectangular oil tank is a metres long, b metres wide and c metres deep. How many litres of oil can it hold, and how many cylindrical drums each x cms. in diameter and y cms. high can be filled from it?

If $a=2$, $b=1·6$, $c=1·3$, $x=40$ and $y=60$, find the number of cylindrical drums.

44. A grain of gold beats out into a leaf of 54 sq. in. and a cub. ft. of gold weighs 1211 lbs. Avoir. How many leaves are together as thick as a sheet of paper when 175 sheets of paper are an inch high?

45. A teak-wood beam 10 ft. long of square cross-section of 1 ft. side is to be worked down to the maximum regular octagonal pillar that could be got out of it. Find (a) by how much the weight would be reduced, (b) the length of a side of the octagon base.

46. A swimming bath whose sides are vertical is 50 ft. long and 25 ft. wide. It is 3 ft. 6 in. deep at one end and slopes gradually to a depth of 7 ft. 6 in. at the other end. Find (a) the volume of water it will hold in gallons, if a gallon weighs 10 pounds and a cub. ft. of water weighs 1000 oz.; (b) the cost of lining the sides and the bottom at 4 as. a sq. ft.

47. A canal is dug having a cross-section 8 ft. wide at the top, 4½ ft. wide at the bottom. The sides slope uniformly making angles of 60° with the top, to the bottom bed. What will be the depth of the canal? If a mile length is to be dug, how many cub. ft. will have to be excavated?

If water be running in the canal to a height of 2 ft. at a velocity of 120 feet per second, how long will it take to raise the level of water by 1 foot, in a tank 70 yards by 60 yards ?

48. When heated 100°C., a certain metal expands ·002 of its length and a certain liquid ·03 of its volume. If a box 5 cms. deep, 6 cms. broad and 10 cms. long made of this metal is filled with the liquid and then heated to 100°C., how much liquid will overflow ?

49. An iron plate, rectangular in shape, is x ft. long, y ft. broad and c in. thick. It is beaten into a rectangular sheet whose length is 3 inches and breadth 4 inches greater than the original. If t denote the thickness of the beaten sheet, find the relation between t, x, y and c. If $x = 1\frac{1}{2}$, $y = 1\frac{1}{3}$, $t = 0{\cdot}18$ find c.

50. A rectangular casket with a lid 11 inches long, 7 inches wide and 6 inches deep, outside measurements, is made of metal of uniform thickness $\frac{3}{8}$ of an inch, and plated inside and out with silver $\frac{1}{150}$ of an inch thick. Find the weight of silver on the casket. (A cubic inch of silver weighs 4·5456 ounces Troy.)

51. The price of a steel cash box depends upon the extent of steel plate used to make it. The prices of six different sizes are given below. Plot them on squared paper and draw a graph to shew the relation between the quantity of steel plate used and the corresponding price of steel box.

SIZE.	PRICE	SIZE	PRICE.
	RS. AS.		RS. AS.
A 16 × 11 × 6″	75 - 0	D 16 × 10 × 7″	113 - 4
B 18 × 12 × 7″	82 - 8	E $14\frac{1}{2} \times 10\frac{1}{2} \times 4''$	45 - 0
C 20 × 13 × 8″	106 - 0	F $16\frac{1}{2} \times 11\frac{1}{2} \times 4''$	55 - 0

52. (a) If a bath 27 yds. long, 15 ft. wide and 11 ft. deep be dug in a garden 100 yds. by 13 yds. and the earth taken uniformly spread over the remaining portion of the garden, by how much will the level be raised?

(b) If 20 men could finish the work in 9 days working 6 hours a day, how many must be employed to finish the work in 4 days of 9 hrs. each ?

53. (a) Sand lies against a wall, covering a strip of ground 4 feet wide. If the sand will just rest with its surface inclined at 30° to the horizon, how much sand may lie on this strip per foot length of wall?

(b) If the entire length of the wall be 60 feet and sand lies against the whole length, what would have been the cost of getting the sand at 3 cartloads for a rupee? A cartload being the contents of a rectangular box 6 ft. by 1·75 ft. by 1·5 ft.

54. The volume of timber in the trunk of a tree is roughly estimated by multiplying the length by the square of the mean quarter-girth. The trunk of a certain tree, if cut so as to be 27 feet in length, would have a mean girth of 10 ft. 6 in. and if it were cut 10 ft. longer the mean girth would be 9 ft. 9 in. In the former case the tree would cost Rs. 80. What would it, in the latter?

55. (a) Describe a circle with centre O and radius 2 in. Draw two diameters AOB and COD such that angle ADC = 60 degrees. Join AC and BD. If the two segments AC and BD be cut off, what fraction of the area of the circle would remain?

(b) If the resulting figure represents to a scale of 1 in = ½ ft. a section of a cylindrical beam 14 feet long, two planks having been sheared at the sides, what will be the weight of the timber if 50 cub. ft. go to a ton?

56. A stone mortar is made of a cylindrical block of stone, 2 ft. high on a base of 1·4 ft. in diameter thus:— A cylindrical hole of diameter 3½ inches is dug out through the middle of the top to a depth of 4·8 inches. To reduce the weight, a cylindrical hole is scooped out from the bottom. If a thickness of 2 inches all round be left out, what should be the height of the cylinder dug out, if the weight of the mortar is to be only 0·6 of the original block of stone?

57. The cross-section of an iron wire is 1 sq. mm. A cub. centimetre of the wire weighs 7·9 gm. and the force required to break the wire asunder is the weight of 35 kg. What length of the wire suspended by one end, will just break under its own weight?

58. (a) Find the area of a rectangular brass plate 9·6 cm. long and 7·4 cm. wide.

(b) If its weight is 115 grams and its density 8·4 grams per cubic centimetre, find its volume and thickness.

(c) If this plate be rolled into a cylinder, what will be the radius of the circular base, if the height be (i) 9·6 cms., ? (ii) 7·4 cms. ?

59. (a) Describe a circle whose radius is 2 cms. and place in it a chord whose length is equal to the radius. Find the area of the major segment.

(b) If this major segment represents the vertical section of a completely closed oil drum drawn to a scale of 1 cm. = 1 ft. find (1) the sheeting required to construct the drum, (2) its volume to the nearest cub. ft. (length of drum = 10 ft.)

60. A paper weight is made of teak-wood. It is rectangular 8 inches by 3½ in. by 1½ in. To make it heavier four cylindrical holes are scooped out from the broadest face, each ¾ in. deep and 1 in. across. These are filled with lead flush with the surface. If a cub. foot of teak weighs 43 lbs. and a cub. in. of lead weighs 0·41 lb. find its total weight.

61. The population of the City of Madras in round numbers is 510,000. The water main is 2 feet in diameter. Each individual requires a minimum of 25 gallons per day and water is available by pipes for 12 hrs. in the day. At what rate must the water be forced through the main, given that 1 cubic foot = 6·2 gallons.

62. (1) From a cylindrical block of wood 10 in. in diameter, a rectangular beam of width 6 in. is to be made. If the beam is to be as strong as possible and the strength varies directly as the depth, find the depth of the strongest beam that could be got out of this block.

(2) Find correct to two decimal places what fraction of the weight of the whole block, that of the beam is.

63. (1) A cylindrical box of wood is 6″ high on a circular base 7″ across, in the interior. A lid, also a cylinder is of the same shape but 1 inch high. The interior of the box and the lid are to be lined with velvet; but the particular kind of velvet used can be had only in one rectangular piece 7 inches broad. Find the length required. Also find how much will be wasted.

(2) Draw a rectangle to scale representing the piece you would purchase and shew therein how you would cut it to pieces to line the interior.

64. A certain patent medicine is supplied in bottles whose transverse section is a rectangle 2″ by 0·5″ with semi-circles attached by their bases to the shorter sides. If the medicine inside the bottle stands to a height of 6″ and the price of each bottle is Rs. 3-8-0, find the price of 1 cub. in. of the preparation.

65. An electric current passing through a solution of silver nitrate deposits silver plating on an object in the bath at the rate of 0·001118 grams per second. If a cylindrical tumbler 5 cms. high on a base 3½ cms. across be left in the bath for 3 hours, find, correct to three significant figures the thickness of silver deposit on the vessel, given that a cubic centimetre of silver weighs 10·5 grammes. The deposit inside and outside is uniform and the thickness of the material of which the vessel is made, may be neglected.

66. A cubical block of metal of 7·84″ weighs ·25 lb. per cub. inch. A hole of circular section is to be made right through the centre in order that the weight of the metal left may be 1 cwt. Find correct to two decimal places, the diameter of the circle.

67. A piece of cardboard $\frac{1}{10}$″ thick was tightly wound round a cylindrical bottle on a circular base 2·8″ diameter and was tightly inserted in a square prismatic case of height 5″. If the bottle be 5″ high and closed, find the volume of space left unoccupied in the case.

68. A kerosine oil drum 2·4 feet high, stands on a square base of 1·2 ft. A pump is used to draw out the oil, and when the piston is continually worked, the oil flows out from the spout whose section is a circle of 0·4″ diameter at the rate of 0·6 ft. per second. How long will it take to pump out the oil in the drum?

69. I wanted to buy 144 feet of round bar iron of $\frac{5}{16}''$ diameter. But I was told that it was sold by the pound. Ready-made tables were not available and I had to calculate from data known :—that specific gravity of iron is 7·7 and that a cub. ft. of water weighs 1000 oz. How many pounds must I buy? Give the answer to the nearest pound.

70. The porous cell in an electric battery is cylindrical and contains nitric acid to a height of 4·5 inches; the diameter of the interior cross-section is 2 inches. A prismatic carbon rod of square section of 1″ side is inserted in the acid. At what height will the acid stand in the porous cell?

71. The end view of the covering of a locally made type-writing machine is a rectangle 1 ft. 3 in. by 9 in. with a semi-circle above the side of 9 inches. Find the iron sheeting required to make a covering of the size, the length required being 1 ft. 3 in.

72. At the Glasgow Exhibition a mercury fountain had at its base a pool of mercury, bounded externally by a cylinder of diameter 6 ft. and internally by a post 4 sq. ft. in section. A cubic foot of mercury weighs 13,600 ounces. How many pounds of mercury did the pool contain when the mean depth was half-an-inch?

73. A cylindrical vessel 8″ high, has water to a depth of 4″. A child picks up a dozen pebbles, each $\frac{3}{4}$ of a cub. inch in volume and throws them into the vessel By how much will the level of the water be raised? (Radius of base = 3″.)

74. Find the total cost of making a circular well. It is to be dug 28 feet deep and 6 feet in diameter at one anna per cubic foot. It is then to be lined with brick and chunam one foot thick at $2\frac{1}{2}$ annas per cubic foot. And finally the sides are to be plastered at the rate of Rs. 3-2-0 per 100 square feet. Given that the ratio of the circumference to the diameter of a circle $= \frac{22}{7}$.

75. (a) The iron roller in engines for metalling roads is a hollow cylinder 3 inches thick and has an internal diameter of 2 ft. 5 in.; 1 cub. ft. of iron weighs 450 lb. The iron roller itself weighs $3017\frac{1}{7}$ lb. What area will it cover in one revolution?

(b) A road is 32 ft. broad and 220 yards long Supposing there is no overlapping, how many revolutions would the roller have made if the whole road had been rolled thoroughly once?

76. A cistern 6 ft. long, 4 ft. wide, and 2 ft. 6 in. deep is filled with pulp for making paper, which on drying shrinks to one-third of its volume. From it is produced a strip of paper 30 in. wide and 4 miles long, which is rolled into a cylinder. Find (a) the thickness of the paper to the nearest hundredth of an inch, and (b) the diameter of the roll to the nearest inch.

77. Supposing that the earth removed by the machine-spades in the excavation of the Panama Canal to be equal in volume to the volume of a circular hole that might be imagined, to run from the North Pole of the Earth to the South Pole with a diameter of 13 feet, find in millions of cub. yds. the volume of the earth excavated by the machine-spades and the cost of the same in millions of pounds at 1 dollar per 1000 cub. yds. (A dollar = 4 s. 2 d. Radius of earth = 4000 miles.)

78. Water is flowing steadily through a pipe, which consists of two parts. The cross-section of the first is an equilateral triangle of side 6 inches and that of the second a circle of radius 2 inches. Find approximately the rate of flow of water in the second part, that in the first being 31·75 ft. per minute.

79. The pressure of the atmosphere is 15 pounds per square inch. When a bicycle tyre, of capacity A cubic inches is inflated by a pump of capacity B cubic inches the pressure of air in the tyre, after n strokes of the pump exceeds the atmospheric pressure by $n \times B \div A$ times the atmospheric pressure. If with a pump 10 inches long and ¾ inch diameter, a tyre be inflated, find the capacity of the tyre and the pressure of air in the tyre after 40 strokes, if the tyre may be taken as a cylinder 88 inches long and 1¾ inches in diameter.

80. A length 10 ft. of a water-pipe, ¾ inch in diameter freezes. In freezing any volume of water expands to 1·087 times that volume. Sometimes this freezing bursts the pipe. Sometimes the ice occupies an additional length of pipe.

What additional length of pipe (in inches) is necessary in the present case?

If no additional length of pipe is available, to what diameter does the water expand in freezing?

81. The section of a loaf of bread is a square of 4 inches side with a semi-circle attached to the top. The length of the bottom rectangle is 6 inches. This is priced at one anna a loaf. What must be the length of a loaf, priced at 3 pies, if the end view is a square of 1½″ side with a semi-circle at the top. Find the length correct to the tenth of an inch.

82. The vertical section of a wagon carrying goods is the shape of a square surmounted by a segment of a circle. If the square is of 8 ft. side and the greatest height of the carriage 9·5 ft. what should be the length of the wagon if it be designed to carry 1780 cub. ft.?

The volume of the curved portion at the top may be taken as $\frac{3}{5}$ of that of a rectangular prism of the same base and height.

83. (a) A square nut of ·48 inch side is ·25 of an inch thick. A circular hole 0·25 of an inch diameter runs right through. Find the weight of a gross of such nuts if the specific gravity of the metal of which it is made be 8·5. (1 cub. ft. of water weighs 62½ lb.)

(b) Find the dimensions of a teak-wood case to pack these nuts if eight are placed along the length and six along the breadth, allowing one inch at the top and half-an-inch at the bottom.

84. (a) Describe a regular hexagon on a base of 0·75″.

(b) On the sides describe semi-circles to fall outside the figure.

(c) If the figure is the shape of a biscuit and if 30 such biscuits each 0·2″ thick weigh a quarter pound how many biscuits circular in shape of radius 0·5″ and of the same thickness weigh a quarter pound?

85. The weight of a foot run of cast iron pipe whose bore is 14 in. across and thickness ⅞″ is 127·5 lb. What would be the weight of a lineal foot of round ba iron $\frac{7}{16}$ in. across?

Also find how many times as heavy as water is cast iron?

86. A gasometer is in the form of a cylinder with a rounded top and the following dimensions are given :—diameter = 28 feet, height at edges, 14 feet, height in the middle 16 feet. Draw a section of the gasometer on the scale of 1 inch to 10 feet and calculate the number of cubic feet of gas that the gasometer will hold. Take the volume of the portion at the top (above 14 feet, the height of the cylindrical bottom) as half the volume of a cylinder of the same base and height and $\pi = 3{\cdot}142$.

87. A lime-kiln is cylindrical in shape and 7 ft. across. The height of the kiln is 7 ft. On slaking, only $\frac{7}{15}$ of the contents is serviceable. If it be sold at 9 as. per kalam, how much will the owner get? $\frac{2}{3}$ of a cart-load which contains $1\frac{1}{2}' \times 1' \times 6'$, measures 20 kalams.

88. A brass cylinder is sent from a foundry in a wooden rectangular box which measures internally 14″, 8″, and 8″ inches and weighs 4 lb.; the length of the cylinder is 14 in., the radius of its base 4 in. and its weight 203·4 lb. The interstices between the box and the cylinder are filled with sand. Find the weight of the whole, given that a cub. foot of sand weighs 110 lb.

89. A very long strip of paper is rolled on a cylinder of wood 1 inch in diameter and the total diameter of the roll is 2 inches. Find what would be the total diameter of the new roll if the same strip of paper were coiled on a cylinder of wood 2 in. in diameter.

90. A plate of metal is beaten to the thickness of $\frac{1}{8}$ of an inch and the weight of a circular medal cut out from it, whose diameter is $1\frac{1}{6}$ inches, is 1·25 oz. If the same plate be beaten to the thickness of $\frac{1}{9}$ of an inch, what will be the weight of a medal cut out of it of diameter $1\frac{3}{4}$ inches?

91. The vertical section of the orifice of a Kalingula is a rectangle 3 ft. by 2 ft., with a sem-icircle attached to the top which is the longer side. Draw a diagram to shew the vertical section.

(*a*) If water flows full through this into a rectangular tank 100 yds. by 80 yds. at the rate of 3 miles an hour, find how long it will be before the level of water in the tank is raised by $1\frac{1}{2}$ feet.

92. The bore for a subway of circular section is d ft. in diameter. If the earth excavated weigh w lb. per cub. ft., how many tons are removed in making 1 yard of the subway?

If air is pumped in at v cub. ft. per minute, what is its speed in feet per second along the subway?

Give numerical answers for the case when $d=10$, $w=65$ and $v=2000$.

93. A fire extinguisher is conical in shape and contains compressed gas. How many cub. ft. of compressed gas does an extinguisher contain given that it is 1·4 ft. high and a sector whose central angle is 60° has been cut out of a sheet of iron to make its curved side?

94. A right cylinder of wood, 10 ft. long and 2 ft. in diameter is sawn along two planes parallel to each other, and parallel to the axis of the cylinder, each plane being at a distance from the axis equal to one half of the radius of the cylinder. Compare the volumes of the logs so formed.

95. The capacity of an oil can may be calculated approximately by regarding the lower part as a cylinder and the upper part as a cone. If the total height is $14\frac{1}{2}$ inches, the height of the cylindrical portion 10 in. and circumference 27 inches, how many gallons will it hold (1 c. ft. $=6\frac{1}{4}$ gallons nearly).

If the circumference be 'a' inches, what would be the circumference of another of the same shape but of double the capacity?

96. Find in the above question how many square inches of tin are there in the two cans mentioned.

97. A circular sector of radius $3\frac{1}{2}$ inches and area 5·5 sq. in. is rolled into a conical cup. Find the depth of the cup and its capacity.

98. A cylindrical tin-measure used by an oil merchant for selling oil has been hammered in at the bottom to assume the form of a cone, its height being $\frac{1}{16}$ of the height of the cylinder. How much per cent. will the merchant gain by using this measure instead of the cylindrical measure without the conical projection. ?

99. A rectangular mat 8·8′ by 5′ is made into a cylindrical barn by bringing its two breadthwise ends together and stitching them. How many kalams of paddy will the barn hold given that 1 kalam = 6¼ c. ft. ?

If when the barn is full, the paddy is removed from it and heaped, what shape will the heap assume ? If the vertical height of the heap be 2½ ft. find the radius of its circular base.

100. (*a*) Draw a square of side 2″ and with its four corners as centres and with 1 inch as radius draw circles to cut the sides. What is the area of the fig. enclosed by the arcs, outside the square ?

(*b*) If the fig. represents the section of a granite block of height 4 ft. drawn to a scale of 1 in. to 1 ft. find its volume.

101. (*a*) Describe a segment on a base AB, 1·5 in. long, the angle in the segment being 60° and find its area.

(*b*) If the segment represents the cross-section of a Kerosine tank 20 ft. long (attached to a Railway Carriage) drawn to a scale of 1 in. to 1 yd., find the volume of Kerosine the tank can hold ? If the tank is only ¾ full, how many tins of Kerosine can be filled from it, if the capacity of a tin is 1¼ c. ft. ?

102. A circus tent is in the form of a cylinder surmounted by a cone. The central post is 30 ft. high and the cylindrical portion 10 ft. high on a circular base of 140 ft. diameter.

Find (i) the area of canvas required for making the tent.

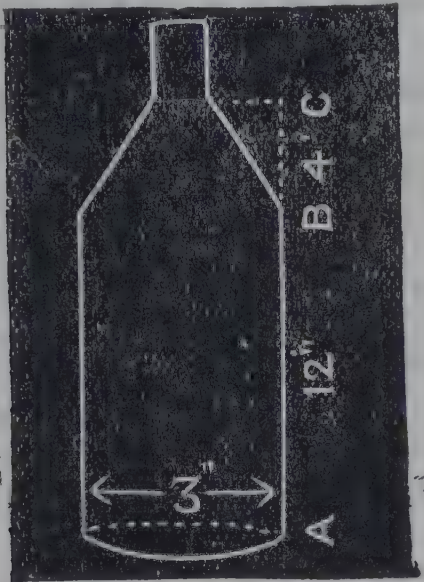
(ii) the number of persons the tent could safely accommodate if each man requires 200 cub. ft. of air per hour and the performance lasts 2 hours.

103. A circular tin-sheet of diameter 28 inches is taken. Two radii are drawn to include an angle of 72° and the circle is divided into two sectors by cutting along these radii. The sectors are then folded, each into a cone. Find the height of each cone and the volume of each.

104. An ounce glass is conical in shape and 1·75 inches in diameter across the top. Find its depth correct to the tenth of an inch, given that the volume of a gallon is 277·272 cub. in. and that 10 such glasses go to a pint.

105. A cylindrical basket 1 ft. 4 in. across and 1 ft. 9 in. high is used to measure pulses. When a person purchases the grain it is heaped up uniformly on to the top to a height of 2⅝ inches; but when he sells, the grain is flush with the top. What is the gain per cent?

106. The top, children play with, is a hemisphere surmounting a cone. If a rectangular piece of wood 3 in. by 2 inches by 2 inches be taken and turned into a top, the greatest length of which is 3 in. and greatest breadth is 2 inches, what fraction of the weight of the wood would the top be? The volume of a sphere is $\frac{4}{3}\pi r^3$ where r is the radius of the sphere.



107. An oil drum 2 ft. high is cylindrical in shape, and stands upon a base of 7" radius. How many bottles of the size given does it contain?

(*a*) the lower portion is cylindrical;

(*b*) the upper portion BC is a cone.

108. A circle is drawn and a sector cut out of it, is 4·4 square inches. If the diameter of the circle be 4·2", find the height of the cone into which the sector may be folded; also find its volume. What will be the volume and height of the cone which the other sector of the circle may be made into?

109. A mahagony pattern made from wood weighing 762 grams per cubic decimetre weighed 2·84 Kilograms. A gun-metal casting of the same shape and size weighed

30·8 Kilograms, the gun-metal employed weighing 8·46 Kilograms per cubic decimetre. Show that there must be a cavity in the casting and find what weight of metal would fill it.

110. A Madras measure is cylindrical 4·5 inches across and 6·8 inches high. An oil-can is in the form of a cylinder surmounted by a cone. The bottom is a circle 3 inches across. The height of the cylindrical part is twice the diameter of the base and the height of the cone is equal to the diameter. How many ollocks (correct to the nearest tenth of an ollock) will the can hold?

Also find how many square inches of tin-sheet will be required to make a can of the dimensions given.

111. (1) The perimeter of a sector of a circle is 21·2 inches and the angle at the centre is 30°. Find (a) the radius of the circle, (b) the length of the arc of the sector, (c) its area.

(2) If the sector be folded into a cone what would be its height? also find the volume of the cone.

112. Three mounds of manure were once bought by a farmer; one mound was conical to a height of 3·6 ft on a base 7 ft. across; the other mound was in a square enclosure of 7 ft. side to a height of 4 ft. and the other in a circular pit 3·5 ft. across to a depth of 5 ft. If in the process of loading and unloading the manure, 5 per cent. be wasted and the remainder is uniformly spread over 100 sq. yds. to what height will the manure be on the field?

113. A cylindrical pontoon, 3 feet in diameter, having conical ends is so far sunk under water, that there is a mean pressure of 2·45 lb. upon each square inch of its surface. Find in tons, the total pressure, the extreme length of the pontoon from the extremities of the cones being 14 feet and the height of each cone being 2 feet. Take $\pi = \frac{22}{7}$.

114. A small bottle known as the one ounce bottle has a circular base 0·6″ in diameter, and contains an ounce which stands to a height of 2″. There is a conical glass funnel and the divisions marked on it are illegible.

Taking the small bottle as the standard, what is the height from the apex of the funnel if the other end be a circle on 1·4″ diameter and it is known to hold 3 oz. when full to the brim?

115. A conical tent, whose slant height is equal to the diameter of the base, is to be made so that the circumference of the base will pass through the angular points of a piece of square flooring, 12 feet along each side. How many square feet of canvas will be required?

116. (*a*) How many tangents can be drawn to a circle from a point outside the circle? (*b*) What do you know about the lengths of the tangents that can be drawn? (*c*) A sphere rests on a table. A right cone placed on a table just covers the sphere. The radius of the base of the cone is 5·5 inches and the height of the cone is 13·2 inches. Find the radius of the sphere.

117. The volume of a sphere is found from the formula $V = \frac{4}{3}\pi r^3$, r being the radius of the sphere and V its volume. A hemispherical bowl is filled with water. A solid cone, the height of which is twice the radius of its base which is 1 foot in diameter is immersed in the bowl and the vertex of the cone coincides with the centre of the hemispherical bowl, the base of the cone being horizontal. How much water will remain in the bowl after the insertion?

Give the answer to the nearest tenth of a cubic inch.

SECTION XIII.

Revision Papers.

I SERIES.

I

1. A train starts from a station *P* and the railway track has *a* rising gradient of 1 in 165. If station *Q* is at the top of the incline at a distance of 5 miles from *P*, how much higher to the nearest foot is station *Q*?

2. In the former of two consecutive years a mirasdar got 600 kalams of paddy which he sold at Rs. 5-8 as. per kalam. Next year the produce fell by 10 per cent By what percentage must he raise the price per kalam, to get the same income as before?

3. A rectangular cistern 8 ft. long and 4 ft. wide is 9 inches deep. It contains pulp for making paper to ⅔ of its height. The thickness of 200 sheets being an inch, how long a roll 30 in. wide can be made from the pulp if ⅓ of the volume be lost in drying? Give the answer to the nearest foot.

4. Two persons compete in a running race. One of them can run a miles an hour and the other can run b miles an hour. The latter is given a handicap of 140 yards. In how many minutes will the former overtake the latter? What is the length of the course if they run a dead heat? $a > b$.

5. Two roads meet at an angle of 60°. A well is between the two roads, 15 yards from one of the roads and 25 yards from the other. Find by careful measurement, the distance of the well from the junction of the roads. Draw a neat figure to scale.

6. The annual rainfall of a district is 49·7 in. Assuming that the fall is distributed uniformly over the district and that a cub. ft of water weighs 62·5 lbs., find the weight in tons of the water that falls throughout the year on a sq. mile.

II

1. The scale of a map is 1 : 528000.

(1) What is the actual distance in miles between two places which on the map are 2·75 inches apart.

(2) What is the area in sq. miles of the district which on the map is enclosed by three straight equal lines each 2·75 inches long.

2. A man buys a house of Rs. 3400. What should be the rate of monthly rent it should fetch, that after deducting Rs. 200 annually for repairs, the man may get 6 per cent. on his investment?

3. Once a boy purchased 17 books, a few of them being English story books and the rest being Mathematics books. Each English story book cost him 14 as. and each Mathematics book Re. 1-4 as. on an average and the total amount he paid was Rs. 17-2 as. How many of each kind did he buy?

4. The average age of a class of p boys was q yrs. Two boys aged m years and n years left the school and the average of the remainder was x. Express x in terms of p, q, m and n.

If $p=38$, $q=13\frac{1}{2}$, $m=16$, $n=13$ find x.

5. A and B are two points 150 yards apart on the bank of a straight river. T is a tree on the opposite bank. The angles BAT and ABT are found to be 30° and 60° respectively. Find (1) the breadth of the river, and (2) the respective distances of T from A and B.

6. A shuttle covers a distance of 25 miles in $1\frac{3}{4}$ hrs. and a passenger train covers 33 miles in $2\frac{1}{4}$ hrs. Find graphically which the faster is.

III

1. In the year 1906 the births in the United Kingdom numbered 935,081 and the proportion of male to female births was 1041 : 1000. Find the number of male children born in that year, correct to the nearest integer.

2. If P be the sum lent, in rupees, at r per cent. compound interest, then the amount A, at the end of n years is given by the formula

$$A=\left(1+\frac{r}{100}\right)^{n}\times P.$$

If $P=1500$, $r=7\frac{1}{2}$ and $n=2$ find A and also find r if $A=1984{\cdot}5$, $P=1800$ and $n=2$.

3. A cyclist has to ride a distance of 42 miles. He rides for a time at 12 miles an hour and then being tired can ride only at 10 miles an hour. If he reached his destination in 3 hrs. 43 min. find when and where he changed his speed.

4. A restaurant keeper buys ice at 4 as. 3 ps. per pound. At what price must he sell a lb. that he may gain 25 per cent. if 15 per cent. of the quantity stocked, be wasted in the melting away.

5. An estate of 267 acres 3720 sq. yds. was bought by a Mirasdar. It was rectangular; the length being $2\frac{1}{2}$ times the breadth. He surrounded it with a fence costing 2 as. 6 ps. per yard of fence. What was the cost of fencing?

6. The following is an extract from the S. I. Ry. Time-Table and two trains run in the times noted below.

Dis. in miles.	Madras Egmore	6-55	Pallavaram	6-0
10	Guindy	A 7-21 D 7-28	Guindy	A 6-21 D 6-29
15	Pallavaram	7-48	Madras Egmore	7-10

Find where there should be a station, to shunt off one of the trains to allow the other to pass.

Find the distance of that station from Madras Egmore.

IV

1. The number of passengers carried by a certain railway in a month is 1,989757 and the fares paid by them amounted to Rs. 7,59368-11-5. Find correct to the nearest tenth of a pie, the fare paid by each on an average.

2. If the simple interest on a sum of Rs. P lent at r per cent. for a period of n years be denoted by I, express each of the letters, $I, P, r, n,$ in terms of the other three letters involved.

Hence find r, given that $P=1780$ and $I=667{\cdot}5$, $n=5$.

3. I bought a house and a garden for Rs. 7400. I sold the house at a gain of 20 per cent. and the garden at a loss of 10 per cent and on the whole gained Rs. 695. What was the price I paid for the house?

4. A carriage drive is to be made round the outside of a circular park whose radius is 585 feet. The metalling is to be 30 ft. wide and 9 in. deep. What will it cost at Rs. 12 per 100 cub. ft.?

5. A candidate fails in his examination by want of 5 per cent. of the required minimum for passing; he has secured 57 marks. What is the required minimum? If this be 40 per cent. of the maximum, what is the maximum?

6. To raise a weight of W lbs. pressure P is to be applied to the handle of a lever. The weight and the corresponding pressure are given below.

W in lbs.	100	200	300	400	500	600
P in lbs.	$12\frac{1}{2}$	20	28	$34\frac{1}{2}$	$43\frac{1}{2}$	51

Draw a graph shewing the relation between P and W and from the graph find the pressure to be exerted to raise 450 lbs. and the weight that could be lifted with a pressure of 32 lbs.

V

1. A booking-clerk in a Cinema office had one day sold f floor tickets at 4 annas each, b bench tickets at 12 as. each, c chair tickets at Re. 1-8 as. each and r reserved seats at Rs. 3-8 as. each and he had p five-rupee currency notes being the whole collection Find p in terms of f, b, c and r. If $f=132$, $b=88$, $c=32$ and $p=147$ find r.

2. The weight of gold that a ton of ore contains is ·0042 per cent. In the process of washing, 2 per cent. of the metal is lost. Find the number of grains that could be obtained from 750 tons of ore. 1 lb = 7000 grains.

3. A race course consists of two parallel sides each a furlong long and two semi-circular ends. If four times round the whole course is two miles long, what is the distance between the parallel sides?

4. Given that 22 knots = 25 miles; draw a graph for converting knots into miles and *vice versa.*

From the graph find the number of miles equivalent to 32 knots and the number of knots equal to $42\frac{1}{2}$ miles.

5. A milkseller buys milk at $2\frac{1}{2}$ measures for a rupee and adds water before selling. Then he sells it at an anna each ollock and gains $41\frac{1}{2}$ per cent. If he had bought 24 measures find (1) the quantities of water added, (2) his gain.

6. An ornamental flower bed is bounded partly by the sides of a square of side 6 ft. and partly by the portions outside the square of four circles, each of radius 18 in. the centres of these circles being at the corners of the square. Draw a figure to scale and calculate the entire area of the flower bed.

VI

1. Students doing practical work in Chemistry were given a reel of copper wire labelled 100 meters or $3\frac{1}{2}$ kilograms. Each one had to dissolve a gram of copper in an acid. How much length should each cut off? How should the cutting be done to be as accurate as possible.

2. Two persons start running from the same place at the same time. One of them runs a miles per hour and the other b miles per hour. (a) In how many minutes will they be 100 yds. apart, b being greater than a? (2) How many yards will they be apart in c minutes?

3. Two persons A and B own equal extents of land. A's is circular, B's is a rectangle whose breadth is $\frac{1}{8}$ of the length. If the area of each plot be $\frac{1}{10}$ of an acre and they have to fence their lands at 2 as. per foot run, how much will it cost them separately?

4. One day the rainfall was recorded as 2·4 in. If on the day $\frac{1}{3}$ of the quantity of rain that fell on an area of 440 acres drained into a rectangular tank 200 yds. by 180 yds., by how many inches would water have risen in the tank?

5. A cubic foot of water weighs 62·426 lbs., a metre = 39·37 in. Find to the nearest tenth of a pound the number of pounds in a kilogram.

6. In a race A runs x ft. per second and B runs at $\frac{3x}{11}$ miles per hour: In 3 minutes A is 240 yds., ahead of B. Find A's rate in miles per hour.

VII

1. A and B start towards each other from two towns h miles apart. A walks at u miles an hour and B at v miles an hour. How far will they be apart in (i) an hr., (ii) d hrs?

2. In the former of two consecutive years the yield of an estate was 432 kalams of paddy, which the owner sold at Rs. 3-12 as. per kalam. In the latter the produce was $12\frac{1}{2}$ p. c. less, but the price per kalam rose by 10 p. c. What was the alteration in the income?

3. When exchange value is 1 s. 4 d. per rupee, how many rupees must a person in India remit that the addressee might receive £ 800?

4. I want to buy 40 sq. ft. of cloth. The variety I want is stocked in rolls of different widths 36 in., 28 in., 26 in. The first variety costs 7 as. per yd, the second 5 as. 3 ps. per yd. and the third 5 as. a yd. Which sort is the cheapest and how many yds. must I buy in that variety?

5. A man starts from a village and walks 3 miles due East and then turns N. E. and walks 4 miles. Then he goes from this place in a N. W. direction and reaches a place due north of his village. Find from a figure

i. the distance of this place from his village.

ii. the distance he walked in the N. W. direction.

6. A motor car running at 12 miles an hour passes a milestone. Another car running 20 miles an hour in the same direction passes the same milestone 8 min. hence. When and where will the latter overtake the former?

VIII

1. A railway truck carries 160 large planks and 80 small planks. 100 of the large planks weigh a ton and the total weight of all the planks is 2 tons 12 cwt. Find the average weight of a smaller plank.

2. The pressure on a vertical wall due to a wind blowing at v miles per hour is $0{\cdot}00492v^3$ lb. wt. per square foot. Find the pressure in tons weight on the front of a house, 36 ft. by 40 ft., when $v=60$.

3. If an alloy is made with 378·17 kg. of zinc and 544·92 kg. of copper, find to the nearest integer, in each case, the number of parts of zinc and copper in 100 parts by weight of the alloy.

4. Two cyclists P and Q leave a station and travel in the same direction, the former at p miles an hour and the latter at q miles an hour. If the latter leave the station s hrs. later than P, find when he will overtake P. (q is greater than p.)

5. 134 bags of rice have to be carried to the market. I can engage double-bullock bandies and single bullock bandies. I find that if I engage 14 of the former, 2 of the latter would carry the rest and that if I engage 10 of the former, 11 of the latter have to be engaged. Find how many bags each bandy can carry.

6. *A* earns Rs. 250 a month and *B's* income is 16 p. c. greater. *A* saves 40 p. c. of his income and *B* finds that he spends 20 p. c. more than what *A* actually spends. By how much per cent is *B's* saving greater than *A's*?

IX

1. The second's pendulum of a certain clock is 1·56 yds. long. Find to two places of decimals its length in metres given that 1 metre = 3 281 feet.

2. A motor-bus runs the distance between two places in 2 hrs. 12 min. If its speed were increased by 5 miles an hour, it would do the journey in 1 hr. and 39 min. Find the distance between the two places.

3. Two persons set out at the same time from two places P and Q to meet each other. One had travelled for a hrs. at b miles an hour and the other for $3a$ hrs. at $\frac{2b}{3}$ miles per hour and they were still a miles apart. Find in terms of a and b the distance between the two places.

4. A merchant buys 20 bags of rice at Rs. 15-12 per bag. The rice had to be cleaned and he paid 4 as. per bag for cleaning and in the process, 2 p. c. was wasted. If a bag contained 50 measures, at how many measures per rupee must he sell to gain Rs. 1-8-0 on each bag he bought?

5. The weight of a gallon of water at the standard laboratory temperature is 10 lbs. and a cub. centimetre of water weighs 0·9976 gram. Find correct to two decimal places the number of litres in a gallon. (1 gram = 15·4323 grains, and 1 lb. = 7000 grains).

6. (*a*) The perimeter of an isosceles triangle is x in. One of the equal sides is a in. shorter than the unequal side. Find in terms of x and a the length of an equal side.

(*b*) Construct the triangle if $x = 8$ and $a = 1·25$. Find the magnitude by measurement of one of the equal angles.

X

1. 40 p. c. of the population of a district can read and write and 25 p. c. of the population of another can do so. The population of the former is however ⅔ of that of the latter. What percentage of the combined population of the two villages can read and write?

2. Buttons are made with lead discs 0·8 in. across and 0·09 in. thick. The volume of a disc a in. across and b in. thick is calculated from the formula,

Volume = $0·785\, a^2\, b$ and lead weight 711 lbs. per cub. ft. How much lead (to the nearest pound) is wanted for a thousand buttons?

3. A cylindrical glass has an internal diameter of 2 cm. and is filled with water to 1 cm. from the top. It is found that 58 steel pellets can be dropped in before the glass is quite full. Find in c. cm. the volume of a pellet; that of a cylinder of diameter d cm. and height h c. m. being $0·7854\, d^2\, h$. c. centimetres.

4. A tradesman effects sales in a week to the value of Rs 750. He had marked goods 20 per cent. above cost price and pays for tax rent etc., 10 per cent. of his weekly sales. Find his net profit.

5. A beam having a square section, is 9 ft. long, and weighs 3½ cwt. A cub. ft. of the substance of the beam weighs 32 lbs. What is the thickness of the beam?

6. A man goes 6 miles N. E. and then 5 miles due N. How far is he from the starting-point? If from this place he walks a certain distance S. W. and is due north of his starting-point. how many miles does he walk in the direction and how far is he from the starting-point?

II SERIES.

I

1. Given that 1·7320508 multiplied by itself gives 3, find correct to three significant figures the result when 0·2679492 is multiplied by itself and the product divided by 4·7320508.

2. A man had in his purse x one-rupee coins and y one-anna pieces. He changed a certain number of rupees into one-anna coins and then the number in each kind was represented by $\frac{y+1}{5}$ and $5(4x-y)$ respectively. The value, in each case, of the contents was Rs. 55-10 as. Find how many coins of each kind he had originally.

3. The sides of two triangles are 9, 12, 15 and 12, 16, 20 centimetres respectively. Show that the triangles can be placed side by side so as to form together a right-angled triangle and find the area of the triangle formed in this way. The three triangles must be drawn as neatly as possible.

4. The population of a district consisted of 13,000 males and 12,800 females. If the average annual increase in male population was 3 p. c. of the population at the beginning of each year and that of the female population 4 p. c., find by how much, to the nearest hundred, the female population would exceed the male population at the beginning of the 3rd year.

5. An alloy of copper and zinc contains 72 per cent. of copper; a second alloy of zinc and tin contains 64 per cent. of zinc and a third alloy of tin and copper contains 48 per cent. of tin. If these three alloys be mixed in the ratio of 1 : 2 : 3, what percentage of copper, zinc, and tin is there in the mixture?

6. In a rectangular piece of level ground 200 feet long and 120 feet broad, a rectangular bath 170 ft. by 90 ft. is dug. The earth removed is evenly spread out on the remaining ground and it is found that the level of the ground is thereby raised by 6 feet. What is the depth of the bath?

7. (i) The cost price of an article is Rs. x and its marked price is Rs. y. The tradesman allows a discount of 20 per cent. on the marked price and yet makes a profit of 10 per cent. Establish the relation between x and y.

(ii) Draw the graph of the equation and from the graph find (*a*) the marked price of an article whose cost price is Rs. 12; (*b*) the cost price of an article marked for Rs. 13-12-0.

II

1. The fifth general census of India was taken on the 18th March 1921. The final census returns "gave a population of 318,942,480, as compared with 315,156,396 in 1911, an increase in the ten years of about—per cent. The census total of 1921 is divisible into 247,003,293, or—per cent. for British India, and 71,939,187, or—per cent. for Indian States." Fill in the blanks with figures correct to one decimal place.

2. A grown-up man's ordinary pace is *a* inches. If he walk fast, the length of each pace is increased in the ratio of 3 : 4 and the number of paces in the ratio of 5 : 7. If at the ordinary rate he takes *b* steps per minute, how much farther would he have walked at the quicker rate than at the ordinary rate in *c* minutes?

3. A cycle-dealer bought 60 cycles at Rs. 175 each. He sold a certain number at a gain of 40 per cent. and each of the remainder at three-fifths of the selling price of each of the former, and thereby gained Rs. 2730. How many did he sell at the lower rate?

4. A refining machine reduces the weight by 7 per cent. of whatever is subjected to it. A certain material had to be refined thrice. If the original quantity weighed 4500 tons, what would be the final weight (to the nearest tenth of a ton) of the material thrice refined?

5. Two posts *PQ* and *RS*, 18 ft. and 16 ft. long respectively, are fixed in the ground to stand vertically at a distance of 8 ft. A length of 2 ft. of each post is below the ground level which is horizontal. A loose string *QTS* connects their tops. A weight attached to the point *T* in the string, draws the two parts straight and it is found that the two parts *QT* and *TS* of the string are at rt. angles and that $QT : TS = 4 : 1$. Find the entire length of the string. Draw an accurate figure to scale, shewing the posts and the string with the weight attached.

6. A cyclist rides at 12 miles an hour from a station A. He has to reduce his rate by 2 miles an hour after every ¾ hour of cycling. A motorist leaves the same station 1¼ hrs. later and travelling uniformly overtakes the cyclist at the 30th mile. Find the speed of the motorist. Illustrate graphically.

III

1. A formula for a photographic developer is as follows:—

Hydroquinone	3·5 gm.
Metol	0·5 „
Sodium Sulphite	25·0 „
„ Carbonate	45·0 „
Water to make up the total to ...	1000 „

Express the weights of the solids in grains when the *added water alone* is 20 oz. (7000 grains = 1 lb.)

2. If *a* apples cost two annas more than *b* oranges, and *c* apples cost twice as much as *d* oranges, find in Rupees, the cost of two hundred oranges. Give a numerical answer when,

$$a = 4, b = 12, c = 8, d = 15.$$

3. Two stations are 3 miles apart. Just when a train leaves the former, it emits a whistle which is heard in the other station at 1 P.M. If the train travels at 33 miles an hour and sound travels at 1089 ft. per second, find when, to the nearest second, the train will reach the other station.

4. Two places A and B are 0·375 inch apart on a map. The scale of the map is 1 in. to 1 mile. A is on a contour line marked 300 ft. and B on sea-level. What is the inclination of the road, to the horizontal, which goes from A to B? Only horizontal distances are mapped. Draw a neat figure to scale.

5. The arc of a sector subtends an angle of 40 degrees at the centre. The perimeter of the sector is 34 cms. Find the radius of the circle and the area of the sector.

6. The money-lender promises a loan of Rs. 2000 at 9 p c. interest, on condition that, if interest be not paid at the end of each year, 10 p.c. shall be charged for the next year on the principal and 18 p. c. on the interest portion. Another wants 10 p c interest, the interest to be added to the capital each year. If the loan is to continue for two years, which is cheaper to the borrower, and by how much is it cheaper?

IV

1. (*a*) A square number which is less than 10,000 is divisible by 2, 9 and 11. Find the number.

(*b*) Find the length in yards, of the side of a square field whose area is $\frac{9}{10}$ of an acre.

2. i. The sides x, y, z of a triangle in inches are given by the relations $x-2y=3z=15$; $3y-2x=2z$. Find the lengths of the sides of the triangle.

ii. Find the magnitude of the greatest angle of the triangle and find the radius of the circumscribing circle of the triangle.

3. A restaurant keeper bought 160 lbs. of ice at 2 as. 6 ps. per pound. A certain portion melted away and having sold the remainder at 4 as. per lb., he gained 40 per cent on his outlay. Find what portion of the whole stock did melt away.

4. A flower-bed is constructed as follows:—On the sides of a square, semi-circles are described to fall outside the square. A pathway is made out all round, 4 ft. wide, so that the interior boundary touches the semi-circles. If the side of the outer boundary be 26 ft. long, find the side of the original square. Draw a neat figure to scale showing the flower-bed and the pathway.

5. The average of p numbers is x. The average of q of them is $\frac{x}{3}$ and the average of s numbers of the remainder is $\frac{2x}{3}$. Find the average of those still left, in terms of x alone, if $p=3(q+s)$ and $q=\frac{s}{3}$

6. Knowing that the values of x and y are both less than 5 and that both are positive, give a neat graphical solution of:—

$$\frac{x}{1\cdot2}-\frac{y}{0\cdot8}=0\,;\quad \frac{x}{3}+2y=8.$$

The corresponding values of x and y that you find, must be such that the points you plot, must lie at the corners of squares in the squared paper.

V

1. The King's prize for aviation was won by F. T. Courtney who covered 810 miles in 5 hrs. 24 min. 51 sec. In the same year the Schneider Cup was won by Lieut. Rittenhouse who covered a distance of 185 miles in 1 hr. 12 min. 26 secs. Find the speed of each aviator in miles per hour, correct to the nearest tenth of a mile.

2. A statuette 10 in. high is to be made of silver. A plaster model 4 ft. in height has been made, and weighs 150 pounds; what will be the weight of the silver statuette to the nearest hundredth of a pound, silver being 12·9 times as heavy as plaster?

3. If 100 oranges be bought for Rs. 23–8 as. and sold in retail, what must be the price of an orange, that 6 per cent. of the amount realised may be profit?

4. A wrought iron plate is made in the form of a quadrilateral ABCD; the length of the sides in feet are as follows: $AB = 10$, $BC = 4·5$, $CD = 10·75$, $DA = 5·5$ and the diagonal $BD = 11·5$. Find the area of the plate. Draw a neat figure to scale. If the thickness of the plate is ½″, find its weight. (1 cub. ft. of wrought iron weighs 480 lbs.)

5. An army 3000 yards from the enemy, advances 2600 yards in 21 minutes. The previous part of the advance, under artillery fire, was at 100 yards per minute, and the latter part under rifle fire was at 300 yards per minute. At what distance was the advancing army, when the other opened rifle fire?

6. In a workshop a man earns a annas a day and a woman b annas a day. There are on the whole p people and the wages a day come to c annas. Find in terms of a, b, p and c (i) the number of men, (ii) the number of women, in the workshop.

VI

1. Some details of the French 75 mm. gun are given below. Take 1 in. = 25·4 mm. and 1 lb. = 453·6 grm. and express the given details in English units.

Calibre	75 mm.
Length	1906 mm.
Weight of projectile ...	7240 Kg.
Muzzle velocity	498 m. per sec.
Weight of gun and limber ...	1520 Kg.

2. (a) A straight line AB is $4(a+b)$ inches long. With centre A and radius $= 3(a+b)$ inches, a circle is described to cut AB at P. Again with centre B and radius $= 2(a+b)$ inches another circle is described to cut BA in Q. Find in terms of a and b the length of QP.

(b) The area of a rectangle is S sq. in. and its length is l inches. Find in terms of S and l (i) its perimeter, (ii) its diagonal.

3. A rectangular cistern is 10·5 ft. long, 5 ft. 4 in. wide and 3·5 ft. deep. It contains 5 tons of water. How high does the water stand? Find also how many bricks (9 in. by 4·5 in. by 3 in.) must be dropped into the water so that it shall just overflow. (A brick is assumed to absorb an amount of water equal to a fifth of its volume: (1 c. ft. of water weighs 62·5 lb.)

4. A field is in the form of a trapezium, ABCD. The parallel sides AB and DC are 120 ft. and 80 ft. respectively, and the sides BC and DA are 40 ft. and 60 ft. respectively. Draw a figure to scale. Bisect the non-parallel sides, and through the points of bisection draw perpendiculars to the parallel sides. Shew that the rectangle obtained is equal to the trapezium. Calculate the area of the trapezium.

5. Sixty per cent. of a certain ore is metal: while another ore of the same metal contains 75 per cent. metal. If 480 tons of the former and 360 of the latter be smelted together, find (i) the percentage of metal in the whole, (ii) the metal actually obtained if 20 per cent of the metal be lost in smelting.

6. (a) Simplify mentally $4001^2—3999^2$. (The process must be explained.)

(b) Two numbers differ by 3 and the difference of their squares is 531. Find the numbers.

VII

1. Two pedestrians walk, one towards the North at 3 miles an hour and the other towards the East at 4 miles an hour.

(a) What will be the distance between them at the end of:—30 min., 40 min., 1 hr., 1 hr. 30 min., 2 hrs.?

(b) Draw a graph from which the distance between them at the end of a given number of hours may be read off.

(c) Find from the graph the distance between them at the end of 1 hr. 45 min., from the time of starting.

2. A manufacturer supplies articles to two firms A and B. On articles supplied to the former he makes a profit of 20 per cent., and on articles sold to the latter he makes a profit of only 15 per cent. as the latter takes a larger quantity. If firm A sells the article at a profit of 20 per cent. and firm B also at a profit of 20 per cent., find how much a person can save by buying at the cheaper firm the articles, for which he had paid Rs 480 at the dearer. Also find the cost of manufacture of the articles he purchased.

3. A tradesman was selling his articles at a profit of 36 per cent. The cost to him of the goods was reduced by 2½ annas per article and he also deducted 2½ annas from the selling price of each article and thus gained 40 per cent. What was the cost to him of an article originally?

4. (*a*) In sinking a well it is found by experience that the depth sunk every week is 0·8 of the depth sunk in the previous week. If a depth *a* feet had been sunk in the first week and the work be stopped at the end of three weeks, what would be the total depth sunk in the three weeks?

(*b*) If 1200 copies of a book were sold in the first year and the no. of copies sold in each successive year be 20 per cent. less than the number sold in the previous year, how many copies would have been sold in three years?

5. *ABCD* is a square; arc *BPD* is drawn with centre *A* and radius = a side of the square and arc *BQD* is drawn with centre *C* and radius = a side of the square. Find the ratio of the area of the figure formed by the two arcs, to that of the whole square. Express the ratio in the form $1 : x$, where x is correct to two decimal places.

6. A town has a population of 400,000 and a water supply of 30 gallons per head, per day, is required. A reservoir is provided to hold supply for 6 months of 184 days with a mean depth of 20 feet. What must, in acres, the area of the reservoir be? Also find the area in sq. miles of the catchment basin if 20 inches of rainfall in the year will yield sufficient water to supply the town. ($6\frac{1}{4}$ gallons $= 1$ c. ft)

VIII

1. The sides containing the rt. angle of a right-angled triangle are $(0{\cdot}5x+0{\cdot}2)$ cms. and $1{\cdot}2(x-1)$ cms. respectively and the hypotenuse $(1{\cdot}3x-1)$ cms. Find the lengths of the three sides of the triangle and the length of the median on the hypotenuse.

2. A merchant bought a bicycle for Rs. 365, which he had to sell at a loss of 20 per cent. He bought another bicycle with Rs. 48 added to the proceeds. He sold this bicycle also and found that on the whole he gained Rs. 29. At what profit per cent, on its cost did he sell the second bicycle?

3. (*a*) A can run x feet in a seconds: *B* in y seconds can run b ft. more than *A*; *C*'s rate is the average of the rate of the two. Find his rate z, in miles per hour in terms of x, y, a and b.

(*b*) If $x = 176$, $y = 10$, $a = 20$, $b = 11$, find z.

4. An army is lying in ambush in a trench 91 yds. long and running east to west. The western end of the trench is 119 yds. and the eastern end 84 yds. from a road along which an enemy is expected to pass. Find from a carefully drawn figure (i) the direction of the road, (ii) the length of that portion of the road within 119 yds. of the trench.

5. A person invested Rs. 5000 in a Marwari Bank. The Marwari promised 9 per cent. interest on the money, in case no withdrawal was made till the end of 2 years from the date of investment or 6 per cent. interest in case the whole amount or a fraction thereof was withdrawn before the expiry of the term. The person

wanted Rs. 1500 at the end of a year. The Marwari offered a loan at such a high rate of interest that the person found he might *as well* to equal advantage, withdraw the amount he wanted, from the bank. What was the rate of interest demanded, if all accounts were closed at the expiry of the term of two years?

6. A man travals a certain distance and finds that if he had gone one mile per hour faster he would have saved an hour and a half; but, if he had gone slower than he did by half a mile per hour, he would have taken one hour longer. Find the distance and his rate.

N.B.—If his normal rate be denoted by x, in miles per hour, and the hour he took at that rate by y, then distance is denoted by xy. Take this hint and proceed.

IX

1. If p is the pressure in a thin pipe of outside diameter d and thickness t, the great tensible stress being f, then

$$t=\frac{p\cdot d}{p+2f}$$

A formula not quite so correct, but which is all the same in use is

$$t=\frac{p\cdot d}{2f}$$

Calculate from both formulæ the thickness of a pipe, such that $f=4000$, $d=8$, $p=500$. What is the percentage error in taking the second value?

2. Two boys have together 37 marbles; one arranges a few of these in rows of five each, and another arranges the remainder in rows of three each. Altogether there are nine rows. How many rows are there in the arrangement of the former?

3. A parcel of books had cost me Rs. 41. This included Rs. 24 *worth of books* on which a discount of 6¼ per cent. had been allowed, no discount having been allowed on the remainder. What would the parcel have cost me if—(i) the same rate of discount had been allowed on the remainder also; (ii) no discount had been allowed at all; also find (iii) correct to one decimal place, what rate of uniform discount allowed on the whole, would cost me the same Rs. 41.

4. On a base 4 cms. long, describe an equilateral triangle. Again on the same base and on the same side of it, describe a rectangle equal in area to the equilateral triangle. Find the height of the rectangle. Find also the ratio of the perimeter of the equilateral triangle to that of the rectangle. (Express the ratio in the form $1:x$, where x is correct to three decimal places.)

5. A person invested Rs. 2750 in a bank paying 4 per cent. interest per annum, the interest being added to the capital at the end of each year. At the end of 2 years 9 months from the date of investment, the bank failed and declared a dividend of 9 as. 6 ps. in the rupee. How much to the nearest rupee did he get from the bank?

6. The following table gives the error in a speedometer for distances recorded :—Exhibit the relation graphically

Distance recorded in miles	250	350	450	550	600	1000	1500
Error in miles	1½	2	2·5	3	3·25	5·25	7·75

(i) Read the probable error in a distance of 850 miles.

(ii) If the error is found to be 2·4, what is the distance recorded ?

X

1. A certain number is divided by the factors 11 and 7 and the successive remainders are 5 and 2. What will be the successive remainders if the same number were divided by the same factors in the order 7 and 11 ?

2. (*a*) When a train is in motion brakes are applied, and these reduce the speed at the rate of y miles an hour, per second. If the train be running at x miles per hour, when will it come to a stop if brakes be applied ?

(*b*) A railway passenger is charged x as. per mile by the first class, y pies per mile by second class, and z pies per mile by third class. A gentleman who goes a distance of 192 miles takes 2 first-class tickets, 2 second-class tickets and three third-class tickets. If A be the amount he paid in Rupees, establish a relation between A, x, y, and z.

3. In a recent motor-car race over a distance of 5½ miles, the winner had 1 min. 6 sec. start and won by a quarter of a mile from the next car, which had 2 sec. start. The average speed of the winner was 54 miles an hour ; find that of the second car.

4. A person bought a house, paid in cash Rs. 1000 and executed a mortgage bond for the balance at 7½ p. c. interest, the interest to be added to the capital each year. He paid the balance together with the interest thereon at the end of two years and found he had paid in all Rs. 3,889–1 a. Find the purchase value of the house.

5. A mass of ore weighed 7 tons. A certain mass was smelted and it yielded 78 per cent. of its weight of lead and 8 oz. of silver per ton of ore smelted ; the remainder 75 per cent. of its weight of lead and 7½ oz. of silver per ton. The total yield of silver was 55 oz. Find the average percentage of lead got from the whole mass on smelting.

6. A cyclist goes to a station 14 miles off, in 1 hr. 10 min., and returns without delay at 20 miles an hour. Find from a graph,

i. when he reaches home again ;

ii. the times when he was 5 miles from home.

III SERIES.

I

1. A cubic foot of	fresh water	weighs	62½ lb.
„ „	Timber-ash	„	49 lb.
„ „	Cork	„	15 lb.
„ „	Clay	„	125 lb.

If the weight of a cub. ft. of water be represented by 1, find correct to three significant figures, the number that should represent the weight of a cub. ft. of each of the other substances given above.

2. If the sides of a triangle are a, b and c inches, the length m inches of the median drawn to the side a is given by the equation

$$2m^2=b^2+c^2-\tfrac{1}{2}a^2$$

Find the lengths of medians drawn to the longest sides in each of the triangles whose sides are 7, 8 and 9 in.; 19·5, 18·9 and 4·8 in.

3. A cabinet maker finds that the cost of making furniture is 30 per cent. of the cost of materials used. At what price must he mark his goods that by allowing for cash sales one anna off each rupee of marked price, he may gain 25 per cent. on an investment of Rs. 300, on *materials alone*?

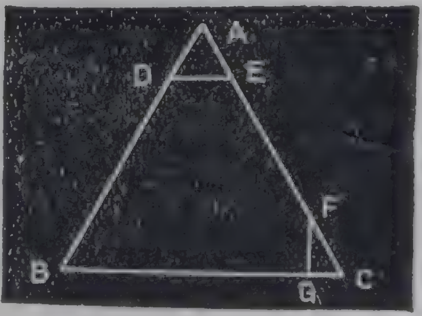
4. Two officers have the same annual income. The first saves 20 per cent. of his income, while the second spends Rs. 1100 more than the other. At the end of three years, the second has incurred a debt of Rs. 960. Find the annual income of each officer.

5. Passengers by a Motor Bus are allowed a certain weight of luggage free and the rest is charged for at a certain rate per pound. Two passengers having with them 116 lb. of luggage, are charged Rs. 2-7-0 and 1-9-6 respectively for excess. A third passenger carrying 77 lb. of luggage is charged Rs. 2-14-6. Find how much each passenger is allowed free of charge and how much per lb. is charged for excess.

6. Petroleum for Motor Buses is sent in cylindrical drums 8 in. across. Each drum is said to contain 4 gallons. Given that a gallon = 277·274 cub. in. and that $\pi = 3{\cdot}1416$, find the height of the drum to the nearest tenth of an inch.

II

1. *ABC* is an equilateral triangle and represents an estate drawn to scale, each side being 36 furlongs; $AD=DE=9$ fur. The portion *ADE* is sold and again the portion *FGC* is also sold. Angle *FGC* is rt. angle and FC = 12 furlongs.



Find by calculation, (i) the perimeter of the remaining portion of the estate correct to the hundredth of a furlong; (ii) its area correct to the nearest sq. yd.

2. A tinker is employed by a merchant to make tin cans. The tinker is to be paid a fixed wage of a annas a day and an extra bonus of b pies for every article finished, over and above m articles. The tinker works h hrs. a day and finishes at the average rate of p cans per hour. Find E his weekly earnings in rupees, if he works throughout the week uniformly.

If $a=11, b=6, m=20, h=10, p=3$, find E by substitution in the relation.

3. A certain kind of brass is made by melting together 25 parts by weight of tin, 160 of Copper and 5 of Zinc. Taking the price of Copper as Rs. 900 per ton, Zinc Rs. 330 per ton and tin Rs. 1770 per ton, find to the nearest anna, the cost of the material of a casting which weighs 130 pounds, it being assumed that in the fusing together, there is neither gain nor loss of weight.

4. At an election A received 250 votes more than B. An illegal proceeding was suspected and a re-election was therefore instituted. In the re-election 10 per cent. of those that originally voted for A and 5 per cent. of those that originally voted for B changed sides and B was elected by a majority of 100. Find the number of votes each secured originally.

5. At the beginning of each season the population of a seaside resort increases by 60 per cent. of the original population. At the close of this season 20 per cent. of the visitors stick to the place and the rest go back to their homes. If the population before the beginning of a season be 10,000, find when the permanent population of the resort will increase by 25 per cent. of the original population? What will that population be?

6. A wire goes once exactly round the edges of a right prism on a hexagonal base of 11 cms. side and also once exactly round a cylinder. If both be of the same height find the ratio of the volume of the cylinder to that of the prism, expressing the ratio in the form $1 : x$ where x is correct to two significant figures.

III

1. How will you proceed, as quickly and intelligently as possible,

(i) to shew that, $684^2+37^2=685^2$?

(ii) to find out the total cost of 15 viss of coffee seeds at Re. 1-15-6 each viss, and 30 viss of sugar at 15 as. 9 ps. per viss?

(iii) to find the interest correct to the nearest pie, on Rs. 375 for 6 months and 15 days at 7½ p. c., per annum. (1 month $=30$ days).

2. (a) If a number be increased by 20 per cent. of itself and if another number be diminished by 30 per cent of itself, the results are equal. Find the relation between the two numbers.

(b) If the sum of the two numbers be 380, find the two numbers geometrically by drawing to scale.

3. *A* wants to dispose of a diamond ear-ring which *B* wants to buy. They take the jewel to an appraiser *C*. Each of *A* and *B* without the knowledge of the other, bribes C. *A* offers *C*, 50 per cent. of whatever amount he by his valuation can save him. *C* fixes such a price for the jewel that he gets an equal amount from both. What is the price he fixes? If *A* gains, even after having paid the discount, 20 per cent. thereby, what was the price he paid for the jewel?

4. A litre of sea-water weighs 1026 grammes and contains 27 grammes of salt. A basin contains 200 litres of sea-water. It is evaporated and the remaining volume contains 15 per cent. of its weight of salt. Find the reduction in volume.

5. What is the side of that equilateral triangle whose area cost as much for paying at 8 as. a square foot, as railing the three sides did at Rs. 15-8-0 per yard?

6. A rectangular piece of wood $2'' \times 2'' \times 9''$ is taken and two uniform holes of circular section are scooped out, each to a length of $3\frac{1}{2}$ inches perpendicular to each square face of the piece. The weight of the remaining solid is $\frac{1}{3}$ of the piece taken. Find correct to the nearest tenth of an inch the diameter of the hole.

IV

1. One hundred members are to be elected from the various provinces in India to a certain constituency. The total population is 318,942,480. How many must each of the following divisions send if the number elected is to be proportional to the population?

Madras with a population of		42,318,985
Bombay	„	19,291,719
Bengal	„	46,695,536.

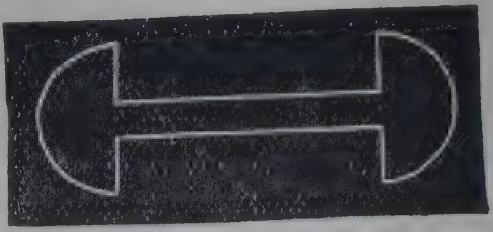
2. A vessel holds *a* c. c. and the weight of the vessel alone is *b* gm. Water is poured to the brim and then a solid of *p* c. c. weighing *q* gm. per c. c. is dropped into the vessel. What will be the weight of the vessel and the contents together, in terms of *a*, *b*, *p*, *q*?

3. At the beginning of each month I invest Rs. 50 in a bank which allows interest at 8% per annum on half-yearly balances; (the interest to be added to the capital at the end of every half-year). How much will stand to my credit at the end of $1\frac{1}{2}$ years from the date of investment?

4. (a) A soldier shooting at a target 570 yards off, heard the bullet strike the target 4 seconds after he fired. Another person standing at a point equally distant from the target and the soldier, heard the shot strike the target $2\frac{1}{2}$ seconds after he heard the report. Find the velocity of sound in feet per second.

(b) If the latter heard the report just as he saw the bulle strike the target, draw to scale a diagram to show the respectiv positions of the target and the soldier and the person.

5. The rungs of a ladder are cylindrical; a transverse section of a side of the ladder (into which the ends of the rungs are dovetailed) is a semi-circle. The entire length of the ladder is 11 ft. There are 20 rungs. Each rung is 9 in. long and 1 in. across and the diameter of the semi-circle is 4 in. Find the volume of teakwood in the ladder. The figure shews a transverse section along the axis of a rung.



6. Describe the quadrilateral $ABCD$, having no reentrant angle, given that $AB = 2{\cdot}4$ in., $BC = 0{\cdot}7$ in., $AC = 2{\cdot}5$ in., angle ACD is a rt. angle, $AD = 6{\cdot}5$ in.

Find, by calculation, the area of the quadrilateral.

V

1. In London there were, in one week, 1577 deaths, equivalent to a death rate of 19 per thousand per annum, while in Greater London (which includes an additional suburban ring) there were 2047 deaths equivalent to a death-rate of 17·8. Find the population of London and calculate the death-rate for that week of the outer ring alone. Take one year = 52 weeks.

2. At sea-level water boils at 212° F. At a height H ft. above sea-level the boiling point is lowered B degrees below 212° where B and H are connected by the relation $H = 520\,B + B^2$. If, at a height water boils at 205·4° F., find the height (to the nearest foot) of the place above sea-level.

3. It was ascertained that 72 per cent. of the weight of a certain ore was metal; but when 485 tons of the ore were smelted, only $287\frac{1}{2}$ tons of metal were obtained. What percentage represents loss in the process of smelting?

4. The population of a city is 48750. It has been found that the population at the end of every 25 years is 30 per cent. more than the population in the previous period. Find the increase in the population. What per cent. of the original, will the population be, at the end of a century?

5. A cigarette is cylindrical in shape and is 9·1 cms. long and 0·4 cms. across. A person who smokes 10 cigarettes a day, throws away $\frac{1}{10}$ of each cigarette. How many cubic cms. of tobacco would he have consumed in a year of 365 days? Also find the area of paper he would have smoked away.

6. Two places P and Q are 48 miles apart. A person leaves P at 7 A. M. and motors at 20 miles an hour to R, 18 miles from P. He then returns at double the rate to P, where business detains him for 9 min. He then motors to Q. Another who leaves Q at 9 A. M. walks to P at 3 miles an hour. If both meet at 10 A. M., find the speed of the motorist graphically. Also find when the motorist will reach Q.

VI

1. (i) The distance between two points whose co-ordinates are $(h, k,)$ $(a, b,)$ is $\sqrt{(h-a)^2+(k-b)^2}$. Find the distance between the two points (17, 21), (12, 33).

(ii) Find the area of the triangle formed by joining the points A (17, 21), B (17, 33), C (12, 33).

(iii) Also find the radius of the circle passing through A, B and C.

2. A contractor promised to supply 800,000 bricks at Rs. 3-4 as. per thousand. He stocked a few thousands which had cost him Rs. 2-14 a thousand and had to buy the remainder at Rs. 3-14 a thousand, with the result that in the contract he lost Rs. 125. Find how many thousands he had stocked.

3. (i) The cost of printing a forme of 8 pages is (i) a sum of money to set up the type, to impose the forme, etc., before printing is begun, and (ii) a certain rate, for every thousand copies to be printed. If a denote cost (i) and b, rate (ii) and if x thousands are to be printed, find y the total cost in terms of a, b and x.

(ii) If it costs Rs. 45 to print five thousand copies and Rs. 94 to print twelve thousand copies find a and b.

(iii) Draw the graph of the relation and find from it the cost for printing nine thousand copies.

4. An annual income of Rs. 2,000 and below Rs. 5,000 is liable to income-tax at 5 pies per rupee of income; while if the income exceeds Rs. 5,000, the tax is 6 pies per rupee. A man who gets an income of Rs. 4980 is, having paid the tax, better off by Rs. 2-11 as. than one who gets above Rs. 5,000. Find the annual income of the latter.

5. The section of the top of a bullock cart is a rectangle with a semi-circle attached to its top. If the height of the rectangle h ft. and the radius of the semi-circle r ft., find in terms of h, r and l, the area of bamboo tatty required for the top, l denoting the entire length of the top.

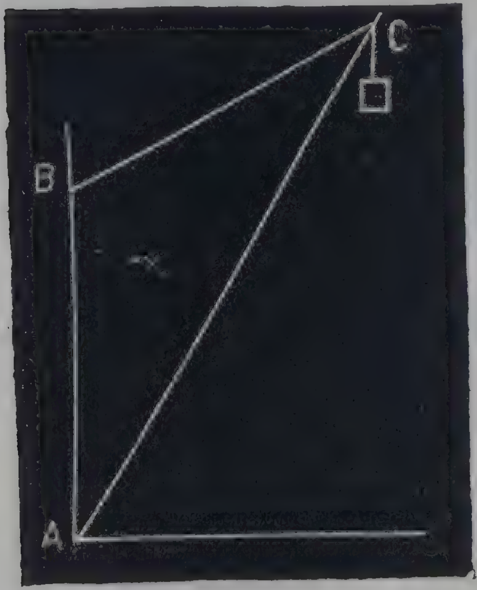
6. A dealer professes to sell ghee at 10 per cent. profit, but really mixes each viss with half viss of cheaper ghee which cost him only ½ the price of the superior sort. How much profit per cent. does he really make?

VII

1. (*a*) At what time after p o'clock will the hands of a clock be q degrees apart?

(*b*) When, between 4 and 5 p. m. will the hands of a clock be (i) at rt. angles, (ii) in a straight line?

2. The figure is a rough sketch of a crane. The vertical post AB is 8 ft. the jib AC, 24 ft. and is inclined at 30° to the post. From a drawing to scale determine the height of C above the ground and the length of the tie bar BC. Also verify results by finding the lengths entirely by calculation.



3. A circular bicycle track, 22 feet wide and a quarter of a mile round if measured 1 foot from the inner edge, is to be laid out in a square field; what will be the area in acres of the smallest suitable field? Take $\pi = 3\frac{1}{7}$.

4. (*a*) A petrol dealer dilutes a number of cans of petrol by withdrawing some petrol from each can and adding water. He gives his customers 10 p. c. discount for ready cash and yet makes a profit of 20 per cent. What percentage of the contents of each can did he draw out?

(*b*) If the marked price of each can be Rs. 3-4 as. how many cans must he sell that he may get a profit of Rs. 45 a day?

5. A farmer borrowed Rs. 200 from a mirasdar on condition that for the use of the money the mirasdar should be given 35 measures of black-gram and 20 bundles of straw. The farmer promised a few days prior to the due date, in full satisfaction, 61¼ measures of black-gram and 10 bundles of straw; but on the day, he was able to give, also in full satisfaction, 45½ measures of black-gram and 54 measures of doll. If in any case the mirasdar got 12½ per cent. interest, find at how many measures per rupee each of the cereals was rated. Also find the price of a bundle of straw.

6. A hollow circular cylinder, the inner diameter of whose base is 3 in. and outside diameter is 4 in. is placed on a cubic pedestal of the same material 6 in. each way. The weight of the pedestal is equal to the weight of the cylinder. Find the height of the latter.

VIII

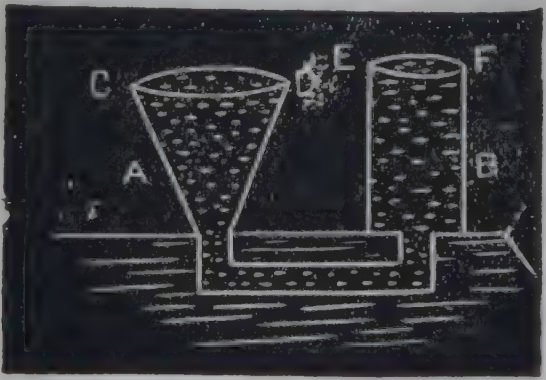
1. A map is drawn to a scale of 1 in. = 12 miles. The area of a country on the map is 12·25 sq. in. How much space will this occupy in a map drawn to a scale of 1 in. = 20 miles? Find the area required without actually finding the area of the country.

2. A school committee sanctioned a sum of Rs. 1120 to make for the school 56 benches and 28 tables. When the work was taken on hand, the committee revised the programme to 48 benches and tables, and thus the expenditure was cut down by Rs. 400. When th

work was in progress, the school received some donation for furniture, which enabled them only to enhance the number of benches to 50 and the number of tables to 25. What was the amount of the donation the school received ?

3. Describe a quadrilateral $ABCD$ with the following data :— AB BC, $AC=5{\cdot}2$ cms., $\angle B=120^\circ$, $\angle D$ is a rt. $\angle$, $CD=4{\cdot}8$ cms. Calculate the area of the quadrilateral. You are to draw the figure using your foot-rule and compasses only. The construction must be outlined.

4. The figure is a representation of the apparatus for shewing that liquids keep their level. A is a cone and B is a cylinder. The diameter CD is 1·4 in. and EF of the cylinder is 1 in. If water be poured so that it stands flush with the top, find the volume of water in the apparatus. The water in the connecting tube may be ignored. The height of each vessel is 2·24 in.



5. A certain liquid is poured into a cylindrical vessel, to half its height, and the weight of the vessel and the water is a gms. When the vessel is full of water, they weigh b gms. What will be the weight of the vessel and the water if the water stands to ¾ of the height of the vessel ?

6. A person invested Rs. 2000 in a business, stocked articles which he sold at 10 p. c. profit. The stock was exhausted in six months and with the money realised he bought fresh stock which he, selling at the same profit per cent. disposed of in the next six months. Another who started business with the same capital on the same day sold his goods at a profit of 8 per cent. but turned the capital and profit over once in 4 months. Who would have got the greater profit and find to the nearest rupee the excess of profit.

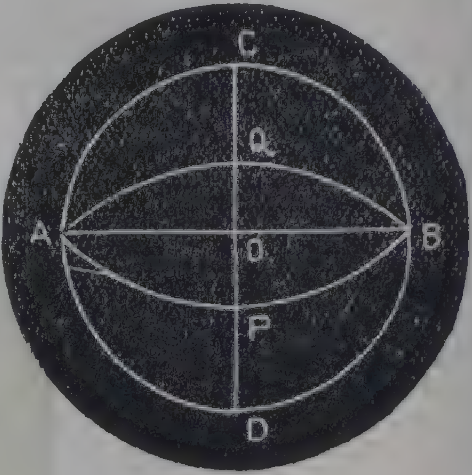
IX

1. A litre of water weighs a kilogram, a litre of another liquid weighs 1·340 kilograms. A mixture of the two weighs 1·270 kilograms per litre. Determine correct to the nearest tenth of a c. c. the volume of each liquid in a litre of the mixture.

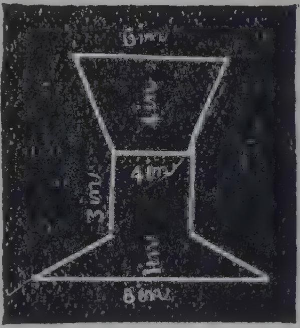
2. The length of a rectangular park exceeds its breadth by a yds. A man whose ordinary pace is b ft. per second, starts from a point in the boundary, and walking along the boundary, reaches the starting point in c minutes. Find the length of the park in terms of a, b and c.

3. An armourer undertakes to supply 2000 swords at 17s. 3d. each. He estimates that if 5 per cent. fail to stand the required test and are not paid for, the profit will be 15 per cent. on his outlay. At the trial 35 per cent. prove worthless and are not paid for. Find his loss.

4. AOB and COD are perpendicular diameters of the circle whose centre is O and radius $= r$. With centre D and radius $= DA$ or DB, arc AQB is drawn. Again with centre C and radius $= CA$ or CB, arc APB is drawn. Find by calculation the areas of the figures (i) bounded by ACB and AQB, and (ii) AQB and APB, (iii) AQB and ADB.



5 The diagram represents the end-on view of an anvil. If the metal weighs 450 lbs. to the c. ft. find to the nearest c. ft. the weight of 1 foot length of the anvil. (*Note*—The anvil is only a certain length of a steel girder.) The necessary lengths are noted in the figure.



6. Of two rival candidates A and B who stood for election, B secured 40 more votes than A; but on scrutinising 5 per cent. of A's votes and 10 per cent. of B's votes were rejected as illegal and A was therefore elected by a majority of 6.

Reduce the conditions to simultaneous equations in two unknowns and hence find the number of votes B secured originally.

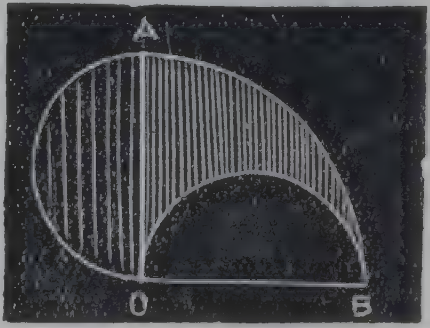
X

1. There are two floors in a building and there are a rooms downstairs and b rooms upstairs. One room downstairs is used by the owner and the two end-rooms fetch only half the rent of the other rooms each of which is let at Rs. m per room per month. The two-end rooms upstairs are let only at half the rate of the other rooms there, each of which is let at Rs. n per room per month. The rent due is collected and the owner has p five-rupee currency notes. Find p in terms of other letters. If $a = 32$, $b = 26$, $m = 6$ and $n = 4$, find p.

2. On a division in the house of Commons, if the number of members for the motion had increased by 50, from the opposition the motion would have been carried by 5 to 3; but if those against the motion had received 60 from the other party, the motion would have been lost by 4 to 3. Did the motion succeed, and how many members voted on the question?

3. In performing a journey of 96 miles, a motorist travels a distance of 58 miles of fine road at 30 miles an hour, 28 miles of rough road at 12 miles an hour and 10 miles of cart-track at 6 miles an hour. Find his average speed in miles per hour correct to the nearest tenth of a mile.

4. *AOB* is the quadrant of a circle. Semi-circles are drawn on the radii *OA* and *OB* as in the figure. Find (1) the area of the shaded portion, (2) the ratio of the perimeter of the shaded portion to that of the quadrant *AOB*. How many figures like the shaded portion can you cut from a circle? Draw a figure to show how you will cut it.



5. A gentleman was able to save in a certain year 35 per cent. of his income. Next year his income rose by 12½ per cent. and his expenses also increased by 25 per cent. with the result that he saved Rs. 36 less than in the previous year. What was his income in the former of the two years?

6. Beyond one end of a tennis court, a wall has to be raised It is to be 45 ft. long and 10 feet high. The bottom is 30 inches thick and the top is 10 in. thick. The interior surface of the wall is vertical and the back slopes uniformly to the vertical. Draw to scale the end-view of the wall at 5 as. per cub. ft.

XI

1. 2,987,750, ½-kilograms of tobacco are shipped in 16,890 bales. Find to the nearest tenth of a cwt., the average weight of a bale, a kilogram being equivalent to 2·2046 lb.

2. *ABCD* is a rectangular enclosure. *AB* = 228 yds., *BC* = 120 yds. *P* is a point 90 yds. from *A* along *AB*. Find a point *Q* in *BC* such that the distance *PD* + *DQ* is less than the distance *PA* + *AD* + *DC* + *CQ* by 136 yds. Also find the lengths of *PD* and *DQ* separately.

3. (*a*) If two trains whose speeds are *T* and *t* miles an hour, start from the same place at the same time, how far will they be apart at the end of *s* minutes from the time of starting (i) if they run in opposite directions, (ii) if they run in the same direction?

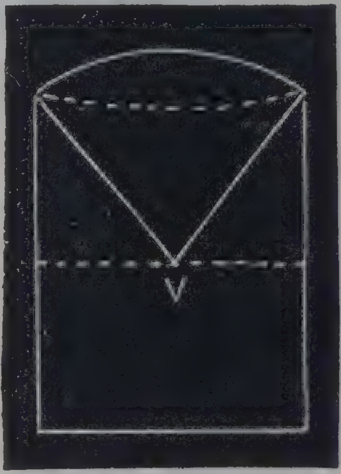
(*b*) Two trains running towards each other are 8 miles apart, in six minutes from the time of crossing. If their speeds are as 9 : 7, find their rates in miles per hour.

4. A circular plate of iron of uniform thickness is 35 cms. in diameter and weighs 17·2 kg. Find what will be the weight of the plate correct to the nearest tenth of a kilogramme after 25 circular rivet holes, each 1 cm. in diameter, have been drilled out of it.

5. A retail dealer bought 90, ¼-viss packets of coffee seeds at Rs. 3-4 per viss. He gently opened the flaps of the packets and drew out from each packet, 2 palams of coffee seeds, and packed the quantity drawn out also into so-called ½-viss packets. If he sold each packet at Re. 1-12 as., find (*a*) his gain, (*b*) his gain per cent.

6. (*a*) If a solid cone whose height is *h* in. and the radius of whose base is *r* inches be put into a cylindrical vessel of the same height and of the same circular base and water poured into the cylinder, how much will it contain?

(*b*) A vessel is made thus:—A cylindrical block of wood 4·2 inches across and 2 ft. high is taken and a cone is scooped out at one end such that the vertex of the cone is in the middle of the cylinder and the circular base of the cone is equal to the base of the cylinder. (The figure shows a vertical section of the vessel through a diameter.) Find the volume of wood in the vessel to the nearest cub. in. without finding the volume of the conical cavity.



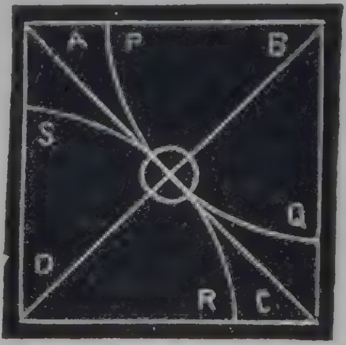
XII

1. A box contains *a* one-grain weights, *b* two-grain weights, *c* ten-grain weights and *d* one-ounce weights. The weight of the box and its contents is *f* ounces. Find the weight of the box alone in terms of a, b, c, d and f. If $a=12$, $b=4$, $c=6$, $d=5$, find f, if the weight of the box alone is 2 lb. (Troy).

2. (*a*) A tradesman has marked his goods at an advance of 20 p.c. on the cost price. The sale-proceeds in a week come up to Rs. 375. If he has to spend 10 per cent. of the proceeds towards rent, establishment etc., find his net profit that week.

(*b*) If, after deducting 10 per cent. of the weekly takings, the tradesman wants to gain 20 per cent., at what profit per cent. must he mark his goods?

3. A person had invested Rs. 12,000 in a business. At the end of a year he got a return of 16 per cent. He gave 5 per cent. of this profit as bonus to his employees and took Rs. 1200 for his private expenses. The balance was added to the capital. Next year there was a return of 25 per cent. on the capital in the business, and of this 25 per cent. was declared as bonus and Rs. 1200 as before taken for private expenses. How much will he have in the business at the end of the two years?



4. *ABCD* is a square, *O* its mid-point. *POQ* is an arc drawn with centre *B* and radius *BO*; *ROS* is an arc, drawn with centre *D* and radius *DO*. Find the area of the figure *POSA*, given that *AB* is *a* in. long.

5. Two men have to go from *P* to *Q*, a distance of 32 miles, and be there at the same time. One of them rides on the cycle at 12 miles an hour and the other begins to walk at 4 miles an hour. The former leaves the cycle at an intermediate village and walks the remaining distance at 3 miles an hour, having taken rest in the village for an hour. The latter reaches the village, takes rest there for an hour and a half and then rides on the cycle at 12 miles an hour, and both reach the destination at the same time. How far is the village from *P*?

6. A granary on a rectangular base is 4 ft. long 3 ft. 4 in, broad and 4 ft. high. This contains paddy to three-fourths of its height. The contents are removed to a cylindrical one, made by bringing together the breadthwise ends of a bamboo tatty 16½ ft. long and 4 ft. broad. How high will it stand?

XIII

1. When x and y are very small we may take

$$\frac{1+x}{1+y}$$

as being very nearly equal to $1+x-y$. What is the error in this when $x=0\cdot02$, $y=0\cdot03$? Give the answer correct to two significant figures.

2. *ABC* is a triangle, *C* being a right angle. The side *AB* is 15·34 in. and the side *BC* is 10·15 in. What is the length of *AC*? What is the area of the triangle in square inches?

If this is the shape of a piece of sheet brass 0·13 in. thick and if brass weighs 0·3 lb. per cub. in., what is its weight?

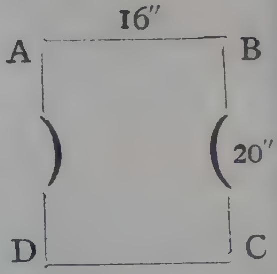
3. A certain quantity of goods is to be carried in carts and wagons and it is found that this can be exactly done with 24 carts and 8 wagons or with 15 carts and 12 wagons, but that 12 carts and 12 wagons will carry 2 tons less than the whole. What is the totalf quantity?

4. The population of a town in 1890 was 12,298, a decrease o 8⅓% of the census of 1880; in 1880 there was an increase of 7½% of the census of 1870. What was the population in 1870?

5. A merchant bought season goods for Rs. 3,000 and marked it at an advance of 25% on the cost price. At this rate he sold $\frac{1}{3}$ of the stock, when he found that two-fifths of the remainder was damaged and was unsaleable. There was however a huge demand, and he therefore enhanced the price of the remainder by 30% of the prices already marked. If the whole stock was cleared, find his gain per cent. on the whole.

XIV

1. From a rectangular piece of paper 16″ by 20″ two equal semi-circles are cut off as shewn in the diagram. If the area of the remaining piece be $281\cdot5$ sq. in., find the radius of the semi-circles cut off.



2. From 1870 to 1880, the population of a town increased 30%; from 1880 to 1890 it decreased 30%. The population in 1870 exceeded that in 1890 by 2781. Find the population in 1880.

3. Two launches race to a buoy and return. The first has a start of 10 minutes, steams at the rate of 6 miles per hour to the buoy and 8 miles per hour on the return. The second steams throughout at $7\frac{1}{2}$ miles per hour and is beaten by one minute. How long is the course?

4. The velocity of sound is given in feet per second by the formula $v = 1090 + 1\cdot1(t-32)$; t denotes the temperature in degrees Farenheit. When the temperature is 72° find v, the velocity of sound. Also find the temperature when the velocity is 1145 feet per second.

5. The distance between the bases of two towers is 50 feet and the angular depression of the top of the first as seen from the top of the second is 30°. The height of the second is 80 feet. Find by calculation, the height of the first and the distance between the tops of the towers.

XV

1. *A* and *B* are two houses on a straight road. The houses are 120 feet apart. *CD* is a road parallel to *AB*, at a distance of 480 feet from *AB*. Locate the well *W* which is as far away from *A* and *B*, as it is from the road *CD*. Draw a neat figure to scale and find by measurement the distance of the well from *A*, *B* or the road *CD*.

2. A fisherman told a yarn about a fish he had caught. If the fish were half as long as he said it was, it would be 10 inches more than twice as long as it is. If it were 4 inches longer than it is and he had further exaggerated its length by adding 4 inches, it would be $\frac{1}{5}$ as long as he now said it was. How long is the fish and how long did he first say it was?

3. A man sets off to a village 18 miles distant walking at 4 miles an hour. At the end of an hour and a half, he gets a lift and reaches the village in another $1\frac{1}{2}$ hrs. He remains in the town 1 hour and returns on his bicycle at 10 miles an hour. Find from a graph the total time he took to return home and also the rate per hour during the second part of the journey to the village.

4. A trader authorised his agent to start a trade with a capital of Rs. 3,000. He bought goods on commission of $2\frac{1}{2}\%$ which he kept for himself. and the trader allowed him $5\frac{5}{9}\%$ commission on the profits. The agent thus got Rs. 94-8 as. What profit per cent. did the trader get? What profit per cent. did the agent realise on the actual amount invested?

5. A square, a circle and an equilateral triangle have each the same perimeter, 15 inches. Arrange the areas in descending order of magnitude.

XVI

1, From which of the conditions given below may we conclude that the triangles ABC and A' B' C' are identically equal? Give the reason for your decision in each case:

(i) $A=A'=80°$
$B=B'=40°$
$a=a'=4$ cm.

(ii) $a=a'=3$ cm.
$b=b'=4$ cm.
$c=c'=5$ cm.

(iii) $a=a'=4$ cm,
$b=b'=2$ cm.
$C=C'=80°$

(iv) $B=B'=55°$
$b=b'=5$ cm.
$c=c'=6$ cm.

(v) $A=A'=35°$
$B=B'=120°$
$C=C'=25°$

(vi) $C=C'=90°$
$c=c'=13$ cm.
$a=a'=5$ cm.

2. The capacity of a double-bullock bandy is 8 ft. by $1\frac{1}{2}$ ft. by $1\frac{1}{2}$ ft.: 180 tons of gravel have to be carried to a roadside from a quarry. 10 bandies are engaged. How many trips must each make to bring in the gravel from the quarry, supposing that all make an equal number of trips and that 20 cub. ft. weigh a ton?

3. An army of 10,000 soldiers had to be raised from three provinces. These respectively had 160480, 194870, 208400 men fit for service. How many should each province supply, if the supply is to be proportional to the population?

4. There are a pages in a book. 3 pages are blank, the title-page has t lines of print; the preface occupies 2 pages and each page of preface has p lines. The rest is the body of the book which has on an average b lines of print on each page. If the total number of lines in the book be l, find a formula for b in terms of a, t. p and l.

If $a=540$, $t=20$, $p=28$ $l=20368$ find b.

5. When two years ago, one rupee weight of pure silver cost 12 annas, I purchased pure silver for Rs. 120 and made into a dish. I had to pay an extra Rs. 10 towards making charges and wastage. If I sell it now for the net weight of silver at 11 as. per rupee weight, how much will I lose if in the two years it had lost $1{\cdot}5\%$ of its original weight? Give the answer to the nearest anna.

XVII

1. The missing digits in the multiplication sum given below are marked by asteriks. Find them, giving brief reason for the digits inserted:

$$\begin{array}{ccccc}
 & & 3 & \ast & \ast \\
 & & & \ast & 4 \\
\hline
\ast & \ast & 4 & & \\
\ast & 5 & \ast & \ast & \\
\hline
9 & \ast & 8 & \ast &
\end{array}$$

2. There are b boys and g girls in a school. The average age of the boys exceeds that of the girls by a months. The average age of all the pupils is p years m months. Find in months the average age of the boys.

3. It is desired to construct a half-mile track. The start and finish are to be straight-ways intersecting at right angles at the goal. The rest of the track is to be an arc of a circle tangent to the two straight-ways. Find the radius of the arc and the length of the arc in feet; also the area inclosed by the track to the nearest tenth of an acre.

4. (*a*) A merchant sells coffee seeds wholesale at Rs. 125 per bag of 50 viss and also retails at Rs. 2-10 as. per viss. If he gains 25% in the whole sale rate, what profit per cent. will he get in the retail?

(*b*) If he sells ⅔ of his whole stock wholesale and the remainder in retail, what average gain per cent does he make?

5. Two hares run round and round a circular course 150 feet along the circumference. When they run in opposite directions they meet every 5 seconds, and when they run in the same direction they meet every 25 seconds. Find the rate of each hare in feet per second supposing their rates to be uniform.

XVIII

1. The formula $w = 3{\cdot}875\ (d_1^2 - d_2^2)l$, gives the weight in pounds of a pipe, d_1 and d_2 being the outer and inner diameters of the pipe in inches and l its length in feet. Find w when $d_1 = 2{\cdot}5$ inches, $d_2 = 1{\cdot}5$ inches and $l = 15$ ft.

2. (*a*) Multiply $5\sqrt{6} + 3$ by $3\sqrt{2} - 2\sqrt{3}$ and express the result in its simplest form.

(*b*). Hence find correct to two decimal places the value of $\dfrac{3 + 5\sqrt{6}}{2\sqrt{3} + 3\sqrt{2}}$ given that $\sqrt{2} = 1{\cdot}414$ and $\sqrt{3} = 1{\cdot}732$.

3. A regular hexagon is reduced to one of smaller size by cutting straight across, from the middle point of each side, to that of the next. What portion of the area is removed ?

4. A carpenter has a certain number of chairs ready made and can make only a fixed number per week. He takes a contract for the supply of chairs and promises to supply regularly 45 chairs per week for 20 weeks or 47 chairs per week for 12 weeks. Find the stock ready on hand and the number he can make per week.

5. A grocer by selling 12 viss of sugar for a certain sum gained 20%. If sugar advances 10% in the wholesale market, what per cent will the grocer now gain by selling 10 viss for the same sum ?

XIX

1. A Municipality planned two roads each 15 yards broad. One was to be 750 yards long and the other 970 yards. If the ground that had to be acquired in that part of the country cost Rs. 2,200 an acre, what was the difference in expense of buying land for each ?

2. An attar merchant gave me specimens of two scented sticks of the same length one of which would burn for 4 hours and the other for 5 hrs. I lighted them at once and after a time I found that what remained of one was exactly 4 times of what remained of the other. How long were they alight ?

3. From Pudukotah to Trichy is a distance of 33 miles. Six people wanted to go to Trichy. They were negotiating with a Bus owner when two more who had to go to Kiranur 14 miles, and 4 more who had to go to Ammachatram 11 miles—both on the road to Trichy—joined them. If the Bus was engaged for Rs. 22-8 as., how much should each pay, if they travelled together and if the fare is to be proportional to the distance travelled.

4 I engaged the services of a boy to weed my garden. I paid him one rupee every three days. He worked only for 8 hrs. a day. The boy left me and another grown up boy offered his services, who did in 20 minutes what the other had been doing in 25 minutes, but worked only for 7½ hrs. a day ? How much per day is he entitled to receive ?

5. Two tangents to a circle from the same point, P, enclose an angle of 60°. If the radius is 4 cm., find *(a)* the distance of P from the circumference, and *(b)* the area of the figure bounded by the tangents and the greater arc.

XX

1. Calculate as shortly as possible, (i) the area of a circular ring the inner radius of which is 2·13 inches and the outer radius is 4·87 inches.

(ii) 93% of 7563 is 31% of what number ?

(iii) If 36% of the strength of a mixed school is girls and there are 736 boys, find the total strength.

2. A train is travelling at 60 miles an hour directly towards a bridge, the arch of which causes an echo. If when 3 miles away from the bridge, the whistle is blown, how long will it be before the driver hears the echo, given that sound travels 1,120 feet per second ?

3. In a certain school 40 boys answered the Eng. History paper at the Public Examination. The school average was 34. If two of the boys had been withheld from sitting for the examination, the average would have risen to 35·5. Find the individual marks secured by these two, if they differed by 3 marks.

4. Find (*a*) the total surface, (*b*) the volume of a block of wood 18 in. square and 3 in. thick, with a circular hole of 14 in. diameter running through its centre.

5. There are two routes from A to B ; one runs due N. from A 3 miles, and then turns N. E. Another runs due E. from A, 4 miles, and turns N. How far is one route longer than the other ?

XXI

1. In the triangle ABC, angle $A=30°$, $AB=48$ feet, and $BC=16\sqrt{3}$ ft. Shew that the triangle is either rt. angled or isosceles and find the length of CA in each case.

2. Of the population of a district 2 % died from all causes and 21·5 % of the deaths was due to cholera. If the deaths from other causes exceeded the deaths from cholera by 912, find the total population.

3. A peg is being driven into a hole by hammering uniformly at the top. Each succeeding blow pushes the peg down to a depth 20 % less than the depth driven in the previous blow. If the first blow has driven the peg to a depth of 4 inches, at what depth will the other end of the peg be, at the end of the third blow ? The answer must be correct to the nearest hundredth of an inch.

4. A number of apples in a basket weighed 60 lb ; *n* children each ate 3 apples and *m* other children each ate 2 apples. If the average weight of an apple is *w* ounces, how many pounds of apples were left in the basket ?

If $w=4$, $n=2m$, and 18 lb. of apples were left, how many children were there ?

5. A rectangular field of 24·2 acres whose length is twice its breadth, is to be fenced all round Assuming that five rows of wire will be required and that the length of each round of wire is 5 % greater than the perimeter, find the length of wiring required.

XXII

1. Given the formula,

$$S=\frac{n}{2}\left[\,2a+(n-1)\,d\,\right]$$

(*a*) Find *d* in terms of the other letters.

(*b*) Find *d* when $s=15\frac{1}{2}$, $a=\frac{1}{3}$, $n=6$.

2. The water in a certain reservoir is 6 ft. deep, but the level is sinking at the rate of 4 in. a day. The water in another reservoir is 3 ft. deep and is rising 5 in. a day. When will the depth of the two reservoirs be the same and what will this depth be?

This question is to be solved (i) by graphs, (ii) by equation.

3. In 1921 a merchant was selling an article at a price 40 % above its selling price in 1914. By how much per cent would he have required to reduce it, to bring it back to its 1914 price? If he reduced the 1921 price by 20 %, how much per cent. would it still be above the price of 1914?

4. A plot of ground consisting of two parallel straight sides and two semi-circular ends, can be so inscribed in a rectangle that the straight sides of the plot lie along the longer sides of the rectangle. If the distance around the plot is half a mile and the longer sides of the rectangle are 1000 ft., find the distance between the parallel sides of the plot and also the lengths of those sides.

5. The model of a solid prism is 2 feet high and stands on a square base of side 8 in. The mid-points of two adjacent sides of the top square are joined by chalk and the block is sawn into two parts vertically down through the line. Draw a figure to represent the top after the cut. Find the ratio of the volumes of the two parts. Find also in cub. in. the volume of the bigger of the two parts.

XXIII

1. A man's annual income is Rs. R. He pays p % of this towards premiums to insurance and this amount is free from income-tax. On the rest he pays tax of 6 pies per rupee. If his monthly household expenses come in the aggregate to Rs. E, find his annual saving S in rupees, in terms of R, p, and E.

2. A rectangular field is 200 ft. by 60 ft. A cow is tied to a peg in the centre of the field, by a rope 60 ft. long. Find what area of the field it can graze through.

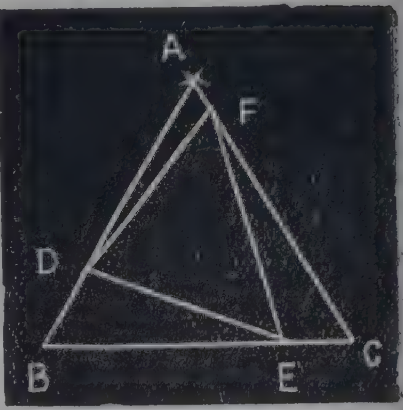
3. A contractor estimated the cost of a building at Rs. 24,000. Two-thirds of this amount was for materials and the remainder for labour. He gave the tender at 15 % above the estimated cost. When a fourth of the work was over, cost of materials rose by 10 % and when half the work was over, wages increased 15 %. What fraction of his estimated profit did he get, supposing he purchased materials as occasion arose? What was the actual gain made?

4. Two casuarina trees, each of 40 ft. height were standing on a level ground 20 ft. apart. Blown by the wind, one of the trees broke without parting asunder and the end of the broken top rested against the other tree 10 ft. below the top of the latter. Find the height of the stump of the broken tree. Draw a figure to scale.

Also shew how without calculation you can geometrically draw the figure and measure the height of the stump.

5. $ABCD$ is a rectangle. $AB = 2{\cdot}8$ in., $BC = 2{\cdot}4$ in. Find by construction a point P, which is equidistant from the sides AB, BC, CD and calculate the distance of this point P, from A and also from the side DA.

XXIV



1. *ABC* is an equilateral triangle, representing a field each side of which is 120 yards long. The sides are divided at *D* in the ratio 2 : 1, at *E* in the ratio 3 : 1 and at *F* in the ratio 5 : 1. Find the area of the triangle *DEF*. What fraction of the area of the field is this area ?

2. In the year 1911 the number of people admitted to workhouses in London was 101300, representing 22·4 per 1000 of population and corresponding figures for 1912 were 105080 and 23·2 per thousand. Find the increase or decrease in population ; what per cent. correct to one decimal place is this increase, of the original population ?

3. With a view to introduce vocational training in schools a board passes the following resolution :—" A sum of Rs. 8000 is to be deposited at the beginning of the 1st year in the Imperial Bank. Each succeeding year this grant will be enhanced by 20 % of the previous year's grant and deposited in the bank." If the bank allows 6 %, interest being capitalised annually, find the amount, to the nearest rupee, that will be available for the purpose at the beginning of the third year.

4. Two towns lie respectively 3 and 8 miles perpendicularly back from the same bank of a straight river. The points on the bank nearest to the two towns are 10 miles apart. A pumping station also on the bank is equidistant from the two towns and is connected with each town by a st. line of pipe. Find from a diagram and also by calculation, the position of the pumping station and the length of piping required for each town.

5. A man rides a distance of p miles and walks back at the rate of q miles an hour. The entire trip took t hours. Find his rate of riding.

XXV

1. Four empty lidless boxes made of teak planks ½″ thick, fit exactly one into the other. If the inside measurements of the smallest is 4″ by 3½″ and depth 2½″, find the dimensions of the biggest. If the boxes be placed one above the other open face downwards, how high will they stand ?

2. A heap of mud is being worn away by the weather. Every month it loses 0·75 % of its weight at the beginning of that month. If it weighed 280 tons 3 months ago, what will it weigh now ?

3. A casuarina tope was auctioned by the sirkar and thehighest bidder for Rs. 3,500 was ordered to cut down all the trees and remove them in 90 days, failing which he shouldpay a penalty of Rs 2-8. as.

for every day beyond the 90 days. The bidder was idle for the first 55 days and then engaged 20 wood-cutters at Re. 1-4 each per day and 10 bandies at Re. 1-8 each per day. He realised a profit of Rs. 590 by selling all the wood for Rs. 6,000. Find the number of days for which he should have paid the penalty.

4. The area of an isosceles triangle is $36\sqrt{3}$ sq. cms. The altitude on the unequal side is 6 cms. Find the area of the circumscribed circle and the length of the arc subtended by one leg of the triangle.

5. A speculator borrowed money at 12 % per annum C. I., interest payable half-yearly. He purchased bicycles. He had to fix prices for cash payment, for six months' credit and for 12 months' credit. If he also compounds interest and principal half-yearly what should be the respective prices for bicycles purchased by him for Rs. 200, Rs. 180 and Rs. 150, prices for cash payments being 20 % in advance. Give the prices to the nearest anna.

XXVI

1. The birth rates of three districts were 12·5, 25 and 20 per thousand and there were 73484, 49382 and 38940 people in those districts respectively. Find correct to three significant figures the combined birth-rate per thousand of the three districts.

2. A and B enter into partnership. A puts in Rs. 3500 into the business but at the end of 3 months withdraws Rs. 450 and again 7 months from this date withdraws Rs. 315 from the capital, while B withdraws nothing. If at the end of a year, they are entitled to equal shares in the profits, how much capital did B provide?

3. P lb. of water per sec. are discharged through a circular orifice, h ft. being the head of water above the orifice. P and h are connected by the equation, $P^2=(14{\cdot}6)^2h$. If $h=18$, find P correct to three significant figures.

4. A man standing on a long straight road saw a motor-car turn a corner into the road and come towards him. He heard the horn of the car $5\frac{1}{2}$ sec. after he first saw the car and the car passed him 3 min. $39\frac{1}{2}$ sec. after he heard the horn. Assuming that the horn was sounded exactly as the car turned the corner and that sound travels at the rate of 1140 ft. per sec., find the number of miles per hour the car was travelling.

5. Plot the points to scale 1 unit$=\frac{1}{5}''$; A, (5, 10); B, (−7, 5) C, (−10, 1). If A, B and C be the corners of a parallelogram, find the co-ordinates of the fourth corner D and find the lengths of the sides

XXVII

1. If the altitude of an equilateral triangle were increased by p in. the area will be increased by q sq. in. Shew that a side of the equilateral triangle is

$$\left(\frac{q}{p}-\frac{p}{\sqrt{3}}\right) \text{ in.}$$

2. A water-tank has vertical sides and the top is a square. A scale up one side shows the amount of water in the tank in thousands of gallons. These marks are $8\frac{1}{3}$ inches apart. Calculate the length of a side of the tank correct to the nearest foot. 1 cub. ft. contains 6·2 gallons.

3. If a water-pipe is l yards long, d inches in diameter, and one end h ft. above the other, the number of gallons which flow through in one minute, is given by the formula $\sqrt{27d^3h \div l}$. Find how many gallons per minute will flow through a pipe, a mile long, $4\frac{1}{4}$ in. in diameter and with a fall of 38 ft.

4. An army has food supplies for a certain number of days. If the army be reduced by m % the supplies will last for n % of the days longer. Show that

$$\frac{1}{m} - \frac{1}{n} = \frac{1}{100}$$

5. Two men measure a rectangular sheet of paper; one finds its length and breadth in inches to be 5·32 and 4·15. The other finds them to be 5·35 and 4·18. Calculate the area in each case. What is the percentage difference of either from the mean of the two areas?

XXVIII

1. A hotel keeper finds that when he has G guests a day, his daily expenditure is Rs. E and his daily income is Rs. R. From the accounts maintained by him over a long period, the following numbers are obtained.

G.	E.	R.
210	167	158
270	194	212
320	216	264
360	234	298

Draw a graph from which his profit may be read off, when there are from 200–400 guests a day. If on a certain day, his income just covered his expenditure, how many guests were there on the day, and find the number of guests when his profit was Rs 5–8 as.

2. Write down algebraically:—"Square a, divide by the square of b, add 1, extract the square root, multiply by w, divide by the square of n." Give the result in an elegant form.

3. The Motor fare between two towns was enhanced by 20 %. As a consequence there was a fall in the number of passengers, though the average receipts continued to be the same. Find the percentage fall in the number of passengers.

4. How much longer is the circumference of a circle of radius 10 cms. than the perimeter of an inscribed regular octagon. Give the result to the nearest tenth of a cm.

5. The steamship *Majestic* in May 1923 covered 3137 knots in 5 days 10 hours and 48 minutes. The steamship *Leviathan* in June 1923 made 687 knots in 25 hours. Find the speed of each per hour.

XXIX

1. The federal income-tax on incomes between \$ 12,000 and \$ 14,000 was in 1919 as follows:—first, a uniform tax of \$ 190, on all such incomes; secondly, an additional tax of 5 per cent on the excess of such an income over \$ 12,000. Write down a formula expressing the total tax t, which a man must pay, whose income x lay between the foregoing limits, and from a graph find the tax that must be paid by a man whose income is \$ 13750 and the income of a man who pays a tax of \$ 230.

2. (*a*) If the rates of four competitors in a race be represented by 1, 0·75, 1·125 and 1·25, find the numbers which must represent the times taken by them respectively to run the whole course.

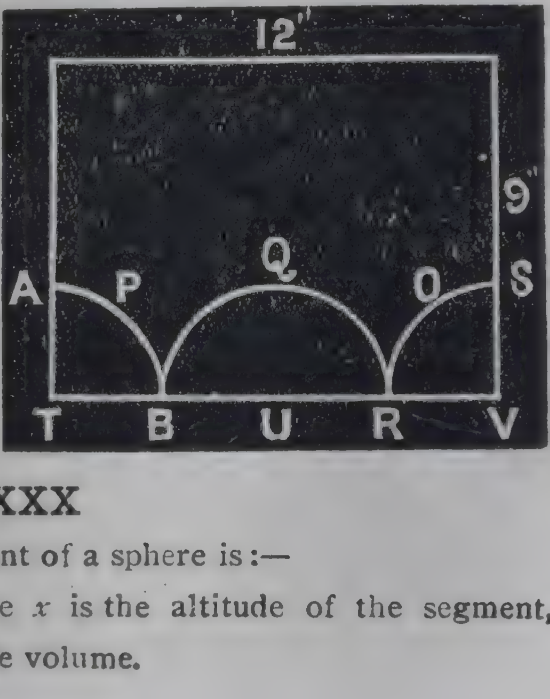
(*b*) Find also what number must represent the speed of a fifth competitor who runs the distance in the average time.

3. Two posts are planted vertically 100 ft. apart. The longer is 90 ft. and the shorter is 60 ft. above the ground. If from the bottom of each post a rope is stretched tightly to the top of the other, find at what height from the ground the ropes will cross each other.

4. A cloth merchant had to clear his goods. He sold silks at a reduction of 37½%, cotton goods at a reduction of 15% and woolen goods at a reduction of 20% on the prices marked and realised Rs. 425, Rs. 1232½ and Rs. 764 respectively. If the marked prices were 25% in advance of the prices he had paid for the goods, find his total gain or loss per cent in the clearance sale.

5. A new grindstone is 18 in. long, 12 in. broad and 9 in. high; To reduce its weight three uniform grooves are cut out along the length so that the end view is of the shape shown in the figure. Find by how much the weight has been reduced.

APB and *ROS* are quadrants of circles; *BQR* is a semicircle. $TB = BV = UR = RV = 3''$.



XXX

1. A formula for the segment of a sphere is:—

$$x^3 + a^2x - \frac{6V}{\pi} = 0,$$ where x is the altitude of the segment, a the radius of the base and V the volume.

(i) Exprees a in terms of x, v and π.

(ii) Find a if $x=2''$ and $V=8{\cdot}71$ cub. in. Take $\pi=3{\cdot}142$.

2. A goldsmith has two alloys of gold first being, $\frac{3}{4}$ pure gold the second $\frac{5}{12}$ pure gold. How much of each must he take to produce 100 ounces of an alloy which shall be $\frac{2}{3}$ pure gold?

3. A brass tube 8 feet long, has an outside diameter 3 inches inside 2·8 inches. What is the volume of the brass in cub. in.? If a cubic inch of brass weighs 0·3 lb, what is the weight of the tube?

4. A tenant takes a house for three years. The rent is Rs. 300 a year and is due at the end of each year. If the rate of interest on money is 9 % per annum, how much must the tenant pay down at once, in order to reduce his yearly payment from Rs. 300 to Rs. 250 (Reckon compound interest.)

5. A circular plot of ground is enclosed by a compound wall. The portion of the wall visible to an observer at a certain point outside the enclosure, subtends an angle of 60° at the eye. Going 30 yards nearer, the portion of the wall now visible, subtends an angle of 120° at the observer's eye. Find from a carefully drawn diagram the diameter of the circular plot of ground.

XXXI

1. A man leaves 60% of his property to be divided between his wife and three brothers in the ratio $1 : \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$ and the remainder to be divided between his nephew and niece in the ratio 3 : 2. Express the share of each as a percentage of the whole property.

2. A merchant estimates that at the end of every year, the value of his unsold goods depreciates to 85% of its value at the beginning of the year. Calculating at this rate, the merchant sold a bicycle for Rs. 144-8 as. two years after the date of purchase. Find the cost of the machine two years ago. If that price is $11\frac{1}{9}\%$ in advance of his cost price, find the total gain or loss per cent. in the transaction.

3. The cross-section of an iron wire is 1 sq. mm. A cubic cm. of the wire weighs 7·9 grams and the force required to break the wire asunder is the weight of 35 kg. What length of the wire suspended by one end will just break under its own weight?

4. A mountain ridge running north and south is 2400 ft. high: On the east side the inclination to the horizontal is 30° and on the west side of the ridge the inclination is 45°. A level tunnel is driven across the ridge. Find its length to the nearest foot. Also find the saving in distance when compared with the shortest route along the sides of the ridge.

5. Solving graphically a simultaneous system, the roots were 2, —3. The coefficient of x in the first equation and the coefficient of y in the second were afterwards rubbed out. The equations being $*x+2y=8$; $3x-*y=9$. Without finding the coefficients, draw the graphs of the two equations, using your knowledge of the answer.

XXXII

1. The value of $2x+3y$ is doubled when x and y are each increased by unity. Again the value of $2x+3y$ is halved when x and y are interchanged. Find the value of x and y.

2. A man walking from A to B, at the rate of 4 miles an hour, starts one hour before a coach travelling 12 miles an hour and is picked up by the coach. On reaching B, he finds that his coach journey had lasted two hours. Find from purely graphical considerations (i) at what distance from A he was picked up, and (ii) the distance from A to B.

3. A cubical gas tank was made to hold 729 c. ft. of gas. It is proposed to enlarge the tank, the depth remaining the same, and the bottom still a square. The enlarged tank is to hold $2\frac{1}{4}$ times as much as the original. Find the dimensions of the base.

4. Two villages on a st. road running W. to E. are 780 yds. apart. One of the villages is 720 yds. and the other is 420 yds. from a st. running river. Find by measurement the distance from the road of that point in the river which is equidistant from the two villages. Check your result by calculation.

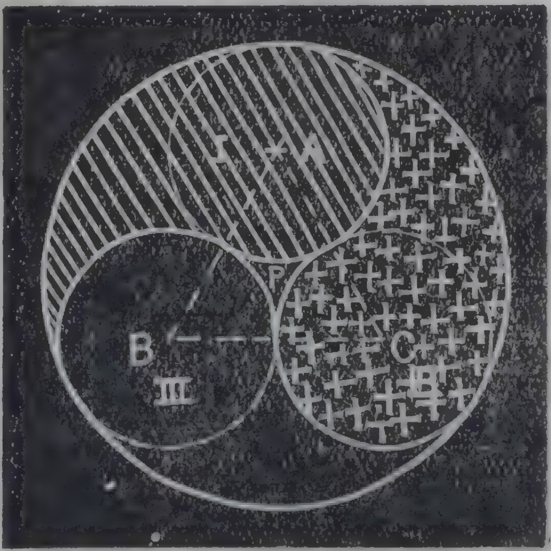
5. The amount of water that can be carried off by a circular drain-pipe whose diameter is D feet is

$$\frac{AD^3}{C+\sqrt{D}} \text{ c. ft.}$$

per minute, where $A = 454{\cdot}3$ and $C = 1{\cdot}005$. Find with as little work as possible the amount correct to the nearest c. ft. of water carried per minute by a drain-pipe 21 in. in diameter.

XXXIII

1. ABC is an equilateral triangle of side 1 in. P is the in-centre. Find the areas of the shaded portions I, II, or III. The whole figure represents a design above a window frame.



2. A merchant marked his goods at an advance of 25% on the cost price. At this price he cleared $\frac{1}{3}$ of his stock when he found that a portion was damaged and could not be sold. He therefore raised the price of the remainder by 10% of the prices already marked and gained 15% on the whole. What fraction of the goods was damaged?

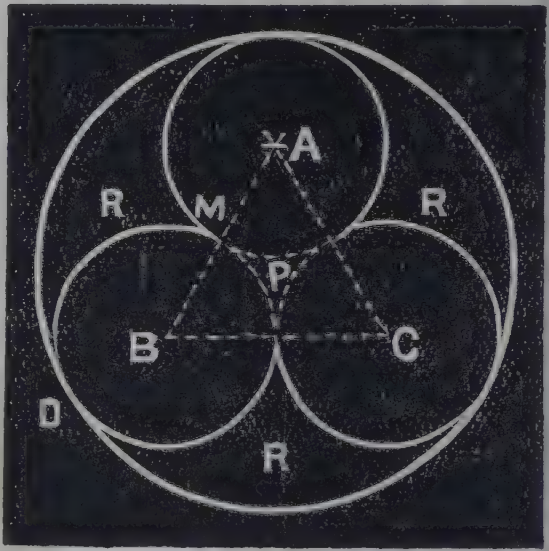
3. A person owes certain sums of money to two creditors and finds that if he pays them respectively sums of money in the ratio 9 : 17, the sums unpaid will be 10% and 15% of the two debts and will together amount to Rs. 52. Find the amount of each debt.

4. A railway embankment is 20 ft. wide at the top, 60 ft. wide at the bottom, and the sides slope uniformly to the bottom making angles of 30° each with the horizontal. Find the height of the embankment and the volume of earth in quarter of a mile length of the embankment.

5. From three stations on a railway *A*, *B* and *C* three trains start at the same time, in the same direction. *B* is 18 miles from *A*, and *C* is 46 miles from *A*. The train from *A* overtakes that from *B* in 1 hr. 12 min. and the train from *C* in two hours, from the time of starting from *A*; when will *B* overtake *C*?

XXXIV

1. *ABC* is an equilateral triangle and *P* is its in-centre. Portions marked *R*, are of red glass and the remainder of green glass. Find the area of each kind of glass required. $AM = BM$ etc. What is the length of *PD*, given that a side of the equilateral △ is 2 in.?



2. Two hundred and sixty feet of round copper wire weigh 30 lbs.; find its diameter if a cubic inch of the copper weighs 0·32 lb. If the same weight of the copper is shaped like a hollow cylinder, 1 in. internal diameter and 2 in. long, what is its external diameter?

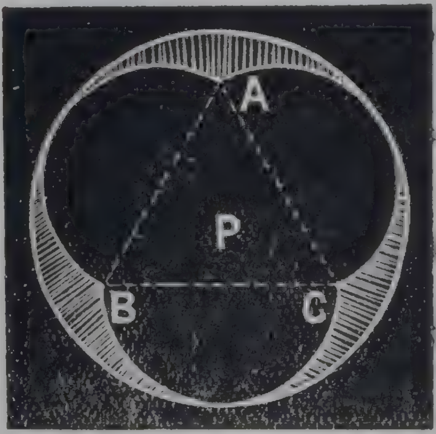
3. The arms of a balance were unequal so that 9 viss in one pan balanced 10 viss. in the other. A customer wanted 40 viss. of the merchant who had this balance. The merchant weighed 20 viss. to his advantage, but being insisted upon by the customer, the other 20 viss. was placed in the other pan. Find his loss or gain per cent. on the whole.

4. *ABC* is a rt.-angled △, the rt. angle being *C*; *D* is the mid-point of *BC* and *E* is the mid-point of *AC*; $AD = 2\sqrt{34}$ in., $BE = 13$ in. Find the sides of the triangle. Also find the length of the third median correct to the tenth of an inch.

5. Four persons, *A*, *B*, *C* and *D* began a game of chance. *A* and *B* played and *A* won half of *B's* money; then *B* played *C* and the former won $1/3$ of *C*'s. *C* and *D* played and *C* won a fourth of *D*'s. They counted what each had and each was found to have 12 annas. With what sums did they begin the game?

XXXV

1. *ABC* is an equilateral triangle of sides $2a$. Semi-circles are described on the sides. With centre *P* (the in-centre of the △) and radius *PQ*, a circle is drawn to touch the semi-circles. The shaded portions are of red glass. Find the area of the shaded portions.



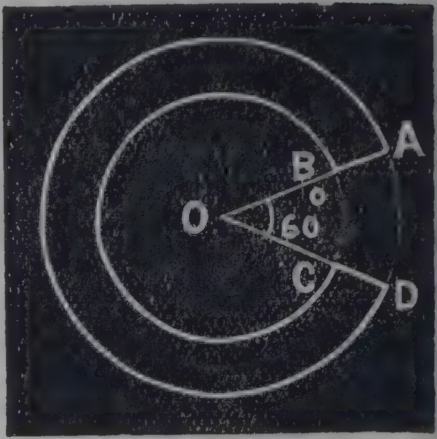
2. The receipts of a mountain railway in a certain year were £35-5 s. on an average for every day the railway was open. In the next year, the railway was open for 18 days more than in the preceding year, and the accounts showed an increase over the results for the preceding year, of £ 1-5 s in the daily average receipts and an increase of £ 857 in the total receipts for the year. What were the total receipts for the 1st year?

3. Three bicyclists *A*, *B* and *C* ride round a circular track half a mile in length, *B* receiving 240 yds. start and *C* 720 yds. If *A* travels at the rate of 18 miles per hour and *B* at 16 miles per hr., find the rate of *C*, if *B* overtake *C* just when *B* is himself overtaken by *A*.

4. A rectangular sheet of tin 18 in. by 11 in. can be rolled into a cylinder in two ways. Compare the volumes.

5. A crown weighing $64\frac{1}{2}$ ozs. was made of gold and silver mixed. It was found by displacement that it contained 8·5 cubic inches. A cub. in. of gold weighs 10·25 oz. and of silver 5·75 oz. Determine the weight of gold and silver in the crown.

XXXVI



1. To make a wooden letter, C, a circular piece of wood of radius $OA = 1{\cdot}5''$ was taken. The inner circle of radius $OB = 1$ in., was removed and the portion $ABCD$ was also removed. $\angle AOB = 60^\circ$. What fraction of the original weight is left?

2. (*a*) I bought an article for Rs. x and selling it at a gain of x per cent. cleared Rs. 36. Find the cost price.

(*b*) I bought an article for Rs. $(x+5)$ and selling it at a gain of $(x+5)\%$ cleared Rs. 12¼. How much would I have gained, if I had bought it for Rs. x and sold it at a gain of (i) $x\%$ (ii) $(x+5)\%$?

3. The price of sovereign in six consecutive days was as follows:—

Monday Rs.	13- 9-6	Thursday Rs.	13-11-3
Tuesday	13-10-6	Friday	13-10-0
Wednesday	13-10-3	Saturday	13- 9-6

If sovereigns were purchased at the cheapest rate how much per cent. will the purchaser gain on the average of these rates?

4. There are two brands of coffee powder tinned. One variety is packed in a cylindrical tin 3¾ in. in height and 2¼ in. in diameter and costs 15 as.; while the other is packed in a cylindrical tin 4¾ in. in height and 2¾ in. in diameter and costs Re. 1-11 as. Which is the better quality coffee? What would be the price of a tin of the better quality, if packed in tins of the size of the inferior quality?

5. A rectangular piece of paper is $8''$ by $6\frac{1}{2}''$. The corners are rounded off, so that they form quadrants of circles. The st. parts left being $5\frac{1}{2}''$ along the length and $4''$ in the breadth. What fraction of the original sheet is left? Express the fraction in the form $1 : x$ where x is correct to two decimal places.

Miscellaneous Examples.

1. Sir Issac Newton made a thermometer with the freezing point of water marked 0° and the heat of the human body 12°. These in the Farenheit scale are respectively 32° and 98°. Find the temperature of boiling water 212°*F.* in Newton's scale. Illustrate graphically.

2. Two roads OX and OY cross one another at right angles. There is a tank A, 50 yds. from OX and 150 yds. from OY. There is also a well B, equidistant from the roads and as far away from A as A is from the crossing. A row of cottages is to be built that each house may be as far away from A as it will be from B. In a figure drawn to scale, exhibit the line of cottages and also find the distance from the crossing, of that house which is equidistant from the well and the tank as also from the two roads.

3. For a music performance only one kind of ticket was sold. With a view to popularise it and to enhance the income the price of each ticket was cut down by 20 per cent. and the number of spectators rose by 30 per cent. Find the increase per cent. in the income.

4. A boy hires a bicycle and pays 8 as. for the use of the bicycle and as. 4 for every hour he uses the bike:

1. Express the cost of hiring in terms of the number of hours used.
2. Draw a graph connecting the amount paid and hours used.
3. From the graph read (*a*) the number of hrs. he can use for Re. 1-8 as., and (*b*) the amount to be paid for using the bike for 5 hrs.

5. The monthly emoluments of an agent to a firm consist of a fixed monthly pay and a discount on the sales he had effected by canvassing orders. Once when he had secured orders for Rs. 450 he drew Rs. 102-8 as. and on another occasion he had canvassed orders for Rs. 550 he drew Rs. $107\frac{1}{4}$.

Find from a neatly drawn graph his fixed monthly pay and his emoluments if he canvassed orders for Rs. 250 only.

Check your results by framing equations and solving them.

6. The distance between the car-stand at Mylapore and my office at Royapuram may be taken as 5 miles. Every day I walk at 4 miles an hour from the stand to the office and get into a bus running at 10 miles an hour as soon as one overtakes me and complete the distance in the car. If t denotes the time in minutes I walk and T the whole time occupied for covering the distance, establish a relation between T and t. Draw the graph of the function and from the graph find (*a*) how long I would have walked if on a certain day the whole journey took 45 minutes, (*b*) the whole time occupied if I had walked for 32 minutes.

7 A person, five feet high, walking along a road with the sun directly behind him, observes that at a place where the road is level, his shadow measures 7 feet and that at a place where the road has an upward gradient, his shadow measures $5\frac{1}{2}$ feet. What is the angle which the sun's rays make with the horizontal? What is the inclination of the road?

8. A party of explorers starting from O, proceed in a direction $22\frac{1}{2}$ degrees north of east and going 10 miles in a straight line reach P. On the second day they go 30° east of north 11·7 miles to Q. On the third day they reverse their *first day's course* as regards *directio* *only* and reach R, from which place they finally regain O, by goin due south. Representing a mile by 1 cm. draw carefully their rout to scale, and find its total length.

9. A person looking through a window in a house on one sid of a street finds that the height of a house just opposite on the othe side of the road, subtends a rt. angle at his eye. He also notes tha the base of the house is 30 degrees below his eye. The window itsel is 10 feet above the ground. Find both by calculation and from carefully drawn figure (i) the height of the house, (ii) the width o the street.

10. A ship sailing towards the North observes two light-houses in a line due East, and after an hour's sailing, one of them is 3 degrees south of East and the other 30 degrees east of South. Draw geometrically the positions of the ship and the light-houses. If a the same rate it made towards the further light-house from the secon position and thence without stopping steamed to the other light-hous direct, how soon would the ship reach it from the time of leaving the second position?

11. The usual prices of certain articles and their bargain price at a season sale of Messrs. Addison and Company are given below Find in each case by how much per cent. the usual prices have bee cut down.

Names of the Articles.	Usual Price		Bargain Price	
	Rs.	As.	Rs.	As
Neptune Fountain Pen ...	7	8	5	12
Wyvern	15	0	9	12
Cameron	9	8	7	14
Sirdar Pocket Watch	25	0	19	8
Silver Tie Clip	5	8	3	12
Hand Blotters	9	8	6	12
14-ct Gold Election W. Watch ...	135	0	79	8

12. In a book of Arithmetic, an example was printed thus:— "If 11 ac.—ro.—8 poles be worth £ 2250 what will 44 ac. 3 ro. 1 po. cost?" (The number of roods in one being accidentally omitted). The answer at the end of the book was £ 8718–15 sh. Find the missing number.

13. The parallel sides of a trapezium, are $1{\cdot}2''$ apart, $AB = 2{\cdot}9''$ $\angle ACD = 30^\circ$; $\angle BDC = 45^\circ$; Describe the trapezium and find the lengths of the three other sides by measurement. Calculate the are of the figure.

14. A cubical block of metal of $7{\cdot}84''$ side weighs ·25 lb. per cub inch. A hole of circular section is to be made right through the centre in order that the weight of the metal left may be 1 cwt. Find correct to two decimal places, the diameter of the circle.

15. A horse van carrying parcels starts from a place A at 5-30 A.M. and goes at 6 miles an hr. At 7 A.M. a motor carrying mails leaves the same place and overtakes the van at the 27th mile and reaches the destination 20 minutes earlier than the van. Find (i) the rate of the motor, (ii) the distance between the two places, (iii) the time when each reaches the destination. Illustrate the problem graphically.

16. A boy was examined in five subjects. The marks he obtained in each subject and the maximum for that subject are given below. Find graphically in which subject he did best.

	English.	Ver. comp.	Ele. Math.	Alg. & Geo.	Phys.
Max.	120	80	150	100	60
Mks.	47	33	61	41	27

17. Three towns A, B and C are such that the road $AB=18$ miles and the road $BC=15$ miles and is at right angles to the road AB. There is another town X such that the roads AX and BX are at right angles and that it is equidistant from B and C nearer to C than to A. Locate the town X on a map drawn to scale. Measure its distance from the three towns.

18. A person at a place A cannot pass directly to another place B due north, but does so in the following manner. He first walks due east 100 ft., then north-east 120 ft., then north-west and reaches B. Find the distance from A to B and draw an accurate figure to any convenient scale.

19. A field 60 ft. square is divided among 4 men. A point 30 ft. from one corner and 50 ft. from the adjacent corner is chosen inside the field, and this point is joined to the corners thus dividing the field into 4 portions. If the price of the whole field be Rs. 900, find to the nearest rupee the price of each part.

20. A shrine on a chain of hills is reached by a path 7 miles 2 furlongs. A devotee has to reach the shrine on foot. From the foot to a place A. $1\frac{1}{4}$ miles up an incline there is a continuous rise of 1 in 176, and this is followed by a falling gradient of 1 in 528 to a place B $1\frac{1}{4}$ miles from A, a level pathway to C, 1 mile distant, an ascent of 1 in 550 for the remaining distance. Find the height of the shrine to the nearest foot, above the base of the hills.

21. (*a*) From a point A, two roads branch off making an angle of 60° with each other at A. Each road is 48 feet broad. A third road whose nearer edge is 300 feet from A cuts these two roads and is equally inclined to these roads. This road is also 48 ft. broad. The area enclosed by these three roads is a nice small park. Find the area of the park. Draw a figure to scale showing the three roads and the park.

(*b*) Equidistant from the three corners of the park is a fountain. Find its distance from A. Locate it in the figure.

22. A railway from a place P to another Q, $1\frac{1}{2}$ miles distant has a falling gradient of 1 in 770 and this is followed by a level to a distance of $1\frac{1}{2}$ miles from which place, the track has a rising gradient of 1 in 385 to station R which is as high above the sea-level as station P. Find the distance of R from P.

23. A picture is hung on a nail by a piece of twine $1\frac{1}{2}$ yards long attached to the ends of the top frame which in the position is horizontal. (a) If a piece of twine 6 in. longer be substituted how far lower will the bottom be? (b) If again a piece 8 in. shorter than the original be substituted how much higher will the picture be? The distance between the ends of the twine attached to the frame is 12 inches.

24. A rectangular field of two acres has a side 80 yards. Find the width of a road which runs round two adjacent sides of the field, if the area of the road be $\frac{1}{8}$ of the area of the field.

25. A trapezoid, the parallel sides of which are as 3 : 4 is to be cut from a rectangular board 12 ft. long and 2 ft. 4 in. broad; find the lengths of the parallel sides that the trapezoid may be three-fourths of the board. Also find by calculation the lengths of the non-parallel sides. Draw a figure to scale and draw the dotted line along which the cut is to be made.

26. The parallel sides of a trapezium are 14 and 8 inches, and two st. lines are drawn parallel to these within the figure and terminated by the other sides so that the four parallel lines are equidistant. Find the lengths of the two lines drawn.

27. To dig a well I employed a number of well-diggers and as many coolies and these did the work in 4 days. My neighbour employed half the number of well-diggers I employed and as many coolies as I did and they dug a well of the same dimensions in 7 days. Supposing that the amount of work done by these to have been uniform throughout, shew that a well-digger is 6 times as good a workman as a coolie.

28. The surveyor's chain must exactly measure 22 yards and is composed of 100 links. A chain equally divided into 100 links was however 10 inches too short of the total length. The length and breadth of a field were measured with such a chain and found to be 4·32 and 3·84 chains respectively. Find the difference between the true area and the calculated area.

29. (*a*) To become the member of a certain tennis club, one has to pay an entrance fee of Rs. 5 and a further monthly subscription of Rs. 2-8 as. If a new member plays only for one month from the time of admission what will be the cost to him of a set at tennis if he can play only on 24 days in the month and on each day on an average can play only two sets?

(*b*) For how many months from the time of admission must the member play that the cost to him of a single set may be only one anna?

30. A lawn is in the shape of a quadrilateral $PQRS$; $PQ = 200$ yds., $QR = 150$ yds., $RS = 100$ yds., $\angle PQR = 105^\circ$; $\angle PSR$ is a right angle. Draw a plan of the lawn to suitable scale.

(*a*) Reduce the quadrilateral to a triangle of equal area and thus find the approximate cost of turfing the field at 8 as. a square yard.

31. Annual incomes of Rs. 2000 and over and below Rs. 5000 are liable to a tax of 5 pies in the rupee, while annual incomes of Rs. 5000 and over and below Rs 10,000 are liable to a tax of 9 pies per rupee. Two officers draw monthly salaries of Rs. 320 and Rs. 480 respectively. If each of the officers is able to save only Rs. 2,500 per annum find the ratio of their expenses other than that of the tax. (Express this in the form of $1 : x$ where x is greater than unity and is correct to two decimal places.)

32. A railway track from A to D has the following gradients :—
rising 1 in 132 for a distance of 2 miles from A to B;
falling 1 in 176 „ „ 1 „ B to C;
rising 1 in 110 „ „ 1½ „ C to D.

If A be 200 ft. above sea-level, what is the height of D above sea-level?

33. A gas meter indicated that 89300 cubic feet of gas had been consumed On being tested it was found that for every 1,000 cubic feet actually consumed 1030·9 cub. ft. had been registered. Find to the nearest cub. ft. the total amount of gas actually consumed and the sum returnable as excess charges at Rs. 3-6 per thousand cubic ft.

34. Work out the following problems employing short-cuts to minimise work.

i. A person gets £ 94-3-6 more by selling a house at a profit of 7½ per cent. than by selling it at a loss of 7½ per cent. Find the cost price of the house.

ii. The average of 47 numbers is 49. The average of the first twenty-three is 44 and of the second twenty-three 51. Find the last number.

iii. Find the rent of 48·75 acres at Rs. 1-5-4 per acre.

iv. A man spends 1 anna 4 ps. in charity, in each rupee of his income and saves $36\frac{4}{11}$ per cent. of the balance. How much does he save in an income of Rs. 51?

v. A man sold 72 yds. of cloth for Rs. 87 and thereby gained the cost price of 11·52 yds. What did he gain per cent.? Find the cost price per yard.

35. If the gold in circulation amounts to 120 millions of sovereigns and of these 25 in every hundred have lost $\frac{1}{50}$ of their weight, while 10 more in every hundred have lost $\frac{1}{30}$ of their weight, what would be the loss to the mint, if it recalled all light gold and issued full-weight sovereigns instead?

36. A few parcels were once entrusted to a Bus driver to be taken to a station. There was no parcel which weighed less than 27·5 lbs. and no one weighing more than 33 lbs. The owner was charged for 14 cwt. 10 lbs. Find the maximum and minimum number of parcels he could have carried.

37. Find—

i. graphically what the remainder is when 173 is divided by 110 by factors.

ii. the value of $2117 \times 1883 - 1113 \times 887$ without direct multiplication.

iii. the cost of 2,448 articles at £ 0-19-11½ each, suggesting the best way for a mental reckoner.

38. A boy multiplied a number by 495 and obtained the product 1925617. He was told that the figures 9 and 7 were wrong and the other figures right. What was the correct product?

39. The population of a country consists of m males and f females. Of the former x per cent. can read and write and of the latter y per cent. can do so. If of the whole population Y per cent. can read and write, express Y in terms of m, f, x and y.

40. Employ the shortest method possible for the following calculations:—

(i) $\{9 \times (4999 - 2 \times 3997)\} \times 9998 \times 701.$

(ii) $46238146 \div 9996.$

(iii) Reduce to equivalent decimal $\frac{73}{1757}$ correct to 3 significant figures.

41. When a stone falls freely from rest under gravity, the distance it falls in t seconds is given by the formula $s = 16\ t^2$. Find corresponding values of s and t at intervals of half a second, draw a curve connecting s and t and from the graph find,

i. the distance it falls through in 2·8 seconds.

ii. the time required to fall through 128 feet.

42. A father finds a piece of cloth will make a coats for his eldest son or b coats for the next younger or c coats for the youngest How many coats can he make out of the same piece for all his three sons who are each to get the same number of coats?

43. The efficiency E of a machine is connected with the load lifted, L lb by the equation, $E = \frac{La}{L+b}$ where a and b are constants. The table shows corresponding values:—

L	110	50
E	37·5	31·25

Find the value of a and b and calculate the efficiency for loads of 5 lb. and 200 lb.

44. To increase the depth of a well 38 cub. feet of rock had to be excavated. When men were employed to chisel out the rock the owner found only 1·24 cub. ft. could be dug out each day. A steam drill was also available, but not as often as he desired; but whenever available, the owner made use of it and it dug out at 1·34 cub. ft. per day and the whole work was done in 30 days. For how many days was the steam-drill used?

45. A square tin of side *s* inches and height *h* inches will hold nearly 0·05 s^2h lb. of jam. A set of square tins are all of 5 in. side. Find their heights if they hold 1, 2, 3, 4, 5, 7, 14 lbs. of jam respectively and illustrate graphically.

A second set of square tins are all 6 inches high. Find their sides if they hold 1, 2, 3, 4, 5, 7, 14 lbs. of jam respectively and show your results in the same diagram.

46. A novice in trade began business with a capital of Rs. 5000. At the end of the year he found he had lost 7% of his capital. With the balance he continued the business and at the end of that year found he had gained on the whole 12 per cent. of the capital with which he started the concern. Find correct to one decimal place what per cent,. of the capital of that year his gain was in the second year.

47. A person invested Rs. 4000 in a venture. He gained 8 per cent. With this amount he started another business in which he lost 5 per cent. of this capital. With the balance he started a third business and on winding it up found he had gained on the whole 7·73 per cent. of the capital with which he commenced business originally. Find what rate per cent. profit he obtained in the last venture.

48. A merchant speculated in three successive ventures. The capital he started with was Rs. 7500. In the first he lost 8 per cent. With the balance he started the second business and he gained 10 per cent., and with this increased capital he began a fresh business in which he gained 15 per cent. What was his profit per cent. on the capital he started with?

49. Two bulls were once purchased by me for Rs. 60 and Rs. 45 and I had afterwards to sell the former at a loss of 15 per cent. and the latter at a gain of 12½ per cent. Find my total loss per cent.

To make up for the loss I bought another bullock and sold it at a gain of 14½ per cent. What did this bull cost me?

50. The duty on a commodity was lowered 20%. By how much should the consumption increase that the revenue may remain the same as before? Find by how much should the consumption increase to increase the revenue by 20%.

51. The following is an extract from the marks register of a certain class.

SUBJECTS.

Names of Pupils.	X	Y	Z	Total.
Mx.	50	40	30	120
A	12	11	9	
B	21	18	12	
C	13	17	10	
D	19	17	11	
E	14	9	12	
F	18	12	10	

Find (*a*) the total marks secured by each student.

(*b*) the boys eligible for promotion if 35% of the total be the minimum required for a pass.

(*c*) the total marks secured by each student, if those obtained in subject Z by each, be raised by 35% and set down to the nearest integer.

(*d*) how many more would pass by the enhancement.

52. A platinum bar expands $\frac{1}{113100}$ of its length at 0°C. for a rise of temperature of 1°C., and copper expands $\frac{1}{53200}$ for the same.

Find the lengths at 0°C. of two bars, one of platinum and the other of copper, such that, whatever be the temperature, the difference of their lengths may be always 50 centimetres.

53. The selling price Rs. 12-10-0 of a teakwood box is made up of :—

	RS.	AS.
Price of teakwood	6	3
" nails, lock, etc.	2	5
Carpenter's wages	3	0
Profit	1	2

Express each of the above items as a percentage of the actua cost of the box, correct to one decimal place.

54. i. Draw a triangle with the following data: $a = 5{\cdot}3$ cms. $b = 4{\cdot}7$ cms. and $c = 3{\cdot}9$ cms.

ii. Describe,

(*a*) an equilateral triangle having the same perimeter as the above triangle;

(*b*) a rectangle whose length is half as much again as its breadth; but of the same perimeter.

Full construction should be shown and foot-rule and compasses alone should be used.

55. A bookseller offers a reduction of p per cent. on the published prices of certain books and still makes a profit of q%. If r denote the published price and c the cost price establish a relation between c, p, q and r. If $p=10$, $q=20$, $r=80$, find c.

56. The prices of tickets for a theatrical performance were in the ratio 20 : 8 : 3 and the people who took tickets in the various classes were in the ratio 13 : 56 : 90 respectively. Find the amount collected in each class and the price of each ticket of each class if the total amount collected was Rs. 611—4 as. and the number that took the costliest ticket be 65.

Max.	28	25	12	15	40	%	Rank.
A	12	13	0	3	16		
B	9	12	3	7	11		
C	6	15	7	6	13		
D	12	16	6	7	a		
E	14	a	6	0	0		
F	26	a	12	6	16		
G	25	7	7	7	12		
H	0	12	6	8	16		
L	7	6	7	9	14		
M	12	11	4	10	10		
N	13	11	6	a	9		

57. The above is an extract from the mark register of a certain class. Fill in the columns left blank. In case of absence the percentage for the boy should be calculated on those maxima for which he was present.

58. Every now and then I have to go to a station 24 miles away As I have inspection business en route I am likely to be delayed for a few hours. I hire a taxi at 10 as. for every mile and 8 as. extra for every hour. If P be the amount in Rupees paid by me and Q the hours I took to reach the destination, establish a relation between P and Q.

(*a*) Draw the graph of the relation and from the graph find the time I took if on one occasion I paid Rs. 16—8 as. and how much I should have paid if I had reached the destination in $5\frac{1}{2}$ hrs. from the time of starting.

59. Three boys meet on a road: the first has a loaves, the second b loaves and the third $a+b$ annas. The loaves are shared equally between the three and the third divides all his money between the other two in payment for his share. How much money should each take?

60. Work the following questions as shortly as you can.

i. Find the cost of 84,127 books at 1 s. 8 d. each.
ii. Simplify $55{\cdot}23 \times 1\frac{1}{8} + 8{\cdot}79 \times 1{\cdot}125$.
iii. Evaluate $7{\cdot}69569$ of 6 s. 8 d. correct to the nearest pence.

61. In a prime number of 3 digits, all different, the digit in the units place is equal to the sum of the digits in the other two places. If the other two digits were interchanged the number is still a prime number. Find the number.

62. Work the following questions as shortly as you can. The short method should be shown clearly :—

(i) A bankrupt pays 17 sh. 6 d. in the pound; how much does he pay in £ 267—6 sh.—8 d.
(ii) Divide Rs. 3288—12—0 among 4 persons in the ratio of 3 : 5 : 8 : 9.
(iii) What will be the cost of 12 beams of wood each 16 ft. long, 16 inches broad and 9 inches thick at Rs. 5-0-8 ps. per cub. ft.

63. Passengers by a motor-bus are allowed a certain weight of luggage free and the rest is charged. Two passengers have with them $80\frac{1}{2}$ lb. of luggage and they are charged Rs. 1-11-6 and 2-1-0 separately. If all the luggage had belonged to one of them he would have been charged Rs. 4-6-6. Find the weight allowed free of charge.

64. The railway between A and B ascends at a gradient of 1 in 275 from A to a place C, $16\frac{1}{4}$ miles distant from A. It descends at a gradient of 1 in 165 from C to B. If the station at B is 28 ft. lower than that at A, find the distance from A to B.

65. A dishonest tradesman marks his goods at an advance of 5 per cent. on the cost price, but uses a fraudulent balance whose beam is horizontal when the weight in one scale is $\frac{5}{11}$ more than the weight in the other. What is his actual gain per cent.?

66. A ruler is supposed to be 6 inches long. It is really only $5{\cdot}88$ in. The divisions into six parts and each part into tenths are however correct. A boy measures the sides of a rectangle with this ruler. He notes down $4{\cdot}8''$ and $3{\cdot}2''$ for the length and breadth and calculates the area. What fraction of the true area is the calculated area?

67. A wall has to be plastered. Slaked lime sells at 9 as. per kalam and this has to be mixed with double the quantity of sand which costs 8 as. per cart-load. The cost of mixing and grinding is half-an-anna per kalam of resulting mortar. I buy two cart-loads of slaked lime and employ 2 masons and 1 coolie for 4 days. The wages of a mason and a coolie being Re. 1 and as. 4 respectively, per day, find how much the plastering will cost me. A cart-load measures 20 kalams.

68. There was a question in an Arithmetic paper :—To find the L. C. M. of three given numbers. One of them was 3425, another 1829, and the third was an even number of 5 figures. One candidate, by mistake, copied down 3245 instead of 3425 but obtained the correct answer. What was the third given number ?

69. (a) A man starts in the direction S. E. and walks for a hours at p miles an hour. From this point he walks in the direction S. W. for b hours at the same rate. How far is he from the starting point ?

(b) i. Find correct to the nearest yard his distance from the starting point if $p=4$, $a=\frac{3}{4}$, $b=\frac{1}{3}$.

ii. Also find how many minutes he might have saved if he had taken the short-cut and walked at his usual rate.

70. In a straight line between two columns, one 60 ft. high and the other 40 ft. high, stands a small statue, the head of which is 35 ft. from the summit of the lower and 80 ft. from the summit of the higher of which the base is 64 ft. from the base of the statue. Find, correct to the tenth of a foot, the distance between the two columns. Draw a figure to scale.

71. Give the answers to the following questions doing as little of work as possible. (It does not mean that the questions should be worked mentally.) :—

(1) Find to the nearest centimetre the difference between 35 yards and 32 metres given that a metre is 39·3708 inches.

(2) A cubic foot of water weighs 1000 oz. A gallon of water 10 lbs. Find the number of cubic inches in a pint.

(3) Multiply 0·00806 by 7·5 and divide 17·175 by 0·0125.

(4) A brick measures 9 inches by $4\frac{1}{2}$ inches by 3 inches. How many will be required for a solid wall 21 feet 9 inches long, $13\frac{1}{2}$ inches thick and 4 ft. 6 inches high, when the thickness of the mortar is not taken into consideration.

73. There are two forts A and B on opposite sides of the entrance to a harbour, 16 miles long. In the harbour there is an island where there is a fort, C, 12 miles from A and 11 miles from B. Fort A has a gun of range 12 miles, fort B a gun of range 11 miles and Fort C, one with a range of 10 miles. Draw a plan of the entrance to the harbour and shade that portion exposed to the range of fire from the three guns at the same time.

74. Show the answer appended to each question is *obviously wrong*. Do not work out the questions.

(i) £ 7348-10-6 multiplied by 240 = £ 1763880-10s.-6d. Ans.

(ii) $\sqrt{\cdot841} = \cdot29$ Ans.

(iii) The income derived from an investment of Rs. 7348 at 6¼% S. I. after paying incometax at 4 pies in the rupee = Rs 459-6-6 ps.

(iv) $(215^2 - 187^2) \div 7 = 1607\frac{1}{7}$.

(v) The number of men in a regiment was such that it could be formed into a solid square etc.........etc. Find the number of men. 73457 Ans.

75. An ornamental grass plot is made as follows:—A square plot of ground is marked. Each side is divided into three equal parts. On the middle parts semi-circles are described to fall outside the square. Again with each angular point as centre and radius equal to a third part of a side, arcs are described inside the square to cut the sides and terminated by the sides. In the centre a flower pot is placed. The corners are covered with white sand and on the remaining surface grass is grown. Find the area of the grass-portion, given that a side of the square is 6 ft. and the flower pot is a cylinder 1 ft. across at the base. Draw a neat figure to scale 1 cm. = 1 ft.

76. A school committee grants a subsidy of Rs. 300 per year for sports in a school. The annual subscriptions from boys amount to Rs. 330. This income has to be equally distributed among the three branches Tennis, Football and Cricket. The cost of running a tennis court is as follows. Price of 15 balls a month at Rs. 20 a dozen. One marker who is to be paid Rs. 6 extra and two pickers who should be paid Rs. 3 extra each a month. Repairs to court Rs. 36 a year. What additional subscription must each player pay, if there are 12 tennis-players?

77. I was asked by a student to give a mathematical solution to the following problem on numbers. Can you suggest one? "The three numbers 480,608, 508,811 and 723,217 when divided by a number leave the same remainder.

Find the divisor and the common remainder.

78. (1) The perimeter of a triangle is 12·5 cms. and the sides are in the ratio of 4 : 3 : 5.

Describe the triangle without calculating the length of each side beforehand arithmetically. The construction of the triangle must be purely geometrical.

(2) Find the length of each side and measuring the altitude on the longest side, find the area of the triangle.

(3) Find also the radius of the circumscribing circle.

79. When 58346 is divided by 47, the quotient is 1241 and the remainder 19. From this deduce the quotient and the remainder—

i. when 583467 is divided by 470;

ii. when 583·467 is divided by ·0047.

80. On a blackboard 5 ft. by 3 ft. the following directions are given to draw a quadrilateral *ABCD* :—

Vertex A is $24''$ from top left corner and $18''$ from bottom left corner.
„ B is $18''$ from top right corner and $32''$ from bottom right corner.
„ C is $30''$ from top right corner and $18''$ from bottom right corner.
„ D is $5''$ from bottom edge, and the angle at this vertex is a right angle.

Show that there can be two such quadrilaterals and draw them.

81. A crowd of beggars consisted of 'm' men, 'w' women and 'c' children. I gave each man p annas, each woman q annas, and each child r annas. Find an expression for T the total amount in rupees distributed.

i. If $m=20$, $w=18$, $c=16$ and $p=3$, $q=1\frac{1}{2}$ and $r=\frac{1}{2}$; Find T.
ii. Find m, when $T=8$, $w=18$, $c=16$, $p=4$, $q=2$, and $r=1$.

82. A recipe directs the mixture of a cu cm. of a substance A, with b cu. cm. of a substance B, but by mistake a gr. and b gr. respectively are taken. Find the proportions of A and B in the mixtures estimated by volume, it being given that p cu. cm. of A weigh x gr. and q cu. cm. of B weigh y gr.

83. A rectangular sheet of paper is a'' long and b'' broad. From a corner a right-angled triangle is clipped off in one straight cut. The sides of the triangle are x'' and y''. Find (i) the perimeter, and (ii) the area of the pentagon so formed.

(a) If $a=12$, $b=8$ and $x=0·5$, $y=1·2$, find in inches the perimeter and the area of the remaining piece.

(b) In the paper of the size given above if $x=4$ what must y be that the area of the remaining piece may be to that of the original piece as 5 : 6?

84. *ABCD* is a rectangle *AB* being greater than half *BC*; $AB=a$ inches and $BC=b$ inches. With centres A and B and radius b inches arcs are drawn to cut one another outside the rectangle at X. Similarly with centres C and D and radius b inches arcs are drawn to cut one another, outside the rectangle at Y. What kind of figure is *AXBCYD*?

What relation must exist between a and r for the figure to be regular?

If $a=6$ cms. construct geometrically a length b, that the above relation may hold between the two lengths and draw the regular figure with these lengths.

85. A boy uses a badly divided ruler supposed to be 1 ft. long and divided into inches But the divisions are such that the first, second, etc., so-called inches are respectively 1·1, ·98, 1, 1·07, ·97, ·96, 1·02, 1·12, 1·08, 1, 1·1 and 1·2 inches. The length of a piece of cloth is measured with this ruler and is found to be 3 yds. 9·6″. Find the real length of the piece. Each inch is equally divided into ten parts.

86. Standing at a distance of 30 yards from a straight running canal with parallel banks I observed a tree just in front on the opposite bank. I then walked a distance of 60 yards parallel to the canal from where I stood and from this point went direct towards the tree and reached the nearer bank having walked 50 yards. Find from a carefully drawn figure the width of the canal and check your measurement by calculation.

87. (*a*) The numbers 4242 and 2903 when divided by a certain number which contains 3 digits, have the same remainder. Find that remainder.

(*b*) Express ·0416 as a vulgar fraction in its lowest terms. Make use of this result in reducing ·375416 to a fraction.

88. (*a*) If the linear side of a cube be increased by 1 inch the surface of the cube would be increased by 66 sq. in. Determine the side of the cube.

(*b*) The linear scale of a map is 1 in 400,000. Find correct to one place of decimal the number of square miles of country represented by an area of 6 sq. in. on the map.

89. (*a*) Simplify to 5 places of decimals:—

$$\sqrt{1024} + \sqrt{102{\cdot}4} + \sqrt{10{\cdot}24} + \sqrt{1{\cdot}024} + \sqrt{{\cdot}001024}$$

(*b*) Find to 3 decimal places

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

90. The ordinary pace of a man is 30 inches and he goes 3 miles 120 yds. in an hour. When he walks fast he increases the length of the pace in the proportion of 3 : 4 and the number of steps in the proportion of 2 : 3. How many paces did he take for a march of one mile half at ordinary rate and half at the quicker rate.

91. A pair of scales is improperly balanced so that one viss in one pan balances 4 seers 6 palams in the other. If a merchant in selling goods places them in the latter pan, how much per cent. does he gain by such a proceeding? How much per cent. will he lose if he places the goods in the other pan?

92. In the centre of a room 21 ft. square there is a square carpet; the rest of the floor is covered with oil-cloth. The carpet and oil-cloth cost respectively 16 s. and 6 d. per sq. yd. and the whole cost is £ 35-4-6 d. Find the width of the oil-cloth border.

93. In a certain school $p\%$ of the boys and $q\%$ of the girls came out successful in an examination so that $r\%$ of the total number passed. If the strength of the school be $100a$, find the no. of boys and girls in terms of p, q, r and a.

If $a=8$, $p=20$, $q=30$ and $r=21{\cdot}875$ find the number of boys and the number of girls.

94. A caravan set out, each one riding a horse and leading another. On the way 17 of the party and thirty horses died and 80 more were sold. If, on their return, one had only the horse he rode on, find how many men were originally in the troop.

95. The price of pure silver per Rupee weight is Re. 1-3-6. The charges for making a silver cup come to two annas and a half for every Rupee weight in the finished cup. If I buy one seer and a half of silver and if 2½% of the silver is wasted in the making, what will be the final cost of the cup, given that a seer weight is equal to that of Rs. 24?

96. A boy was asked to subtract Rs. $y-x$ as. from Rs. $x-y$ as. He, by mistake, subtracted y as x ps., and the result was Rs. 5-1-5 more than what it should be. Find x and y.

97. The area of a verandah 'v' ft. wide inside a lawn 'l' feet square is 'p' sq. ft. Establish a relation between v, l, and p. Find l if $v=1\frac{1}{4}$ and $p=4356$.

98. "The following figures give the times of the rising and the setting of the sun on certain days of the year in the neighbourhood of London."

Date.		Sunrise.		Sunset.	
		H.	M.	H.	M.
January	1	8	8	3	58
February	1	7	42	4	46
March	1	6	49	5	36
March	21	6	5	6	11
April	1	5	39	6	29
May	1	4	35	7	19
June	1	3	50	8	3
June	21	3	44	8	19
July	1	3	48	8	18
August	1	4	23	7	49
September	1	5	13	6	48
September	23	5	44	6	2
October	1	6	0	5	40
November	1	6	54	4	34
December	1	7	45	3	53
December	21	8	5	3	51

Draw in one and the same diagram two graphs to show the variation in the time of (1) Sunrise, (2) Sunset; and find from your graph (1) the length of the longest day, (2) the length of the shortest day.

99. A merchant bought a viss of coffee at Rs. 3 a viss and b viss of sugar at Re. 1 a viss. He sold the former at a gain of $x\%$ and the latter at a gain of $y\%$. Find his total gain per cent. in terms of x, y, a and b.

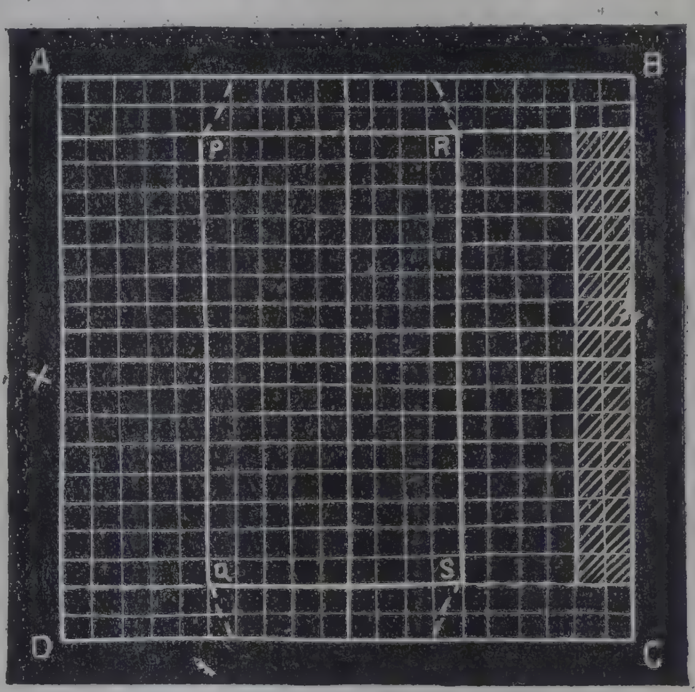
If $a=20$, $b=15$, $x=10$, $y=20$ find the gain per cent.

100. I bought a house site 32 ft. × 60 ft. The site was 1½ feet below the level of the road and I wanted to raise the level such that the ground-floor may be 1 ft. above the road-level. Mud could be had at 3 cart-loads for a Rupee and the mud was brought in carts $8' \times 2' \times 1\frac{1}{2}'$ and even this part was only $\frac{2}{3}$ full. I had to ram the mud to consolidate and the level would then be lowered by ½ foot. How much did it cost me to have the level of the ground-floor as I desired it should be.

101. A Mirasdar has a dishonest agent who generally uses a fraudulent balance in getting the rent from the farmers. Once he had to receive tamarind due to his master and he used a balance whose beam was horizontal only when a ten-seer weight was put in one scale pan for nine seers in the other. This balance he used to his benefit, both when getting the rent from the farmer and when weighing it to his master and thus when tamarind was selling at Re. 1-14 as. a maund, the value of his plunder was Rs. 14-4 as. How many maunds of tamarind were due as rent?

102. If an alloy is made with 378·17 kg. of zinc and 544·92 kg. of copper, find, to the nearest integer in each case, the number of parts of zinc and copper in 100 parts by weight of the alloy.

103. ABCD is a rectangular sheet of thin cardboard. AB = 2 in., BC = 2 in. To make an envelope the corners are scissored off along the dotted lines. What fraction of the area of the whole paper is left? Each division in the figure is $\frac{1}{10}''$, and the figure represents the actual paper drawn to scale of $\frac{1}{10}'' = \frac{1}{2}$ in. If all the flaps be closed along creases PQ, QS, SR, RP, what will be the size of the actual envelope?



104. A tinker was ordered to make a conical vessel on a base 7 in. across and of height 8·4 in. When it was finished he was ordered to make another similar vessel of double the capacity. Find to the nearest sq. in. the area of tin-sheeting required for each vessel. Given that $1{\cdot}26 \times 1{\cdot}26 \times 1{\cdot}26 = 2$.

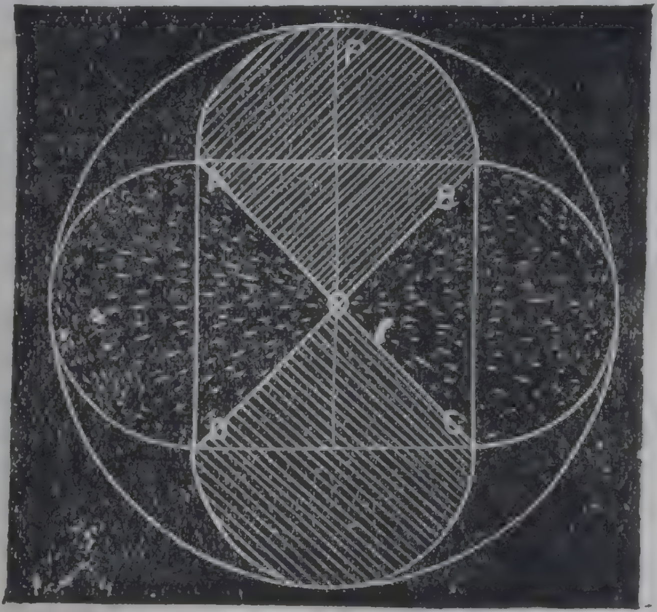
105. If the weight of a certain bulk of water be represented by 1, the weights of an equal bulk of flint glass, copper and gold will be represented by 3·2, 8·9, and 19·258 respectively. By what figures would all the other substances be represented if gold were represented by 1?

106. (a) A touring officer is paid Rs. a for the halt of every day at a station, and b annas for every mile travelled. If the officer had halted, in a month, at stations for y days and had travelled x miles, find P his total income in Rupees.

(b) The officer, during a month, had halted for 12 days and had travelled 400 miles. In another month he had halted for 8 days and had travelled 600 miles. If the respective allowances had been Rs. 130 and Rs. 100, find the rates.

107. A manufacturer invests Rs. 50,000 in a concern. By hand-labour he can realise 10 per cent. profit on the capital. He buys a machine for Rs 10,000 by which his profit on the remainder of the capital is enhanced to 20 per cent. At the end of each year he can invest the profit in either case in the business itself. The machine depreciates each year in value by 20 per cent. of its value at the beginning of each year. Which will be more profitable, and by how much, at the end of three years?

108. *ABCD* is a square and *O* its centre. Semi-circles are described on the sides as in the figure. With centre *O* and radius *OP*, a circle is drawn to touch the four semi-circles. The whole figure represents an ornamental design above a window frame. The shaded portions are of red glass, the dotted portions of green glass, and the dark portions of white glass. If the side of the square be 30 in., find the area of each kind of glass in the design.



109. To determine the calibre of a certain glass tube, mercury was let in and a thread 47 mm. long weighed 585·5 m. gr. Find the internal diameter supposing it to be uniform, throughout the length of the tube. Mercury weighs 18·59 times as much as an equal quantity of water.

110. The monthly income of a self-supporting High School is spent as follows:—

Pay of Staff	Rs.	900
Sty. and Prg.	Rs.	35
Library	Rs.	45
Laboratory	Rs.	45
Provident Fund	Rs.	50
Stipends	Rs.	50

Draw a circle with any convenient radius to represent the total expenses, and to represent each item of expenditure, mark off sectors whose areas are proportional to the figures given.

111. (i) If the speeds of two trains be as $a:b$ $(a>b)$ where will the faster overtake the slower, if both had started at the same time, run in the same direction, the slower from a place c miles ahead of the faster ?

(ii) If $a\%$ of the selling price of an article be the gain, what profit per cent. is actually made ? Find the profit per cent. if $a=90$.

112. A person purchased two lots of land for Rs. 200 each, and sold one at 40% more than cost and the other at 30% less than cost and took a promissory note for the amount of the proceeds of the sale, bearing $7\frac{1}{2}\%$ interest for two years, compounded annually. At maturity he collected the note. What profit per cent. was the amount of the note on the original sum invested in the lots ?

113. A contractor employs 850 coolies, 250 of whom are girls paid an average wage of 5 as. a day, the remainder are men and boys paid 10 as. and 4 as. a day respectively. His daily wage bill is Rs. 415-10 as. How many men does he employ ?

114. The most expensive battleship of the *U.S.A.*, the *Connecticut* cost $ 6,000,000 and the *Louisiana* $97\frac{1}{2}\%$ as much. This was 117% of the cost of the *Vermont* which cost $\frac{1}{4}$ more than the *Kansas*. Find the total cost of this division of the *U.S.A.* fleet.

115. The total cost C of a ship per hour (including interest and depreciation on capital, wages, coal etc.) is $C=a+bs^3$ where s is the speed in knots or nautical miles per hour.

When s is 10, C is found to be £ 5·2
s is 15, C „ £ 7·375.

Calculate a and b each correct to 3 significant figures. What is C when s is 12 ?

116. (i) A ditch has to be made 360 ft. long, 8 ft. wide at the top and 2 ft. wide at the bottom. The angle of the slope at each side being 45°, find the cub. ft. to be excavated.

(ii) If the earth excavated be evenly spread over half an acre, find to within a tenth of an inch, the height of the level raised.

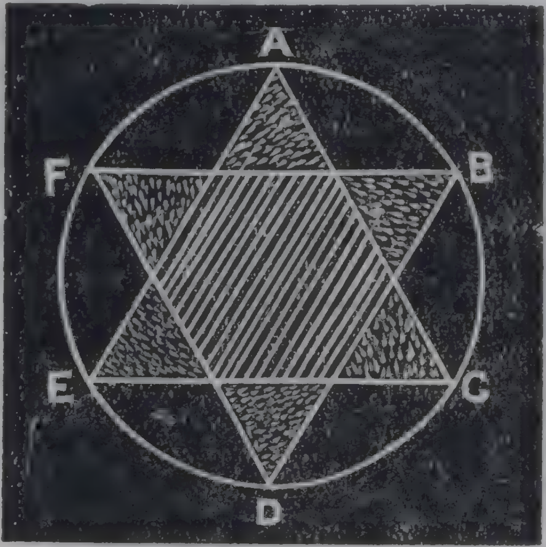
117. From a town T, two roads branch off, TAB and TCD. TAB runs due east and A and B are villages 24 and 32 miles respectively from T. The road TCD is such that the straight short-cuts, CA and DB (from the two villages C and D to A and B respectively) are at rt. angles to the road TCD. Find both from a diagram and by calculation the distances respectively of C and D from T ; if $BD=20$ miles. Also find the distance of C from A.

118. The ice on a circular lake is $1\frac{1}{2}$ ft. thick. If the lake is 1000 yds. in circumference, find the volume of ice to the nearest cub. ft.

119. The income of a railway from first, second and third class passengers is in the ratios 2 : 2 : 3. The corresponding fares per mile are in the ratios 5 : 3 : 2. Find the number of second and third class passengers to every 600 first class passengers.

120. Three trains A, B and C run between P and Q, a distance of 60 miles. One day A leaves P at 9 a. m. and runs at 12 miles an hour. C leaves Q at 9 a. m. and runs at 30 miles an hour. B which leaves P later meets C at 10-40′ and reaches Q, just when A reaches Q. Find graphically the speed of B and the time when it should have left A.

121. *A, B, C, D, E* and *F* are the corners of a regular hexagon inscribed in the circle. The figure represents a design above a window. The shaded portion is of green glass, the dotted parts of red glass and the black portions of white glass. Find the area of each kind of glass used in the design. The radius of the circle is 1·5 ft.



122. The scouts that were invited to be present at a review in the chief town of the district were stationed in four camps with their scout-masters. A sum of Rs. 420 had to be distributed for boarding among the four camps. If each scout-master was allowed to have twice the allowance of an ordinary scout, find how much (i) to the nearest anna, each scout camp received, and (ii) to the nearest pie, each scout was allowed.

Camps.	No. of Scouts.	No. of Scout-Masters.
A	49	3
B	74	4
C	68	3
D	127	6

123. A square reservoir measuring *a* yards each way receives the drainage of *b* acres. It is found that a thunderstorm raised the level of water in the reservoir by *c* inches. What depth of water fell, if it may be supposed to have fallen uniformly over the whole area, and that all the water which fell, found its way into the reservoir? Give a numerical result for the case $a=40$, $b=20$, $c=14$.

124. Two years ago I lent a sum of Rs. 1,200 at 9%, interest to the capital each year. Finding it impossible to collect the amount due, I put the bond in the hands of a professional bill-collector to compromise. He accepts 90% of the amount of the bond, takes by way of commission 7½% of the collections and gives me the balance. How much do I get?

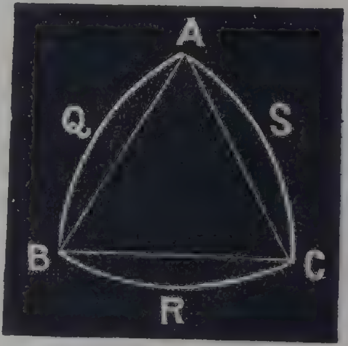
125. In a certain village *A*, the price of land per acre is Rs. 1,210; in an adjoining fertile village the cost is double. I buy a plot 60 ft. long and 90 ft broad in village *A* and another of the same shape but 50 ft. long in village *B*. What will the two plots together cost?

126. A jutka leaves a station *A* at 5-30 a. m. and travels at 6 miles an hour. A cyclist riding at 10 miles an hour, leaves *A* some time after and overtakes the jutka at 10-30 a. m. The cyclist continues his journey, reaches his destination, and halting there for an hour, returns and meets the jutka which is travelling all the time, at a distance of 3¾ miles from the destination. Find—the distance between the two places, when the cyclist left the station *A* and when the jutka will reach the destination.

127. A stone was thrown into an empty cylindrical vessel, which was then filled with water; when the stone was taken out, the water fell 4·75 inches. What was the volume of the stone, the diameter of the vessel being 9 inches?

128. There are two varieties of ghee which a merchant wants to mix for sale. If the varieties be mixed in the ratio of 2 : 3, the cost to the merchant of a viss is Rs. 3-2 as. If mixed in the ratio 3 : 5, the cost of a viss comes to Rs. 3-1-9. Find the cost to him of a viss of each variety.

129. *ABC* is an equilateral △ and the arcs *AQB*, *BRC* and *CSA* are drawn with centres *C*, *A* and *B* respectively. Find the area of the figure *AQBRCS*. Given that the length of a side of the triangle is 4 in.



130. Three partners *A*, *B* and *C* agree to divide the profits among themselves thus: Each is to take 20% of the profits. The balance is to be divided among them in the ratio of 3 : 3 : 2 subject to a maximum of Rs. 600 to *C*. If there is anything left *A* and *B* must divide it in the ratio 2 : 3. How must a profit of (*a*) Rs. 1,800, (*b*) Rs. 2,400 be divided among them?

131. (*a*) How muny data are necessary to determine a quadrilateral?

(*b*) Describe a trapezium *ABCD* given that the parallel sides *AB* and *CD* are 6 and 10 cms. long and that the non-parallel sides *BC* and *DA* are respectively 5·1 and 7·7 cms. Find the distance between the parallel sides and hence find the area of the trapezium.

132. In a farmer's pasture is a triangular house, enclosed by three walls, each side of which is 30 ft. A cow is staked at one corner and the rope is such that the cow can reach a distance of 40 ft. from the corner. What area of the pasture can the cow graze through?

133. The statement below gives the expenses and receipts for a certain book for various numbers printed :—

No. of copies	...	1000	2000	3000	4000
Expenses Rs.	...	80	120	140	160
Receipts in Rs.	...	65	95	165	195

Draw a graph showing the relation between the number of copies produced and the expenses.

In the same figure exhibit the relation between the receipts and the number of copies.

Estimate as nearly as possible the number necessary that the receipts may be equal to the expenses.

MADRAS SECONDARY SCHOOL LEAVING CERTIFICATE PUBLIC EXAMINATION.

1911.

1. Find graphically what the remainder is when 189 is divided by 56 by factors.

2. A rectangle measures 10·05 cm. by 9·93 cm.; find the area of the rectangle without directly multiplying these two numbers together.

3. Find the cost, to the nearest anna, of a school library as follows:—

No. of Books.	Published price of each.	
	s.	*d.*
128	7	6
200	5	0
20	3	6
236	3	0
256	2	6
104	2	6 net.
350	2	0
220	1	0 net.

(a) 12 as. to the shilling.

(b) 6¼ per cent. discount allowed off the published prices, except for net books.

(c) Packing and carriage 2 per cent. of the published prices of the books.

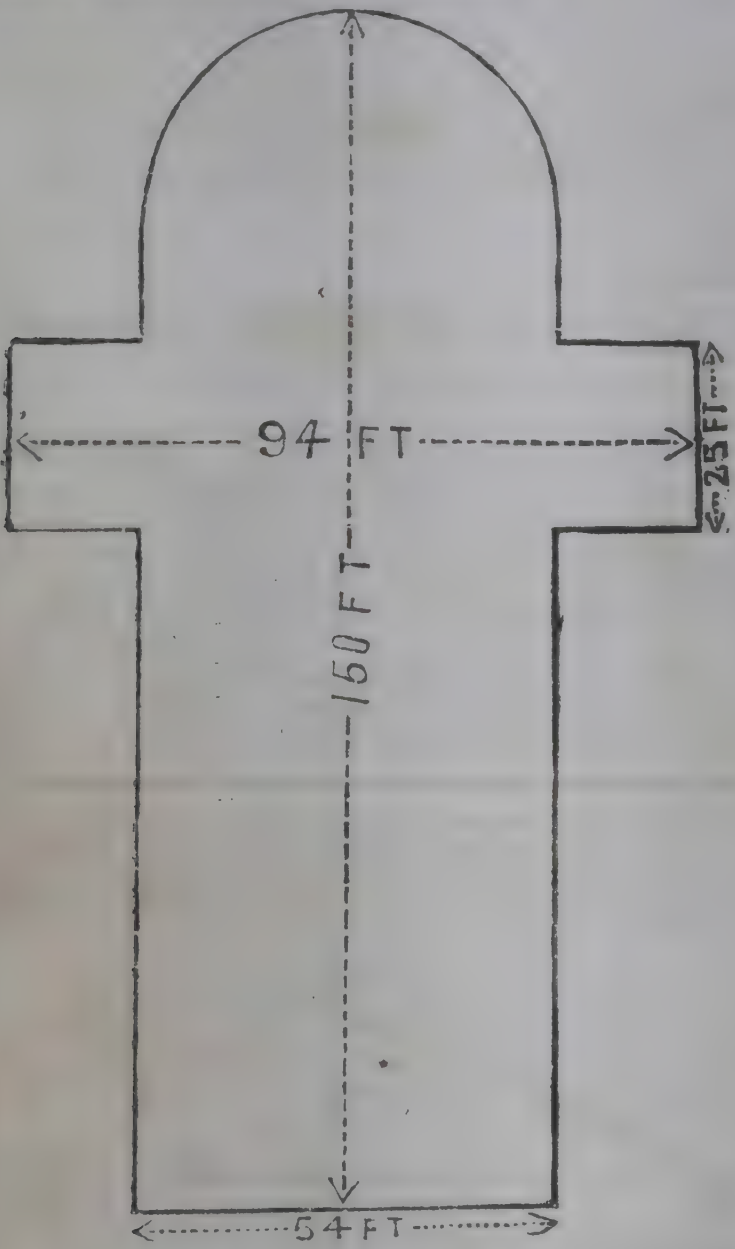
4. The following are the results of a recent examination:—

Centre.	No. of Candidates.	No. of Passes.
A	24	10
B	119	60
C	327	106
D	218	49
E	196	37
F	623	214
G	257	98
H	69	44

Find, correct to ·1, the percentage of passes for the whole examination; and give the percentage of passes at the most and at the least successful centres.

5. I bought a house for Rs. 6,500 ten years ago. I charge Rs. 40 a month for rent, and pay the house-tax (2s. 8d. a month) and do the repairs (average Rs. 60 a year) myself. It has been let for an average of 9½ months in every year, and while it stood empty I had to pay a man Rs. 5 a month to look after it. At the same time a friend of mine deposited Rs. 6,500 in a bank and has drawn annually interest at the rate of 4½ per cent. per annum. Which of us has benefited the more from his investment, and by how much?

6. This is the plan of a lecture-hall (not drawn accurately to scale), one end terminating in a semi-circle.



Find the area of the floor. The average height is 30 ft.; each person requires for health 200 c. ft. of fresh air every hour. If a lecture lasts $1\frac{1}{2}$ hours and the air in the hall is not renewed, how many people can be accommodated without injury to health?

7. At some School Sports the mile race is run on a perfectly circular course, the track being 8 yards wide. Four times round the course, on the inner edge of the track, is an exact mile. What is the length of the outer edge of the track?

8. A boy uses a badly divided ruler supposed to be six inches long and to be divided into inches. As a matter of fact it is so divided that the first, second, etc., so-called inches are ·98, 1·03, 1·01, ·96, ·94, and 1·01 inches (each 'inch' being divided into 10 equal parts). He measures a line and finds it, according to his ruler, 3 ft. 4·6 in. What is the real length of the line?

9. A boy hires a bicycle by paying one rupee for the use of the bicycle plus eight annas for every mile that he rides:—

(1) Express the cost of hiring in terms of the number of miles ridden.

(2) Draw a graph representing the relation between the cost and the distance.

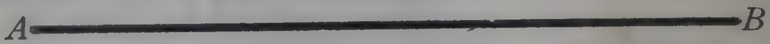
(3) From the graph find how many miles he can ride for Rs. 5.

10. The distance from Madras to Tuticorin is 440 miles. The following trains run:—

(1)	Monday, Madras dep.	6 P.M.
	Tuesday, Tuticorin arr.	4 P.M.
(2)	Monday, Tuticorin dep.	8 P.M.
	Tuesday, Madras arr.	6 A.M.
(3)	Monday, Tuticorin dep.	11 A.M.
	Tuesday, Madras arr.	7 P.M.

Represent these three journeys graphically in one diagram. With no help but the diagram say (*a*) how far each train is from Madras at 11 o'clock on Monday night; (*b*) at what time the train from Madras (No. 1) passes the trains (Nos. 2 and 3) from Tuticorin (omitting stoppages and assuming each train to travel uniformly.)

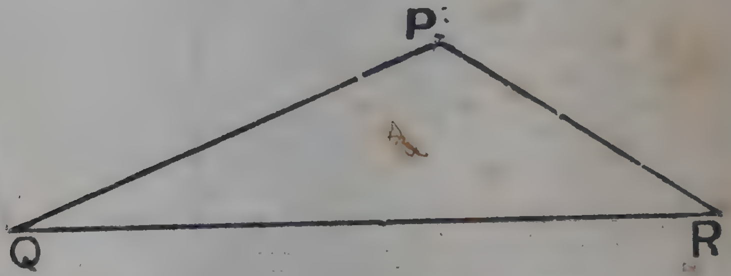
11.



(1) Draw a line exactly equal in length to *AB*.

(2) Draw another line $\sqrt{3}$ times the length of *AB*.

12. Measure, as accurately as you can, each of the angles and each of the sides of *PQR* and find its area.



13. On a blackboard measuring 4 ft. by 3 ft. is drawn a triangle so that:—(1) one vertex is 18 in. from the top left corner of the blackboard, and 24 in. from the bottom left corner. (2) Another vertex is 24 in. from the top right corner and 18 in. from the bottom right corner. (3) The third vertex is 24 in. from the bottom right corner and the angle at this vertex is a right angle. Draw a plan showing the position of the triangle relative to the sides of the blackboard. State your scale below the plan. What is the length of the hypotenuse (or side opposite the right angle)?

14. A goat is tied by a rope to a post 15 ft. from a long straight fence. The rope is such that the goat can reach with its mouth 23 ft. from the post. Along what length of the fence can it nibble?

Draw to scale, choosing and stating your scale.

15. The full pension granted to an officer retiring from Government service is half his average salary for the last three years of his service; but in the case of an officer who enters service after the age of 30, the full pension is reduced by one-fortieth for every year by which his age at admission into service exceeds 30. Express the pension of such an officer in terms of a full pension and of his age at admission into service.

Determine what pension a man who enters service at the age of 35 receives upon retiring, if his salary per mensem during his last 3 years of service is Rs. 375 for 10 mos., Rs. 350 for 3 mos., Rs. 370 for 8 mos., Rs. 380 for 9 mos., and Rs. 400 for 6 mos.

1912.

	ELEMENTARY MATHEMATICS.										Total.	p. c.	Rank
Max...	20	10	6	16	20	22	14	10	14	18	150		
A ...	14	8	3	13	13	16	12	4	10	12			
B ...	6	1	1	9	5	7	8	2	6	5			
C ...	2	2	0	6	1	4	5	1	3	2			
D ...	20	10	6	16	19	22	14	7	13	18			
E ...	13	3	2	12	12	14	11	3	9	11			
F ...	4	4	0	7	3	6	6	1	4	3			
G ..	1	5	0	5	0	2	4	1	2	1			
H ...	18	9	5	15	16	19	14	6	12	17			
L ...	*a*	4	1	11	*a*	11	10	*a*	8	8			
M ...	5	3	0	8	4	7	7	1	5	5			
N ...	7	2	1	10	6	8	9	2	7	6			
O ...	16	7	4	14	14	17	13	5	11	13			

In a table of 4 columns give—(1) the names (*A*, *B*, &c.) of these boys in alphabetical order; (2) the total of each boy's marks; (3) his percentage (in whole numbers) of the maximum; (4) his rank (determined by his percentage).

In case of absence the mark in col. 3 should be the boy's percentage of those maxima only for which he was present.

2. To celebrate the accession of Edward VII a certain town subscribed a sum of money which was spent as follows :—a procession, Rs. 57; feeding the poor, Rs. 214; fireworks, &c., Rs. 136; sports, Rs. 143; entertaining children. Rs. 184; local hospital, Rs 250; office expenses, Rs. 36 If Rs. 1,445 had been subscribed last year and spent upon the same objects in the same proportion, how much (to the nearest anna) would have been spent upon each?

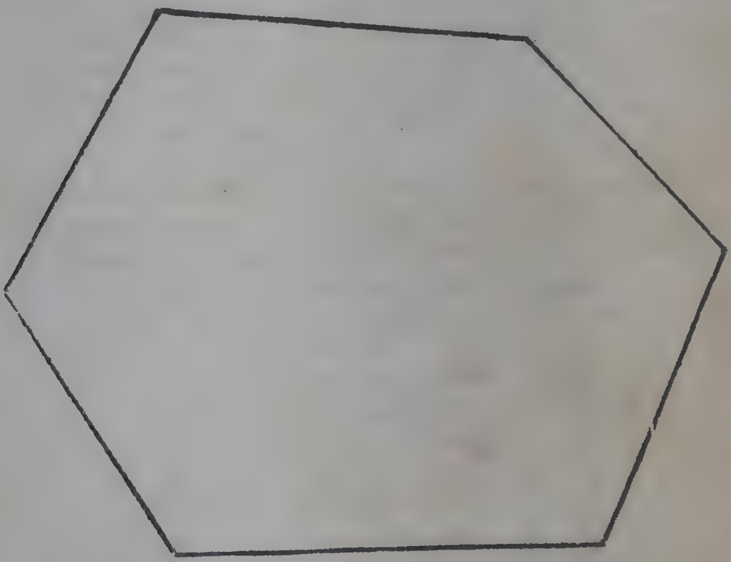
3. (1) A man invests Rs. 6,980 in 3½ per cent. Government securities at 89; find his annual income.

(2) Find the simple interest for 9 months and 20 days on Rs. 1,264 at 10 as. per Rs. 100 per mensem.

4. Allowing an average of 23 inches for each boy to stand in, if 198 boys are arranged in a circle, what will be its radius?

If they are arranged in a rectangle of which each long side is double each short side, what will be the length of its diagonal in feet, correct to five significant figures?

5. If this figure is an accurate plan (on a scale of $\frac{1}{10}$ in. to 50 yds.) of the foundary of an estate, find the area of the estate in sq. yds.

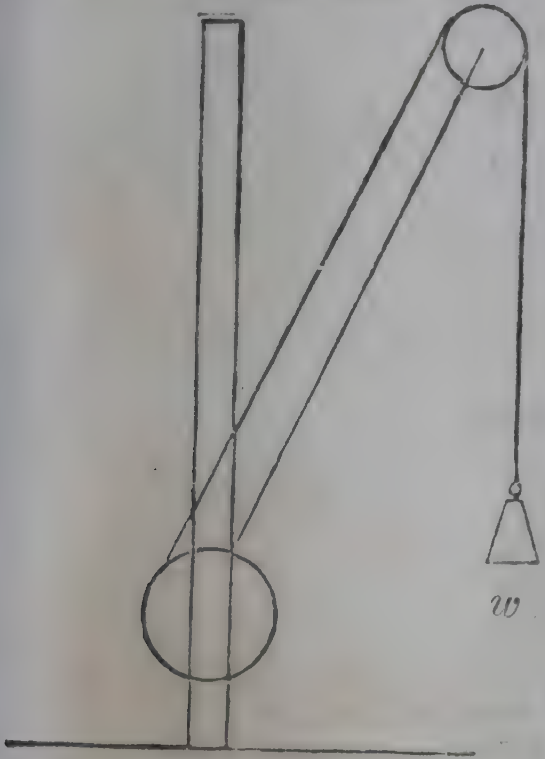


6. The temperature of a patient taken every three hours was found to vary as follows:—

Monday, 4 p. m.	100·2°
„ 7 „	101·4°
„ 10 „	102·8°
Tuesday, 1 a. m.	103·6°
„ 4 „	102·7°
„ 7 „	100·5°
„ 10 „	100°
„ 1 p. m.	100·1°
„ 4 „	100·3°

Draw on squared paper a curve indicating these changs;e from it deduce what was probably his temperature at midnight on Monday and at 8-30 on Tuesday morning.

7. This figure is intended to represent a crane, the arm of which can be completely revolved, around the central post. The bottom of the weight (w) can be made to touch the ground and can also be raised 20 ft. above the ground. The distance of the vertical rope from the central post is 12 ft. and the radius of the central post is 6 in. If the weight is moved into every possible position by the crane, find the area of the surface over which a point on the bottom of the weight and vertically under the rope moves.



8. Draw a line 4·6 cm. in length and upon it describe an isoceles triangle having the angle at the vertex 30°.

Draw another triangle of the same form, the sides of which are to the corresponding sides of the first triangle as 7 is to 3.

9. If a certain substance weighs 4 tolas for every cubic inch, find the weight of a solid right circular cone of the substance, of which the height is 20 in. and the diameter of the base 14·6 in.

10. Draw a circle of 3·3 cm. radius; insert in it a chord of 4·2 cm. length and bisect the smaller arc of the circle.

11. Construct a graph showing the relation between so many rupees a month and its equivalent in pounds sterling a year. From it deduce how many pounds a year are equivalent to Rs. 40 a month; and also how many rupees a month are equivalent to £ 60 a year.

12. If a metre = 39·37 in. and a kilogram = 2·2 pounds, express a pressure of so many pounds per sq. ft. in terms of its equivalent in grammes per sq. cm.

13. The estimated cost of an irrigation project is 2 lakhs, and it is expected to yield, after deducting for maintenance charges, a net return of 5·2 %. Without such deduction, however, the return would be 6·4 %. Assuming the maintenance charges to be 12 as. per acre, find (1) the extent of land proposed to be irrigated, and (2) the water-rate per acre proposed to be levied.

14. The time (t) in seconds of an oscillation of a pendulum is given by the formula

$$t=\pi\sqrt{\left(\frac{l}{g}\right)}$$

where l is the length of the pendulum. Find the length in cm. of a pendulum which oscillates 89 times in a minute.

$$g=981\,;\ \pi=3{\cdot}142.$$

15. The income and expenditure of a certain school are as follow :–school-fees, Rs. 12,672-12 ; fines, Rs. 9-7 ; stationery fees, Rs. 226-8 ; library fees, Rs. 318-6 ; laboratory fees, Rs. 116-8 ; entrance fees, Rs. 56 ; miscellaneous receipts, Rs. 27-7. Salaries, Rs. 12,028-8 ; contingencies, Rs. 125-4 ; stationery, &c., Rs. 314-8 ; library, Rs. 367-3 ; class books, Rs 63-8 ; furniture, Rs. 281-5 ; magazine, &c., Rs. 213-6 ; laboratory Rs. 187-8 ; prizes Rs. 129-3 ; annual repairs, Rs. 253-7.

If Government gives a grant of half the deficit, find the cost to the proprietors per boy, there being 540 boys.

1913.

1. (1)

Cities.		*Population in 1911*	*No. of females to 1,000 males.*
Madras	...	518,660	946
Madura	...	134,130	999
Trichinopoly	...	123,512	1,006
Calicut	...	78,417	920
Kumbakonam	...	64,647	1,064

From the above table find (i) the total population of these five towns, and (ii) the number of males in each town.

(2) Find correct to three significant figures.

$$0{\cdot}0923\times13{\cdot}215\times2{\cdot}654364.$$

2. (1) What is Rs. 698-12-0 in English money if exchange is at 1*s.* $4\frac{15}{16}$ *d.* to the rupee ?

(2) A man placed Rs. 1,500 as a fixed deposit in a bank paying 6% interest, the interest being added to the capital every year. Four years afterwards the bank failed and paid a dividend of two annas in the rupee. What did he get from the bank ?

3. If a metre is 39·3708 inches, find, correct to the nearest foot, the difference between 83 kilometres and 51 miles.

4. Solve the following equation algebraically, and check the result by means of a graph :

$$3x-\tfrac{2}{3}y=10\,;\ \tfrac{3}{4}x-1\tfrac{1}{3}y+1=0.$$

5. In a certain bicycle the hind-wheel revolves two and a half times for one revolution of a pedal. If the outside diameter of the hind-wheel is 28 inches, express the distance travelled in terms of the number of revolutions of a pedal ; and find the distance when the number of revolutions is 500.

6. (1) Of two cylindrical vessels the diameter of the second is double that of the first, and the height of the second is half that of the first. Compare the capacity of the second with that of the first.

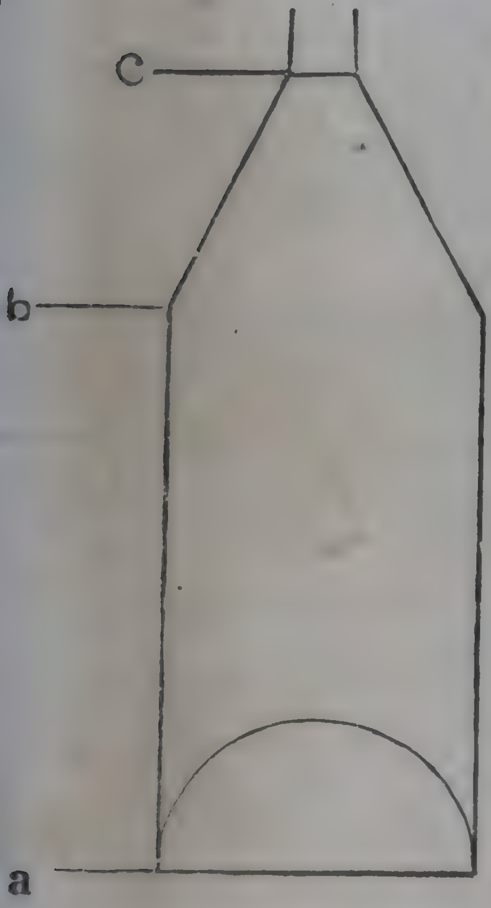
(2) If each has some water in it and a stone is immersed first n the one and then in the other, compare the height by which the water would rise in the second, with the height by which it would rise in the first.

7. (1) Draw four parallel straight lines such that the distance between the first and second is 1 in., between the second and third $\sqrt{2}$ in., and between the third and fourth $\sqrt{3}$ in., finding $\sqrt{2}$ in. and $\sqrt{3}$ in. geometrically.

(2) Draw a straight line cutting these four parallels and naking an angle of 45° with them. What length of this line is intercepted between the second and third parallels?

8. (1) Draw a triangle whose sides are 1·8, 2·4, and 3 in. respectively; give the measurement of each angle, and state what is the area of the circumscribing circle.

(2) Draw a right-angled triangle and draw the square on each side; then show, by dotted lines and by instructions given below our figure, how to cut up the squares on two sides and, by superposition of the cut pieces upon the third square, how to prove that the area of two of the squares is equal to that of the third square.

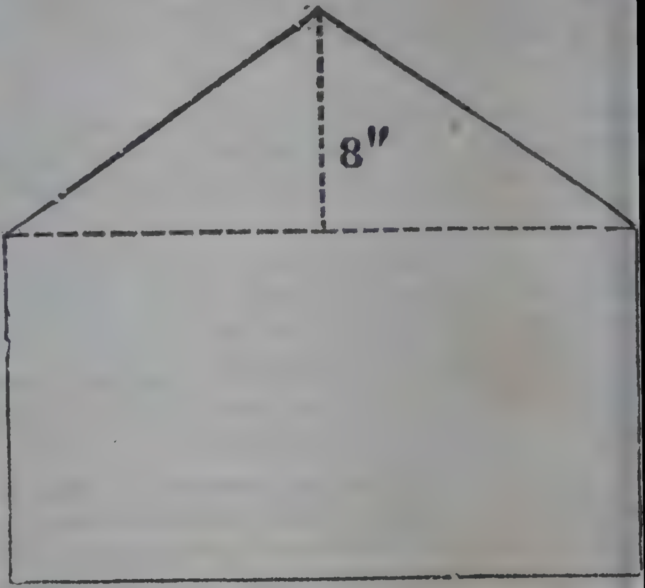


9. The figure represents a bottle, cylindrical in shape, with a hemispherical bottom thrust in as shown, and with a shoulder which is the frustum of a cone; diameter of the cylindrical portion 6 in., diameter of the bottom of the neck (or top of the shoulder) 2 in., height from *a* to *b* 12 in., and height from *b* to *c* 5 in. Find the capacity of the bottle up to the bottom of the neck.

10. For every acre of wet land a man pays a land revenue tax of Rs. 8; and for every rupee of land revenue, he pays a local cess of 1 a. 6 p. Draw a graph showing the total tax paid upon land varying in extent from 20 to 40 acres. What would be the total tax on 35 acres?

11 Find the cost of whitewashing a room which is 40′ by 18′ with walls 14′ high, allowing for *half* the area of the following openings: two doorways, each 7′ by $4\frac{1}{2}$′; four windows, each $3\frac{1}{2}$′ by $4\frac{1}{2}$′, and 2 circular ventilators, each 2′ in diameter; the wall at each end of the room is as in the figure.

Whitewashing : 2 a. 6 p. per 100 sq. ft.



12. An open rectangular cistern, of which the internal lengt breadth, and depth are as 4 : 3 : 2, is made of metal 1·5 cm. thick an 8 times as heavy as water. If the capacity of the cistern is 12,28 litres, what is its weight in kilograms when empty?

13. A bookseller is allowed by a publisher 20% discount on t nominal price of a book and 5% on this last amount for cash. F every dozen books he gets an axtra copy for nothing. The booksell allows his cash customer 4 annas in the rupee on the nominal pric Find (to one decimal place) the bookseller's profit on cash sales.

1914.

1. The following table gives the number of workers in Textil in the Madras Presidency in the years 1901 and 1911. Fill in t percentage (correct to one decimal place) column and the row totals:—

Name of textile.	*No. in 1901.*	*No. in 1911.*	*Percentage of increase (+) or decrease (−)*
Cotton (ginning, &c.) ...	24,089	32,553	
Cotton (spinning). ...	623,783	580,321	
Jute	533	2,428	
Rope	42,376	58,794	
Other fibres	7,886	9,252	
Total ...			

2. Show by a rough approximation, without working the questions in detail, that the answers affixed to divisions (1) and (2) are bviously wrong, and that those affixed to divisions (3) and (4) are pproximately correct.

(1) Find the cost per metre of cloth at 12 as. a yard; given one netre = 39·37 in. *Ans.* 12 as. 6 p.

(2) Find the income derived from the investment of Rs. 6,220 t 4 per cent after paying income-tax at 4 pies in the rupee.

Ans. Rs. 249-4-7 p.

(3) Simplify $30{\cdot}12 \times 19{\cdot}93 - (10{\cdot}01)^2$. *Ans.* 500.

(4) Simplify

$$\frac{1{\cdot}8 \times 25\frac{1}{14}}{201 \times 398} \times \frac{4042}{1523{\cdot}5}. \qquad \textit{Ans. } \tfrac{1}{60}.$$

3. The average of m numbers is x and the average of n of those numbers is y; what is the average of the remainder?

The area of the Madras City is 17,280 acres and the average population is 30·3 per acre. The area of the thickly populated divisions is 2804 acres and the corresponding average population is 135·3 per acre. Find the average population per acre for the rest of the City.

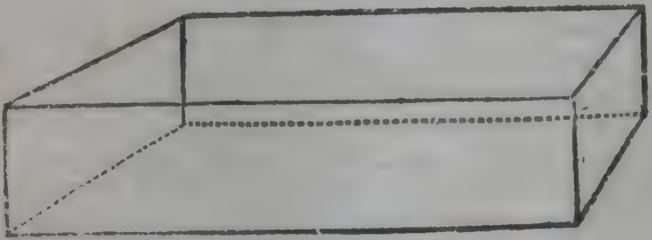
4. A milkseller pays 5 as. per measure for the milk he purchases and adds water before selling. He also defrauds by using, while selling, a false ollock measure which is $\frac{1}{16}$ short of a true ollock. If he sells at $\frac{1}{2}$ anna an ollock and makes 12% profit find the proportion of water to milk in his mixture. [A measure = 8 ollocks.]

5. A manufacturer of an article sells it to a merchant at a gain of $p\%$ on the cost of making. The merchant sells it for Rs. r at a gain of $q\%$. What is the cost of making? Use your result to find the cost of making, when $p = 10$, $q = 12\frac{1}{2}$, and $r = 495$. Verify your result in any way.

6. A circular arch over a river rises 4 ft. at its highest point above the level of the tops of the vertical piers. If the distance between the piers is 28 ft., find graphically or otherwise, the radius of the circle of which the arch forms a part.

7. $ABCD$ is a quadrilateral field; $BC = 20$ yds., $AD = 24$ yds., the diagonal $BD = 32$ yds.; the angles A and C are right angles. Draw a figure to scale (1″ = 10 yds.), and determine the area.

8. How many sq. in. of metal are required to make a tray of the shape shown in the diagram? The adjacent sides of the base measure 9 in. and 15 in. and the corresponding sides of the top measure 10 in. and 16 in., and the width of the strip making the sides in 3 in.



9. In order to find the diameter of a tube of uniform bore, some mercury was poured into it and the height of the column measured. The weight of the mercury was 19.72 grm.; the height of the column 20·3 cm. What was the diameter of the tube? (A cub. cm. of mercury weighs 13·6 grm. and $\pi = \frac{22}{7}$.)

10. In 1911 the population of the Madras City was 551,660. Since then the birth-rate per annum has been 42 per thousand and the death-rate 39 per thousand; and the town has attracted people from outside at the rate of 17 per thousand every year. What will be the population in 1914?

11. Find, by graphs, which of the following will form the best and which the worst investment:—

2½% Consols at 86.
Transvaal 3% at 94.
Indian Railway 3½% at 102.
Japan 4½% at 102.
[Assume that all are equally secure].

12. The following table gives the water level in the Red Hills Lake on the 15th of every month:—

MONTH.	LEVEL.	MONTH.	LEVEL.
1911.		1911 (*contd.*)	
April.	42·1	Nov.	39·1
May.	40·5	Dec.	44·0
June.	39·0		
July.	37·9	1912.	
Aug.	36·9	Jan.	44·3
Sept.	36·6	Feb.	43·4
Oct.	37·3	March.	43·0

Exhibit the variations in level by means of a smoothly drawn graphe and find approximately when the level was (1) the highest, and (2) the lowest.

1915.

1. In a certain year the United Kingdom imported 61,325,694 cwt. of wheat valued at £ 22,934,523 from the foreign countries, and 41,568,705 cwt. of wheat valued at £ 16,837,425 from British possessions. Find, to the nearest penny, the value per cwt. of imported wheat (1) from foreign countries, (2) from British possessions. Also find, to the nearest integer, what percentage of the total value of the imported wheat is the value of the quantity imported from British possessions.

2. An English sovereign weighs 123·3 grains and contains 91·67 per cent. of pure gold. Find the value, to the nearest tenth of a penny, of the gold in a sovereign at £ 3-17-6 d. for 480 grains.

3. I want to buy 20 yards of certain iron wire which is 0·14 of an inch in diameter, but I find it is sold only by the pound. Calculate for me the weight of the wire which I want, given that iron weighs 470 ℔. a cubic foot. [Use the formula $v = 0{\cdot}785d^2l$; where v is the volume in cubic inches of a piece of wire l inches long and d inches in diameter.]

4. At a show the following kinds of tickets are sold. Green tickets at 9 pies, white at 3 pies, pink at 6 pies, and yellow at one anna. If g green, w white, p pink, and y yellow tickets were sold on a day, construct a formula for T, the total collection in annas for the day.

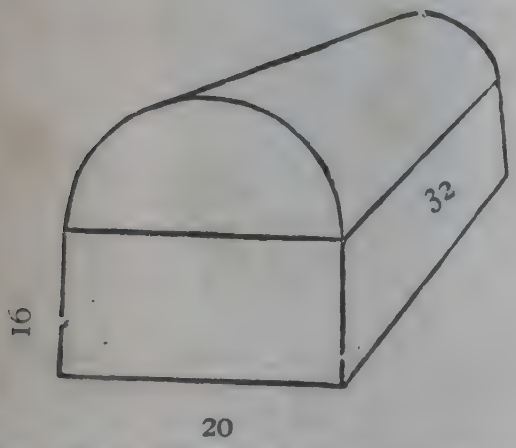
Use your formula to find T if $g=360$, $w=1760$, $p=480$, $y=80$. Also find g, if $w=920$, $p=440$, $y=72$, and $T=762$.

5. A steamer goes a miles upstream in b hours, and c miles lownstream in d hours. The stream flows e miles an hour, and the vessel could go f miles an hour in still water. Express a and c in erms of b, d, e, f. If $a=16$, $b=2$, $c=18$, $d=\frac{3}{2}$, what are e and f?

6. A ladder 12 feet long leans against a wall making with it an angle of 25°. Draw the ladder in position representing a foot by one-fifth of an inch. Then draw the ladder in a new position with its ower end 2 feet farther from the wall, and find, by measurement, ow much lower the top of the ladder will then be.

7. Take any straight line and mark a point A at a distance of cm. from it. Now find another point B which is 3 cm. from A and cm. from the line. Find also a point C 2 cm. from A at which the ine AB subtends a right angle. Find, by measurement, the distance f C from the first line.

8. A hay-shed (shown in the adjoining figure) consists of a rectngular space 32 ft. long, 16 ft. high, and 20 ft. wide, surmounted by a overing roof whose section is a semi-circle. Calculate how many ons of hay can be stored in it, if a ton of hay occupies 300 cubic feet.



9. A merchant sells silk of two qualities which cost him Rs. 4 d Rs. 3-4-0 per yard respectively. The selling price of the latter is o-thirds that of the former, but the quantity sold is double. He ins 25 per cent. on the whole. Calculate the selling-price per yard each.

10. In a certain factory the machinery was valued at Rs. 32,500. it depreciates each year at the rate of 8 per cent. of the value which had at the beginning of the year, find, to the nearest rupee, what its lue would be after 4 years.

11. I live three-quarters of a mile from my railway station and take 15 minutes to walk the distance. I am accustomed to meet at a certain point some children going to school in the opposite direction. The children take 30 minutes to the mile. One day (when I am at my usual time) I am 220 yards farther on when I meet them. How much later will the children arrive at school that day than on ordinary days, provided they keep their ordinary pace?

Draw three graphs on one diagram to show the position of the children and myself at any time on ordinary days and on the special day mentioned, taking the usual meeting-place and time as the origin.

12. Passengers on the railway are allowed a certain weight of luggage free, and the rest is paid for at 5*s.*-7*d.* per cwt. If a traveller with luggage weighing 256 lb. had to pav 8*s.* 4½*d.* for his luggage, find the weight of luggage allowed free, and draw a graph connecting the amount of luggage carried and the charges paid.

1916.

1. The fallowing table is a classification of the number of peopl in thousands, supported by agriculturists and actual workers in agriculture for the years 1910 and 1911. Find to the nearest integer the decrease or increase per cent. in the figures for 1911, as compare with those for 1910 under each head:

Description.	Supported by agriculturists.		Actual workers in agriculture.	
	1910	1911	1910	1911
Non-cultivating Landowners ...	34	30	23	19
Cultivating Landowners ...	461	512	426	484
Non-Cultivating Tenants ...	6	2	4	1
Cultivating Tenants ...	225	167	207	151
Labourers ...	279	284	340	345

2. On March 1, 1913, a person borrowed from *A*, Rs. 1,450 at per cent. simple interest, and lent the sum on the same day to *B* 8 per cent. compound interest: if both the transactions were close on March 1, 1915, find his gain.

3. Determine by graphs which of the following is the most pr fitable investment:

(1) 3½ per cent. Government of India paper at 23;

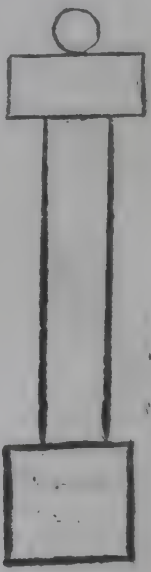
(2) 4 per cent. Bank deposit at par;

(3) Rs. 20 Bank Shares yielding 10 per cent. dividend Rs. 48.

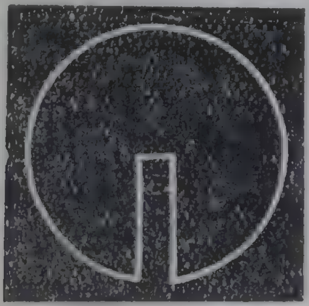
4. A school grants its games Association Rs. 216 a year, which is distributed among its three branches, Tennis, Football, and Badminton, in the proportion of 14 : 6 : 1. The Tennis branch is 14 strong, and each member pays a subscription of Rs. 2 per half-year. If the branch pays Rs. 6 a month to the pickers and spends Rs. 32 per annum on repairs to the court, find how many balls a month at Rs. 12 per dozen the branch can supply itself with, so as not to work at a loss.

5. If a bookseller fixes his prices so as to include a profit of 30 per cent on the cost price and offers a reduction of 10 per cent on the published price for cash payments, find his gain per cent. If the cost price of 34 copies of a book be Rs. 50, what is his average gain per copy, when all are sold for cash?

6. Find the volume of a gate pillar consisting of a cylindrical column 6′ high and 1′ in diameter, placed on a cube of 2′ edge and surmounted by a square slab 1′ thick and 2′ side with a sphere 6″ radius placed on the top of it.



7. A cast-iron cylindrical weight is 18″ in diameter and has a slot 2″ wide and 10″ long as in the figure. Find, correct to 2 places of decimals, its thickness in inches when its weight is 100 lb. (One cubic inch of cast-iron weighs 0·26 lb).



8. The following table gives the power (P lb.) to be applied in order to move the weight (W lb.) for certain machine:

W	50	100	200	300	400	500
P	106	122	152	184	216	246

Draw a graph showing the relation between P and W, and find to the nearest integer the value of P when $W=350$, and of W when $P=158$.

9. If a person buys p dozen mangoes at a as. per dozen, q dozen mangoes at b as. per dozen, and r dozen mangoes at c as. per dozen and sells $\frac{2}{3}$ of the whole lot at d as. per dozen, and the rest at $\frac{1}{2}a$ as. per dozen, find his profit. If the profit is Rs. 4-6-0 when $p=12$, $q=5$, $r=13$, $a=20$, $b=12$, $c=10$, find d.

10. Solve $2x+3y=12$, $0{\cdot}5x-y=4$.
Compare your answer with results obtained graphically.

11. Construct a quadrilateral $ABCD$ having $AB=AD=4$ cm., $AC=8$ cm., and $CB=CD=6$ cm. Measure BD and the angles B and D.

12. Draw an equilateral triangle and a regular hexagon so as to lie between the same parallels, and compare the areas of the two figures.

1917.

1. (1) The H. C. F. of three numbers is 7 and their L.C.M. is 12012. Two of the numbers are 364 and 84. Find the least value of the third number.

(2) You are given that $461010=77\times5987\frac{1}{7}$. Show how you can write down (correct to five places of decimals) the quotient when 0.046101 is divided by $(14\times0{\cdot}55\times0{\cdot}001)$ without actually going through the ordinary process of division.

2. The following statement gives the figures under each head of contingent charges, both estimated and actual, for the year 1913-14:—

ITEMS.	Estimated.			Actual.		
	RS.	A.	P.	RS.	A.	P.
Printing	15,650	0	0	10,595	6	0
Stationery	12,550	0	0	6,610	2	0
Travelling expenses	10,630	0	0	12,939	7	0
Postal and other charges ...	6,670	0	0	7,483	9	7
Examination charges	3,450	0	0	3,362	10	1
Contribution to the Library ...	5,550	0	0	5,550	0	0
Cost of University Lectures, &c. ...	6,600	0	0	6,434	8	0
TOTAL ...						

Fill in the row of totals, and find by what percentage (correct to one decimal place) the total of actuals have fallen short of the total of the estimated charges.

3. Two partners invest Rs. 2,700 and Rs 1,500 respectively in their business, and arrange that 40 per cent of the profits should be divided equally between them and the remaining profits treated as interest on capital. If one partner's share of the profits is Rs. 90 more than the other's share, what was the whole amount of the profits?

4. A man buys a field for Rs. 3,750 and lets it at a rent of Rs. 150 payable at the end of each year. The rent is invested at 5 per cent compound interest per annum. At the close of three years he sells the field for Rs. 4000 and draws out the accumulated rents and interest. Find the total he will have and the annual percentage (simple interest) he makes by the whole transaction.

5. A railway track starts from station *A* with a gradient of 1 in. 200, *i.e.*, a vertical rise of 1 yard in 200 yards measured along the track. The gradient remains the same for ½ mile, and is followed by rising gradients of 1 in 150 for ¾ mile, 1 in 120 for ½ mile, level for 1 mile, and falling gradients of 1 in 110 for ½ mile and 1 in 330 for ¾ mile, when station *B* is reached. If station *A* is 179 feet above sea-level, find to the nearest foot the elevation of station *B*.

6. The average heights of a man and a woman and their corresponding weights are given in the following table :—

HEIGHT.	5′ 0″	5′ 2″	5′ 4″	5′ 6″	5′ 8″	5′ 9″	5′ 10″	6′ 0″
Average weight of a man in pounds.	...	126	139	145	155	162	169	178
Average weight of a woman in pounds.	103	112	126	139	152	156	...	...

Draw graphs in one and the same figure showing the relation between the average height and weight for each sex. Find from your graphs the difference between the average heights of a man and a woman who each weigh 10 stone, and the difference in weight if they are each 5′ 6″ in height.

7. A cylindrical oil tank is 12 feet in diameter and has ends which are hemispheres of the same diameter. Its total length is 40 feet. If it is lying in a horizontal position, how many gallons of oil will be required to fill it a depth of 6 feet? 1 gallon is equivalent to 277½ cubic inches nearly).

8. A child is instructed as follows :—‘ Think of a number. double it, add 11 to the result, multiply the sum by 3, subtract 12, divide the remainder by 3, give me the answer.’—Find a simple rule by which the number first thought of can be readily deduced from the answer given by the child.

9. *A* and *B* engaged in play : when *A* had lost Rs. 20 he had only one third of the money which *B* had ; but by continuing to play he not only won back his Rs. 20, but also Rs. 50 more with it, and then found he possessed half as much again as *B*. With what sum did they respectively begin?

Reduce the conditions of the problem to simultaneous equations in two unknowns, and solve the problem.

10. *A* is the centre of a circle of 5 cm. radius, and *B* is the centre of a circles of 2 cm. radius. *CD* is a direct common tangent to the two circle and touches them at *C* and *D* respectively. If *CD* is 4 cm. long, find the distance between the centres. Draw a neat figure to scale.

11. Divide a straight line AB, 6·4 cm. long, internally at X and externally at Y, in the ratio of 5 : 3. Verify the result of your construction by measuring the lengths of the segments.

Also show that they satisfy the formula:

$$2/AB=1/AX+1/AY.$$

12. Show by means of a carefully drawn diagram how you will verify by paper-cutting the truth of the proposition that the square on the hypotenuse is equal to the sum of the squares on the sides of a right-angled triangle.

1918.

1. The fee income from the secondary department of a school in a certain month is given in the adjoining table:

Form.	Strength.	Amount. Rs.	As.	Ps.
VI	79	414	0	0
V	80	404	4	0
IV	78	383	8	0
III	76	240	9	0
II	74	191	3	0
I	125	326	8	0

Find

(1) the average free paid by each boy in the whole department; and

(2) the percentage of the I Form collections to the whole income.

2. A merchant bought 40 candies of paddy at Narasapur at Rs. 38 a candy, and sent the same to Tanjore, paying Rs. 10 towards boat hire as far as Nidadavole and 17 times as much for railway charges from there to Tanjore. His agent sold the whole quantity at Tanjore at Rs. 2-4 as. a kalam. Find the amount of gain or loss t the Narasapur merchant, if he paid his agent a commission of 5 pe cent. on the sale-proceeds.

[A kalam = 24 Madras measures. A candy = 800 seers eac of which is ⅔ of a Madras measure.].

3. On each page of an Atlas the border lines are $7\frac{1}{2}$ in. length-wise and 6 in. breadthwise. The map of the Indian Empire is draw to a scale of 300 miles to an inch, and occupies $\frac{4}{9}$ of the space withi the border lines. Find the area of the Empire in square miles.

On another page of the same Atlas, with similar border lines there is the map of the Madras Presidency, whose area is 153,00 square miles, excluding Native States, occupying $\frac{17}{72}$ of the spac enclosed by the border lines. Find the scale to which this map i drawn.

4. The following is a rough extract from a railway Guide:

Trains from Madura.

Distance from Madras in miles.	Stations.	Boat Mail to Ceylon.
348	Madura *dep.* ...	11–45
448	Ramesvaram *arr.*	16–30

Trains from Rameswaram.

Distance from Madras in miles.	Stations.	Boat Mail from Ceylon.	Rames-varam Express.
448	Ramesvaram *dep.* ...	9–45	12–30
348	Madura *arr.* ...	14–30	19–30

[*Note.*—Time is calculated for 24 hours from midnight to midnight.

Represent the motion of the three trains graphically. From the graph find when they are 40 miles from Madura, and when and at what distance from Madura the Boat Mail from Madura meets the other two trains.

5. In the Post Office, War Loan Certificates were issued on the following terms: For a payment of Rs. 7-12 as., a certificate was issued which entitled the holder to get Rs. 8-1 a., Rs. 8-14 as., and Rs. 10, at the end of one, three, and five years respectively. What rate per annum simple interest does it come to in each of the three cases?

Given also that the certificate-holder is entitled to receive Rs. 8-7 as. and Rs. 9-6 as., at the end of two and four years respectively, show graphically how the money grows during the five years.

6. A cylindrical granary with a conical top is 14 ft. in diameter on the inside. The wall is 11 ft. high, and the total height to the top is 14 ft. It was full of grain. Half of it was sold at Rs. 5-12 as. a bag, and the other half at Rs. 5-4 as. a bag. Find the amount realized, given that a bag of grain measures 8·25 c. ft.

7. Describe a regular octagon on a straight line 2 cm. long, an nd its area by any method.

8. At a Public Examination in the Madras Presidency, x candidates who had taken up Optional Mathematics scored in Elementary Mathematics an average of m marks and the remaining y candidates for the examination scored an average of n marks. What was the Presidency average in the subject?

If $x=2{,}500$, $y=6{,}428$, $m=53$, and $n=30$, find the Presidency average.

9. A village postman attached to the Post Office at P has to go to five villages A, B, C, D and E, and return to P. A is 5 miles due north of P, B, C and D are on the eastern side of the road from P to A; B is 8 miles from P and 6 miles from A; C and D are 5 and 6 miles respectively from P on a straight road PCD which is at right angles to PB. E is 3 miles from P, its position being to the west of the road PA, and the roads PE and AE being at right angles to each other. Draw a diagram [scale. 1 cm. = 1 mile] and indicate the shortest route for the postman. What is the length of his circuit?

10. (1) Construct a quadrilateral $ABCD$, given that $AB=3$ cm., $DA=4$ cm., $A=120°$, $C=70°$, and AB is parallel to CD. Measure BC.

(2) The middle points of the sides of a scalene triangle are joined so as to form a new triangle. What fraction is this of the given triangle? Explain from a figure how your answer can be shown to be true experimentally by superposition.

11. The distance from Madras to Ootacamund by railway is 358 miles, part of which is ordinary railway and the other part mountain railway. The second class fare over the former is 9 ps. per mile, and over the latter 1 a. 9 ps. per mile. If the total fare be Rs. 18-10-6, find the length of the mountain railway.

12. A reservoir which supplies water to a town has two inlets whose diameters are 30 in. and 16 in. respectively, through which water pours in throughout day and night. Water is to be taken into the town through six pipes, all of uniform diameter. But they will remain open only from 5 a. m. to 9 p. m. daily. What should be the diameter of each of the latter, if the quantity conveyed to the town every day is to be equal to the daily supply into the reservoir, assuming that the rate of flow is the same through all the eight pipes?

1919.

I. In Form I of a certain school there are four divisions. The adjoining table gives the strength and the total marks gained by the students of each division in a certain subject.

Division.	Strength.	Total Marks.
A	34	1372
B	35	1339
C	36	1489
D	37	1275

Find, correct to one decimal place, the average marks for the whole Form, and ascertain which division gets the highest average and which the lowest.

II. A merchant cleared his goods by selling them at reduced prices. He sold certain goods whose catalogued sale price was Rs. 760 at a reduction of 15 per cent. another set of goods whose sale price was Rs. 880 at a reduction $12\frac{1}{2}$ per cent. and the remaining goods whose sale price was Rs. 480 at a reduction of 25 per cent. He however, gained in all 11 per cent. on his original cost price. Find the original cost price, and also what percentage he would have gained had he not made any reduction in the catalogued sale prices.

III. $ABCD$ is a quadrilateral field in which $AB = 198$ yards, $BC = 264$ yards, and $AC = 330$ yards. D is equidistant from A and C, and at a distance of 214·5 yards from the diagonal AC. (1) Draw a plan of the field to a scale of 1 cm. = 33 yards. (2) Find the area of the field in acres.

VI. The distance from P to Q along a road is 65 miles, A person A starts from P at 7 a.m. and travels to Q on his motor-cycle at the rate of 10 miles an hour, taking rest for 10 minutes at the end of every hour of cycling. B starts from P at 8-45 a.m. and goes to Q on his motor-car at a uniform rate of 15 miles an hour. Represent their journeys graphically.

Find from the graph (1) when each of them will reach Q, (2) when and where B will overtake A, and (3) by what distance they will be apart at 9-15 a.m.

V. A person borrows Rs. 1,800 at 8 per cent. per annum simple interest, and forthwith lends it as follows: (1) Rs. 800 to A at $10\frac{5}{8}$ per cent. per annum simple interest; (2) Rs. 600 to B at 10 per cent. per annum compound interest; and (3) the remaining sum to C at $12\frac{1}{2}$ per cent. per annum during the first year, and 12 per cent. per annum during the second, the interest being compounded with the principal at the close of the first year. At the end of two years, he collects the amounts due to him from A, B and C, and pays his dues to his creditor. How much does he gain?

VI. (1) Show by a geometrical construction how a length which is $\sqrt{3}$ times a given length may be found.

(2) Describe a regular hexagon on a straight line 1·5 inches long and find its area.

VII. A stone road-roller should be four feet long, and should weigh 4 tons 8 cwt. What should be its diameter, given that the weight of the stone is 441 lb. per cubic foot?

VIII. There are some beggars consisting of men and boys. If the former be given 5 as. 6 ps. each, and the latter 3 as. 6 ps. each, a sum of Rs 12-10 as. will be required. If each man is given 6 ps. less and each boy 6 ps. more, the total sum required is 2 as. less. Reduce the above conditions to equations, and find the number of men and of boys.

Represent the equations graphically, and verify the result from the graphs.

IX. A garden is 240 yards long and 180 yards broad. A moat 5 yards wide and 2 yards deep has to be dug close to it on the outside all round, and the earth dug out spread evenly over the garden.

(1) Find in inches, correct to one decimal place, the height by which the level of the garden will be raised thereby.

(2) A certain contractor undertook to finish the work in 30 days and employed 36 men. At the end of 10 days, only 1 ft. 6 in. of the depth of the moat had been dug all round. How mnny additional men should he employ from the 11th day to finish the work in the stipulated time?

X. A certain boy took a circular piece of card board having a radius of 17·5 inches, and made an angle of 144° at the centre. He then cut out and then removed from it the sector piece containing the angle 144° and made the bounding radii of the remaining piece coincide. What shape did he get? Calculate its height.

XI. A merchant buys m maunds of coffee seeds at Rs. a per viss, and n maunds at Rs. b per viss, and mixes them. He sells the mixture at a uniform price, and gains c per cent. on his total outlay. Find the selling price, of one viss of the mixture,

If $m=4$, $n=5$, $a=1\frac{1}{4}$, $b=1\frac{1}{2}$, and $c=12\frac{1}{2}$, find the selling price of the mixture per viss.

XII. In a certain meadow there is a hut whose ground-plan is a square, each side of which is 28 ft. A cow is tethered to a post at one of its corners by a rope 35 ft. long. Find the area that the cow has for grazing.

Re-Examination, 1919.

1. The strength of Form VI in a certain school is 105. The Form average in Elementary Science was exactly 39 in a certain year. The boys scored marks thus in the subject:—15 boys 54 marks each, 22 boys 45 each. 28 boys 38 each, 25 boys 33 each, and 13 boys 24 each. Of the remaining two boys A and B, A scored 20 marks more than B. How many marks did each of them get? Also find their rank.

2. A grocer wants to mix 80 lb. of tea worth 15 as. a lb. with some tea worth Rs. 1-6-0 a lb. so that, by selling the mixture at Rs. 1-4-0 a lb., he may gain 20 per cent. on his outlay. How many pounds of the superior sort should he take to form the mixture?

3. In a 100 yards race A beats B by 20 yards and C by 8 yards. If B and C run over the same course of 100 yards, find who will win and by how many yards to the nearest integer. Verify the result graphically.

4. An open rectangular box is to be made out of a rectangular piece of cardboard 21 in. long and 10·5 in. broad, by cutting out four equal squares at the corners of the rectangle and folding up the four rectangles that result so as to form the vertical sides and ends of the box. If the ends of the box are squares, find the cubical contents of the box.

Draw a plan of the cardboard, to a scale ot 1 cm. to 1 inch, as it will look after the square corners are removed and before the rectangles are folded up.

Find the maximum number of such cardboard boxes that can be packed in a cubical box whose edge on the inside is 2 ft. 4 inches.
[**N.B.**—*The thickness of the cardboard is to be neglected throughout.*]

5. According to the catalogue of a London firm the cost of the books that are to be bought for a High School in Madras is £ 60, of which 25 per cent. is marked net. In the case of the books so marked the packing is free. For the remaining books, a discount of 20 per cent. is allowed, but £ 1-4-0 is charged for packing. The freight to Madras for the whole consignment comes to £ 2-16-0. A Madras bookseller undertakes to supply the books marked net at Re. 1 a shilling and the rest at as. 14 a shilling. Which is more profitable, to get the books from London or to buy them in Madras, and by how many rupees? [Current market rate, £ 1 = Rs 19].

6. Men, women and children are employed in a factory in the proportion of 1 : 2 : 3, and the wages of a man, and woman and a child are as 6 : 3 : 2. When the number of men employed is 50, the total weekly wages of all the hands amount to £ 225. Find the weekly wages paid to a man, woman and a child respectively.

7. Describe a quadrilateral $ABCD$ with the following data:— $AB=8·5$ cm., $BC=7·5$ cm., $AC=6$ cm., $CD=5$ cm., and the angle $D=90°$.

Find its area correct to two decimal places.

If it represents a field drawn to a scale of 1 cm. = 30 yards, find the area of the field in square yards, correct to one decimal place.

8. A person borrows a certain sum of money at $13\frac{1}{3}$ per cent. per annum simple interest, promising to pay the amount at the end of 4 years and 6 months. But the loan had to be extended for a period of 1 year and 6 months more. Thereupon the creditor demanded not only that the interest that had accumulated till then should be added to the principal but also the rate should be raised to 16 per cent. per annum for the period extended. On account of this the person had to pay Rs. 107-5-4 more than he would have paid had the sum continued at simple interest for the whole period at the original rate. Find the sum borrowed.

9. The following table gives the population in thousands, of two towns A and B at the beginning of each of the years specified :—

Year ...	1835	1845	1855	1865	1875	1880	1890
A ...	24·4	26	29·5	34	40	43	50
B ...	34	36	38·4	41·1	43	44·8	46·7

Draw graphs on the same diagram to represent the above, and estimate therefrom the population of each of the towns at the beginning of 1870. In which year was the population approximately the same in both the towns, and what was it?

10. An Indian merchant arranges to buy tweed in France at 4·4 francs per metre. The cost of getting the same to India averages 0·1927 francs per metre. If he sells it at Rs. 3 per yard, find his gain or loss per cent. [Re. 1 = 1·68 francs, 1 metre = 1·0935 yards].

11. The length of a park is to its breadth as 4 : 3, and its area is 77 acres, 88¾ square yards. Find the length of its diagonal in yards.

12. To walk one mile, A takes m minutes and B, n minutes.

1. What distance does A walk more than B in one hour?

2. Find in terms of m and n, what time A takes to walk 3 miles more than B.

3. Express to the nearest minute in what time A walks 3 miles more than B, when $m=16$, $n=21$.

1920.

I. The sea-borne trade of the Madras Presidency for the year 1918-19 shows that the value of foreign imports under 'Tobacco, raw and manufactured,' was Rx. 188,652 for 1918-19, and Rx. 113,000 for 1917-18, the average for the preceding four years up to 1916-17 inclusive being Rx. 49,394. Find (1) the increase per cent. in the value of the imports in 1918-19 over that in 1917-18, and (2) by how many rupees the value of the imports in 1918-19 was in excess of the average value for the preceding five years, i.e. up to 1917-18 inclusive. [1 Rx. = 10 Rs.]

II. The cross-section of the canal running across the Godavari at the Gannavaram Aqueduct is a trapezium (a quadrilateral with two of its sides parallel) 24 ft. at the top and 19 ft. at the bottom, the depth being 6 ft. If the slopes are of equal inclination, draw the cross-section to a scale of 1 cm. = 2 ft. Measure the angle at which the sides are inclined to the bottom of the canal.

Given that the length of the Aqueduct is 810 yards, find in cubic yards the maximum quantity of water that can be contained in the whole length of the Aqueduct. and also in how many minutes a boat 15 yards long and going down the canal at 3¾ miles an hour will pass the Aqueduct.

III. In the catalogue of a certain merchant the selling prices are given as shown in the accompanying table. He usually allows a discount of 5 per cent. on silks and 10 per cent. on checks. Owing to the war conditions he has raised his prices on all his goods by 20 per cent., but allows still the same rates of discount on the increased prices. A customer buys 6 yards of Tassur silk, 7 yards of Nugur silk, 8 yards of Turban silk, 14 yards of Calicut check, and 16 yards of Holland chek. What sum has he to pay the merchant?

Goods.	Price per yard.		
	Rs.	A.	P.
Tassur silk ...	4	11	0
Nugur silk ...	3	2	0
Turban silk ...	2	5	6
Calicut check ...	1	9	0
Holland check ...	0	12	6

IV. To find the height of a certain tower a surveyor fixes a pole vertically at a distance of 10 yards from it, the height of the pole above the ground being 15 feet. On going 5 yards from the pole and away from the tower, he sees the tops of the pole and of the tower in the same straight line with his eye. Find the height of the tower, given that the height of his eye is 5 feet from the ground.

V. A person bought two horses for different prices. By selling the first at a profit of 10 per cent. and the second at a profit of 15 per cent. he gained 13 per cent. of the total cost price. Had he sold the first at a profit of 15 per cent. and the second at a profit of 10 per cent., he would have got Rs. 6 less on the whole. Find the cost price of each of the horses.

VI. A road runs from P to S passing through Q and R. PQ is 10½ miles and QR, 7 miles. A starts from P at 6 a. m., and walks to S at a certain uniform rate, halting at Q for half an hour and at R for one hour. B starts from P at 7 a. m., on his cycle, his rate of cycling being 7 miles an hour. He passes A at 7-45 a. m., and goes on to R, where he stops one hour. He returns to Q, where business detains him for an hour and a half. He then goes straight to S, which he reaches at 4 p. m. Find the distance from P to S, A's rate of walking, and the time when he reaches S.

Represent the above graphically, and find from the graph when and where A and B will be together.

VII. Describe an acute-angled triangle ABC, given that $AB = 3·2$ in. $BC = 2·6$ in., and angle $A = 50°$. Measure AC and the angle C. Find the area of the triangle in square inches by any method.

If it represents a field drawn to a scale of 1 in = 1 furlong, find the area of the field in acres.

VIII. When performing a certain experiment in Science the temperature at the end of 10, 20, 30, 40, 50 and 60 seconds was found to be 15°, 30°, 43°, 56°, 58°, and 45° respectively, it being observed also that 60° was the maximum reached. Draw a smooth curve, showing the changes in the temperature, and find from it the temperature at the end of 25 seconds, and also the time when the temperature was 50°

IX. If m men can do $\frac{1}{n}$ of a piece of work in p days, find an expression for number of men required to do the whole work in q days.

Find the number when $m = 15$, $n = 4$, $p = 1\frac{1}{2}$, and $q = 20$.

X. When a certain weight was cast it was found to weigh 1·85 lb. more than was intended. Thereupon a cube, with an edge of 1·5 in. was scooped out from the bottom of it. But thereby the weight was diminished only by 1·08 lb. It is proposed to diminish the rest by scooping out a cone with a bottom circumference of 5½ in. What should be the depth of the cone? [$\pi = 3\frac{1}{7}$.]

XI. One litre of water weighs I kilogramme (kg.), and one litre of pure milk 1·032 kg. A person bought 12 litres of milk, and found it weighed only 12·288 kg. Find in cubic cm. the quantity of water in the milk. [1 litre = 1 cubic decimetre.]

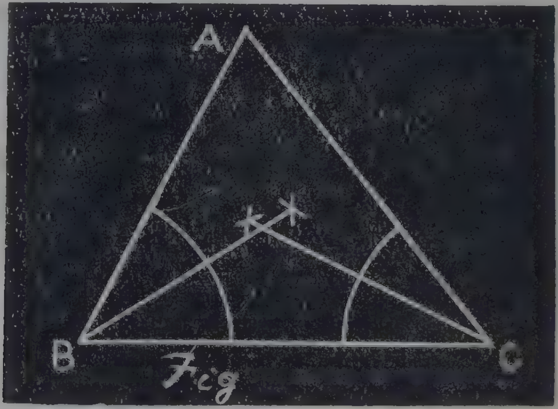
XII. A War Savings certificate costs 15*s.* 6*d.*, and is worth at the end of the fifth year £ 1. After that period 1*d.* is added to it every month, and at the end of the tenth year from the commencement, a bonus of 1*s.* is added to the total value of the Certificate. What rate of simple interest per annum does this work out for the whole period of ten years?

XIII. Forty-eight Englishmen employed in India saved each a sum of Rs. 2. 8 as. 4 ps. daily from March 1 to September 2. both inclusive, and with the total sum thus saved arranged to buy War Savings Certificates in England. How many certificates did they buy in all, and what total sum would they get in England by converting the Certificates into cash at the end of ten years after purchase? [Re. 1 = 1*s.*–4*d.*]

1921.

I. A metre is a ten millionth part of the arc of a meridian of the earth from the equator to the pole. The earth's semicircumference measured along a meridian is 10,800 nautical miles. Find the number of feet in a nautical mile, If 1 metre = 39·370 inches.

II. How far is the following statement correct?—Three measurements are necessary and sufficient to prepare an exact copy of a given triangle.



Measure the side correct to the nearest tenth of an inch, and the angles correct to the nearest degree, of the given triangle ABC.

Construct a triangle PQR, of the same shape as ABC, but of perimeter 9·6″.

N.B.—The data required for the construction of the triangle PQR should all be obtained geometrically.

III. AB is a diameter of a circle, whose centre is C. The circle is folded so that the points A, B, are brought to the centre C, thus causing two creases. Show that the points A, B, and the four ends of the two creases are the corners of a regular hexagon.

Calculate the area of the hexagon to the nearest hundredth of a square inch, if the radius of the circle is exactly 2·4 inches.

Check by drawing an accurate figure.

IV. Here is an extract from a statement of Educational Progress in the Madras Presidency for three years:—

	1914–15	1915 16	1916 17
Total population.	41,404,405	No change.	No change.
Total scholars in all institutions.	1,542,955	1615,159	1661,012
Grand Total of expenditure in thousands of rupees.	19,353	20,652	21,688

Find from this statement.—

(1) The ratio of the number of scholars to the total population for 1915–16, expressed as a percentage.

(2) The increase per cent in the number of scholars in 1916-17 over that of 1914-15.

(3) The increase in the average expenditure per scholar for 1916-17 over that for 1914 15. Answer to the nearest anna.

V. When sovereigns were selling at Rs. 18-12-0 a gentleman purchased a watch-chain made of 12 sovereigns (including a wastage of $6\frac{1}{4}$ per cent). The making charges were 14 as. per sovereign. A few days later he had to sell it, when the price of the sovereign fell to Rs. 16 4 0. The purchaser would take the chain at its net weight only and would pay only 75 per cent of the making charges per sovereign on the net weight of the chain. What did the gentleman lose by the transaction?

VI. A High School Co-operative Society was supplied by a Bombay firm with Mathematical books at a discount of $33\frac{1}{3}$ per cent, and History books at a discount of 40 per cent. of the published prices. Subsequently it was found that the society would have paid $2\frac{1}{2}$ per cent. of the published prices less, if all the books had been purchased from a Madras bookseller, who offered a uniform discount of $37\frac{1}{2}$ per cent. What percentage of the purchase, calculated at the published prices, was of Mathematical books?

VII. The Secretary of a Chit Fund was authorized to buy a cash box. Every year of use was supposed to decrease the value of the box by $12\frac{1}{2}$ per cent of what it was worth at the beginning of that year. At the end of two years, the Fund was dissolved, and the box was sold by auction for half its original value. By how much per cent was this less than its value, as calculated at the above rate of depreciation?

VIII. At the time of the German reverse (1914), a body of Germans began their retreat at the rate of 24 Kilometeres a day from a place 36 Kilometres distant from the Allies' Battle Front. The Allies began their pursuit a day later, at the rate of 32 Kilometres per day. After three days' marching they were delayed in their pursuit for a day to construct a bridge across a river; they then continued the march for three days more and were again delayed for half a day to repair another bridge blown up by the enemy. After how many days from the beginning of the retreat were the Germans overtaken?

Draw a graph to illustrate the movements of the armies.

Find from the graph,—

(1) How far the armies were apart seven days after the beginning of the retreat.

(2) The time when the armies were 32 kilometres from one another.

IX. A circus tent is in the from of a cylinder surmounted by a cone. The cylindrical portion of the tent is of diameter 96 feet, the height of the top of the tent is 36 feet from the ground, and the height of the cylindrical part 16 feet. Find the total area of the canvas required for the tent. Answer to the nearest square foot ($\pi=3{\cdot}1416$).

X. The alloy of which the rupee is made consists of silver and copper melted together in the ratio of $r:1$. The alloy is worth a Rs. per ℔. and pure silver s Rs. per ℔. Express the value of copper per ℔. in terms of a, s and r. What does copper cost per ℔, if $a=30\frac{3}{4}$, $s=40$, and $r=11$?

XI. For an entertainment to be given to the Principal of a College on the eve of his transfer, the students of the B.A. classes agreed to raise among them a certain sum, levying a uniform rate of subscriptions. The intermediate students joining them subsequently, the subscription per student was reduced in the ratio of $7:4$, although the estimated expenses were increased by 50 per cent. Compare the strength of the B.A. and Intermediate classes.

From your result find the strength of the College, given that the number of students in the Intermediate classes was 182.

XII. The following tables give a series of soundings (measures of depth) taken across a river 42 yards broad at a ford:—

Distance in yards in a bank.	0	5	8	10	14	16	19	23	26	30	32	35	38	42
Corresponding depth in feet.	0	2.8	4.4	5.0	7.2	8.0	10.2	9.8	8.4	7.3	6.0	4.8	3.1	0

Draw a diagram to represent a section of the river at the ford. From it find—

(1) The probable depth in the middle of the stream.

(2) How far a man who will be safe up to a depth of 5 ft. 2 in. can venture into the river from either bank, without danger.

1922.

I. Here is an extract from Indian Railway Statistics:—

Year.	Miles of State Railways.	Gross earning in Rupees.
1903	21,131	36,00,82,035
1912	25,125	61,65,07,005

Calculate—

(1) The earning capacity of the Railways per mile in the two cases, correct to the nearest rupee.

(2) The percentage of increase in that capacity for the period 1903-12, correct to the first decimal place.

II. Two persons standing at P and Q 300 yards apart on a straight seashore, wish to find the distance of a boat B from the shore. They observe the angles BPQ and BQP to be 50° and 70 respectively. Obtain the distance of the boat from the shore.

III. Assuming that 15 centimetres = 5·9 inches, draw a graph to convert inches into centimetres and *vice versa*. From the graph, get (1) the number of inches (to the nearest tenth) (i) 4 cm., (ii) 10 cm.; and (2) the number of centimetres (to the nearest mm.) in (iii) 2·8 inches, (iv) 4·5 inches.

IV. The following is one of the rules of a Sangeetha Sabha:—

Each member shall pay Rs. 1-4-0 every time he attends a performance, *or* pay an advance annual subscription of Rs. 15 and also 10 annas at each performance.—performances being weekly).

A member X elects the first method of payment, and another member Y the second method. Both attend the performances regularly After how many performances does Y gain an advantage over X?

Verify the solution of the question by drawing two graphs (in the same figure) to show the amounts paid by X and Y.

V. In connexion with a street improvement scheme, a Municipal Corporation purchased the whole block of houses on one side of the street and the ground (708 feet by 108 feet) on which they stood. The houses alone cost Rs. 2,54,000, and the land cost Rs. 45 per *kuli* (144 square feet). The houses were pulled down and the materials auctioned away for Rs. 95,000. The street (708 feet long) was then widened by 12 feet, and the remaining land was sold at Rs. 185 per *kuli*. Find the net cost of the improvement scheme.

VI. From a rectangular sheet of brass 32 inches long and 20 inches broad, a circular piece of diameter 20 inches is cut out, and the remaining part is melted and beaten into a rectangular piece of the same shape as the original sheet and of the same thickness. Find its length and breadth to the nearest inch. [$\pi = 3·1416$].

VII. Pure gold is said to be 24 carats fine. Mint gold contains 22 parts of pure gold to every 2 parts of alloy and is hence said to be 22 carats fine. The gold nibs of Waterman Fountain Pens are 14 carats fine. To make such nibs, 89 ounces of pure gold and 48 ounces of mint gold are melted together with a certain quantity of alloy. Find this quantity of alloy.

VIII. At a 'Grand Clearance Sale," a firm say that they have reduced their catalogue prices 20%, and also allow their customers 5% off their bills for immediate cash payment. If their profits even after such reductions on cash sales come to 14%, with what rate of profit in view were the original catalogue prices fixed.

XI. I purchased a bungalow two years back for Rs. 50,000, and spent 10% of this sum to effect certain alterations and repairs. The bungalow has remained unoccupied till now, and during this interval I reckon 5% per annum, compound interest, on my outlay. If I want to realize a profit of 20% on the total amount, at what price should I sell the bungalow now?

X. The owner of a lead mine collected 216·15 tons of stuff from which lead was to be extracted. It contained $x\%$ of lead. After washing, in which process no lead was lost the stuff was found to contain $y\%$ of lead. Show that $\frac{y-x}{y}$ 216·15 tons was washed away.

Find the quantity of lead obtained from the stuff, if $x=15{\cdot}9$ $y=87{\cdot}45$.

XI. Draw a circle of 3 inches radius, and divide it into six sectors whose areas are proportional to the six continents given in the adjoining tabular statement:—

Name of Continents.	Areas in square miles to the nearest tenth of a million.
Asia ...	17·0
Europe ...	3·8
Africa ...	11·4
Australia ...	3·0
North America.	8·0
South America.	6·8

(1) Indicate in your figure the values of the angles of the sectors, to the nearest degree.

(2) Enter neatly the names of the continents in the corresponding sectors.

XII. Two cylindrical rollers of radii a and b feet lie side by side on a level ground, touching each other along their lengths. Express in terms of a and b the distance between the lines along which the rollers touch the ground. Find the distance, if $a=2{\cdot}25$, $b=1.69$.

1923.

1. The following statistics are taken from the Report on Public Instruction in Madras for 1920–21 :—

Year		Grants from Provincial funds to		
		Schools under private management. RS.	Local board schools. RS.	Municipal schools. RS.
1919—20	...	40,54,293	33,39,703	6,53,466
1920—21	...	42,96,756	34,61,324	4,31,880

Find—(1) the grand total of grants-in-aid in 1919—20 and 1920—21 ;

(2) the increase per cent. in grants for 1920—21 as compared with those for 1919–20 for Local Board schools ;

(3) what percentage the grants in 1920—21 for private institutions was of the total grant for the same year.

II. (1) If a piece of iron plate 2·1 metres long, 1·13 metres broad and 2·3 cms. thick weighs 425 Kilogrammes, find, to the nearest gramme, the weight of a square metre of sheet iron 2 mms. thick.

(2) Two cyclists start abreast at 3 o'clock to race on a circular track. If one goes round in 3 minutes 12 seconds and the other in 3 minutes 30 seconds, find at what time they will again be riding abreast past the starting point. Find also the number of complete circuits they will then have made.

III. A tradesman marks certain goods 30 per cent. above cost price ; he allows cash customers a discount of 12 per cent. and credit customers a discount of 5 per cent. ; five-eighths of his goods thus marked are sold for cash and the rest on credit. When the whole stock has been sold and paid for, what is his percentage of profit ?

IV. The Boy Scouts of a High School have a Savings Bank of their own and one of them had a balance of Rs. 6 at the bank on the first January 1922. At the beginning of February and of each alternate month following he deposited Rs. 3. In the middle of April when the school closed, he withdrew Rs. 5 for his journey. If the interest be allowed at $6\frac{1}{4}$ par cent. per annum, what interest should be added to his account at the end of June 1922, each month being calculated as one-twelfth of a year ?

V. A Railway contractor purchased 360 tons of cement at Rs. 95 per ton two years before it was actually required and in conveying the cement from Madras to the nearest Railway Station, paid a railway freight of Rs. 9–14—0 on each ton. The unloading and conveying charges amounted to 4 annas for a bag of cement (10 bags of cement weighed one ton). On the above outlay he reckoned 5 per cent. compound interest per annum. During the two years the cement lay idle he had to stock it in a building paying rent of Rs. 25 per mensem. When after two years the cement was wanted he charged the Railway a higher rate and thus realized a profit of 60 per cent. on the total amount spent. Find to the nearest rupee the rate he charged for a ton.

VI. A contractor employs 20 men working 8 hours a day in order to finish building a house in 25 days. At the end of 14 days only one-third of the work is finished. He engages more men and all his men now work 9 hours a day. What is the least number of additional men that he must engage if the work is to be done by the specified time?

VII. Two boys *A* and *B* had their heights measured on the first days of 6 consecutive years and the measurements are given below:—

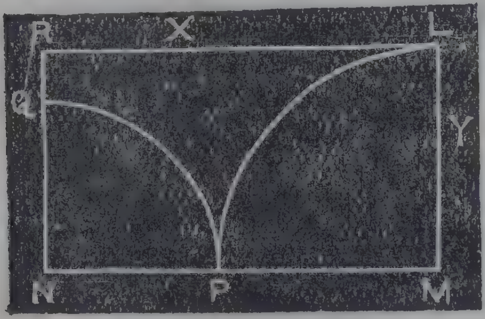
——	1916	1917	1918	1919	1920	1921
A	FT. IN. 3 0	FT. IN. 3 4½	FT. IN. 3 7¾	FT. IN. 3 10¾	FT. IN. 4 1	FT. IN. 4 2¾
B	3 2	3 3½	3 5¾	3 8½	3 11¼	4 4

Draw, on one diagram, graphs to illustrate their growth, and find the dates when they were of the same height

VIII. If the speed of an aeroplane is v miles per hour, find its average speed from *A* to *B* and back, if the wind is blowing all the time from *A* to *B* at u miles an hour.

Find the average speed of the aeroplane if $v=60$ and $u=5$.

IX. I stand on the road six feet from a lamp-post and notice that the end of my shadow just reaches a point at the foot of a wall. Find, by drawing to scale (1 cm. to represent 1 foot), the distance of this point of the wall from the lamp-post, if the light is 10 feet above the ground, and my height is 5 feet 6 inches. Find this distance also by calculation.



X. In the accompanying figure *LMNR* is a rectangle; a quadrant of a circle is drawn with *M* as centre and *ML* as radius, meeting *NM* at *P*; another quadrant with *N* as centre and *NP* as radius meets *RN* at *Q*, *P* being between *N* and *M* and *Q* being between *R* and *N*. If the curved path *LPQ* from *L* to *Q* is 20 feet less than the path *LMNQ* and 10 feet less than the path *LRQ*, find the sides of the rectangle ($\pi=3{\cdot}1416$).

XI. Draw a right-angled triangle with equal sides and with hypotenuse equal to 2·3 inches. On the other side of the hypotenuse construct a square.

Compare the area of the triangle with that of the square. If the figure thus drawn represents the end view of a hay stack whose breadth is b feet and length l feet, what is the volume of the stock in cubic feet?

Calculate the volume in cubic feet when $b=20$ and $l=50$.

* XII. A boy fills with moist sand a cylindrical bucket which measures internally 12 inches in diameter and 16 inches in height. He empties the bucket, and builds a conical mound with the sand on a base whose diameter is 18 inches. Find the height and volume of the mound and the area of its curved surface.

1924.

I. The following statistics taken from the census of 1921 give the population, in thousands, of Hindus, Muhammadans and Christians in six districts:—

—	Population in thousands.			
	Hindus.	Muhammadans.	Christians.	Total.
Guntur (M) ...	1,514	131	153	
North Arcot ...	1,893	118	36	
Tanjore (M) ...	2,105	130	89	
Trichinopoly (M) ...	1,746	63	92	
Ramnad ...	1,524	116	85	
Tinnevelly ...	1,600	109	192	

(1) Find the total population for each district (horizontally) and for each community (vertically).

(2) Express the Muhammadan population in each of the districts marked (M) as a percentage of the total population of that district correct to two significant figures, and state which of them has the highest percentage.

II. (1) Find, in pounds and ounces (correct to the nearest tenth of an ounce), the weight of a cubic foot of water, given that 1 metre = 39·371 inches 1 kilogram = 2·2046 lb. and 1 cubic metre of water weighs 1,000 kilograms.

(2) A railway company increased its railwey fare by one-half and again increased it by ¾ of its new value. Find the original cost, to the nearest anna, of a ticket which after the two increases costs Rs. 19—4—0.

III. (a) The capital of a Co-operative Publishing House is fifteen thousand rupees; one-third of it is borrowed from a Bank at 6 per cent, and the remainder is held in shares. The working expenses of the House are 40 per cent. of the gross receipts in a year. Find what the gross receipts must be in order to pay the shareholders 9 per cent. at the end of the year.

(b) Three vessels, each capable of holding 32 gallons contain mixtures of wine and water. In the first, which is half full, the quantities of wine and water are equal; in the second which is three-quarters full, there are 9 gallons of wine; and in the third, which is full, there are only 4 gallons of wine. If the second is now filled up from the third, and then the first filled from the second, how many gallons of wine and water will be found in the first vessel?

IV. For a certain football tournament twelve teams (each consisting of six players) entered, paying an entrance fee of Rs. 10 each. During each of the first four days of the tournament the spectators who numbered 500 on an average were charged 2 annas each, and on the fifth day, when the finals were played, the attendance was half as much again as on any previous day and tickets were sold at 4 annas each. The committee in charge spend 38 per cent. of the total income in fencing the ground, 25 per cent. of the remainder for ground-rent and Rs. 30-7-0 on miscellaneous charges. They further set apart Rs. 200 for Reserve Fund after purchasing a silver cup for the winning team and six silver medals for the runners-up, the cost of each medal being one twelfth of that of the cup. Determine the cost of the cup and of a medal.

V. A dealer offers to sell a motor-car for Rs. 5,000 cash or for twelve monthly instalments of Rs. 500 each, the first being payable at once. Calculate the rate of simple interest charged on the supposition that the Rs. 5000 invested at that rate for 11 months gives the same amount as the total of the amounts of the instalments invested at the same rate.

VI. Among the elected members in a certain Legislative Council, the ratio of the ministers' party to the rest was 5 : 2. Eighteen members left the ministerial party and joined the opposition and consequently the ratio was reduced to 26 : 23. Find the total number of elected members.

VII. A man standing 12 yards from a river bank observes a post on the further bank directly opposite to him. He walks 30 yards parallel to the bank and then walks directly towards the post, reaching the bank after walking 15 yards. Find, by drawing to scale (1 cm. to represent 3 yards), the breadth of the river, assuming the banks to be straight. Find this distance also by calculation.

VIII. A surveyor gives the following measurements of a quadrilateral field $ABCD$ (having no re-entrant angle): $AB = 1{,}980$ links, $BC = 920$ links, $AD = 1{,}700$ links, $\angle BAC = 35^\circ$, $\angle BAD = 81^\circ$, C and D being on the same side of AB. Draw a plan of the field to a scale of 1 inch for 400 links stating the steps of your construction. Find the area of the field in acres given 100 links = 22 yards.

IX. The following table gives the times of sunrise and sunset at a certain place from July to December 1923:—

(1) Plot graphs (in two different diagrams) showing for the given period (a) the time of sunrise, (b) the duration of sunshine, on successive days.

(2) Deduce from the graphs the time of sunrise on November 22, and the duration of sunshine on August 8.

Date.	Sunrise.	Sunset.
July 1.	H. 5 M. 47	H. 6 M. 39
July 15.	H. 5 M. 50	6 39
Aug. 1.	H. 5 M. 54	6 36
Aug. 15.	H. 5 M. 56	6 31
Sep. 1.	H. 5 M. 58	6 21
Sep. 15.	H. 5 M. 58	6 12
Oct. 1.	H. 5 M. 58	6 0
Oct. 15.	H. 5 M. 59	5 51
Nov. 1.	H. 6 M. 2	5 44
Nov. 15.	H. 6 M. 8	5 39
Dec. 1.	H. 6 M. 16	5 40
Dec. 15.	H. 6 M. 23	5 45
Dec. 31.	H. 6 M. 31	5 53

X. A shop-keeper uses a badly made balance so that a body really weighs $a+b\,Q$ ℔. when the balance shows only Q ℔., a and b being numbers which are the same for all weight. If the balance shows 1 ℔. for a real weight of 1·1 ℔. and 2 ℔. for a real weight of 1·9 ℔., what are the values of a and b and what will the balance show for a real weight of 2 ℔.? Draw a graph to show the relation between the real weights and the weights shown by the balance.

XI. A canal whose cross-section is a trapezium is 8 feet deep, 18 feet wide at the surface and 12 feet wide at the bottom. Find the area of the cross-section and the number of gallons of water that pass any spot in an hour when the canal is full and water flows at 2 miles per hour. [Given one cubic foot of water = 6·25 gallons.]

XII. A right circular cylindrical log of teak is 12 feet long and of diameter 14 inches. Find its volume and curved surface.

If it is planed down to the least amount necessary to form a prism whose cross-section is a regular hexagon, what would be the volume of the prism in cubic feet, given $\pi=\frac{22}{7}$ and $\sqrt{3}=1{\cdot}73$.

What percentage (correct to an integer) of the whole log would be wasted in planing down?

1925.

I. The following table shows the population, in thousands, of five Indian provinces according to the census taken in the years 1911 and 1921 :—

	Population in thousands.	
	1911	1921
Bengal	45,483	46,696
Madras	41,405	42,319
Bombay	19,626	19,348
United Provinces	46,807	45,376
Punjab	19,579	20,685

Find, correct to one decimal place :—

(1) the percentage of increase or decrease of the population in each of the first three provinces.

(2) the percentage of increase in all the five provinces taken together.

II. Three men, *A*, *B* and *C* have together completed a piece of work for which they receive Rs. 64-14-0. *A* has worked for 4 days of 10½ hours, *B* for 11 days of 7 hours, and *C* for 6 days of 9 hours. How much money ought each to receive?

III. A goldsmith undertook to make a jewel casket out of certain quantities of gold and silver, together weighing 104 ounces and costing £ 230-4-0. Instead of using the quantities of gold and silver agreed upon, he interchanges them. If gold costs £ 3-17-6 and silver 5s. 6d. per ounce, find by how much the value of the casket has been reduced.

IV. A clerk's salary was £ 80 for each of the first three years. Then it was raised at the rate of £ 10 a year for 12 years, after which it remained stationery. When he retired from work he found that his average salary per year had been £ 174-10-0. How many years did he work?

V. Exactly three years ago a man borrowed Rs. 1,890 from a bank at 6¼ per cent. per annum. At the end of the year he paid the interest for that year and a part of the loan, amounting altogether to Rs. 500. Similarly he paid Rs 700 at the end of the second year. What sum must he now pay to clear off the debt? (Answer to the nearest anna).

VI. Three cyclists, *A*, *B* and *C* ride round a circular track at 8, 9 and 10 miles per hour respectively. They start together from the same point, but *C* rides in an opposite direction to *A* and *B*, and meets A one minute after passing *B*. What is the length of the track?

VII. If the manufacturer of a machine sells 36 machines to a wholesale dealer at 25 per cent. above cost price, and gives one extra machine free; and if the wholesale dealer sells each machine to a shopkeeper at 5 per cent. more than what it cost him, taking into account the machine given free; and if the shopkeeper sells it to a customer for £ 34-13-0, making a profit of 10 per cent. what did it cost to make?

VIII. A rectangular metal plate has a circular hole to it. The length of the plate is 1·3 metres, the width is 80 centimetres and the thickness is 2 millimetres. If the plate weighs 32·25 lbs. find the diameter of the whole correct to a centimetre, given 1 c. cm. of the metal weighs 8 grams and 1 lb. = 453·0 grams.

IX. The following table shows the number of grams of alum which can be dissolved in 100 grams of water at different temperatures.

Temperature in degrees C.	0°	10°	20°	30°	40°	50°	60°	70°
Grams soluble in 100 grams of water.	3·9	9·5	15.1	22·0	30·9	44·1	66·6	90·7

Draw a graph and find at what temperature 50 grams of alum could be dissolved in 100 grams of water.

X. A straight rod *AB*, 2 feet 8 inches long, is held under water, *A* being 2 feet 6 inches and *B* 1 foot 2 inches below the surface. Find, by drawing to scale, the distance below the surface of a point *C* on the rod which divides *AB* so that *AB* = [illegible] *BC*. Find also the angle which the rod makes with the surface of the water.

XI. The cost of making a certain machine consists of an initial expense plus a fixed sum for each machine. To make 25 machines costs £ 885; to make 40 machines costs £ 1335. Find the initial expenses and the fixed sum.

XII. *ABCD* is a quadrilateral field in which the diagonals *AC* and *BD* are at right angles. *AB*, *BC*, *CD* and *AC* are 600, 700, 800 and 1,000 links respectively. Draw the quadrilateral to scale and hence find the area of the field in acres. (100 links = 22 yards).

XIII. A vessel in the form of a hollow right circular cone, whose height is 1 foot 6 inches and radius of base 6 inches, is placed with its axis vertical and vertex downwards. Find how many gallons of water it will hold (1 cubic foot = 6·24 gallons). Find also the cost of tinning the inner surface of the vessel at 4 annas per square foot.

1926.

PART A.

I. The following table shows the values in lakhs of rupees of exports of merchandise from the Madras Presidency to some of the foreign countries during the years 1920-21, 1921-22 and 1922-23:—

		1920-21.	**1921-22.**	**1922-23.**
		LAKHS.	LAKHS.	LAKHS.
(1)	United Kingdom	839·77	613·96	824·11
(2)	Other countries in the British Empire	667·37	757·54	743·34
(3)	France	151·06	426·14	519·67
(4)	United States	92·59	105·14	150·08
(5)	Belgium	137·97	108·08	121·24
(6)	Germany	36·55	93·82	118·36

Find—

(1) the total value of the exports from Madras to all the countries during all the three years taken together;

(2) the percentage of increase or decrease between 1920–21 and 1922–23 in case of each of the countries (1), (5) and (6).

II. A person going from one place to another travels 90 miles by steamer, 330 miles by rail and 30 miles by horse. If the journey occupies 30 hours 50 minutes and the rate of the train is three times that of the horse and one and a half times that of the steamer, find the rate of the train.

III. The external length, breadth and depth of a rectangular closed metal vessel are 14, 10 and 9 inches respectively, and the thickness of the metal half an inch. When the vessel is empty it weighs 1,500 ounces and when filled with water 2,041·6 ounces. Find the weight of a cubic foot of water.

IV. Each railway passenger is allowed to take a certain weight of luggage free, and is charged for excess over that weight. Three passengers had to pay Rs. 1½, Rs. 3 and Rs. 4 respectively for 70 ℔s., 100 ℔s. and 120 ℔s. Find graphically (*a*) what a passenger should pay for 145 ℔s, (*b*) how much can be taken for Rs. 6, and (*c*) what amount of luggage is allowed free for passenger.

V. ABCD is a quadrilateral field such that AB = 520 links, BC = 390 links, CD = 250 links, DA = 600 links, and the angle ABC = 90°. Draw the quadrilateral to scale and state what the distance AC is and what the angle CDA is. Find also the area of the field in acres correct to the second decimal place.

VI. A rectangular sheet of drawing paper measures 15 cm. by 25 cm. It is rolled into a cylinder 15 cm. high, the two edges which meet being stuck together by stamp paper. How many c. cm. of sand will the cylinder hold if it stood upright on a table?

PART B.

I. A and B became partners in a business and supplied £ 15,000 and £ 4000 capital respectively. There was a profit during the year of £ 1,975 of which B as manager received 30 per cent and A and B received the same greatest possible integral rate per cent on their capitals which could be paid by the remainder of the profit. How much did each receive and what sum was left over?

II. When exchange was at 1s. 8d. per rupee a Madras merchant imported cloth for £ 500 at 6s. 8d. per yard. The cost of importation was 5d. per yard, and the cost of landing and other charges 4 annas per yard. One-fourth of the cloth, being damaged, was sold at a loss of 25 per cent on its total cost to the merchant. At what price per yard did he sell the remainder if he gained 20 per cent on the total outlay?

III. A man had two creditors, his debt to the one being double his debt to the other. After paying his larger creditor 4s. in the £ and the other creditor in full, he had £ 10 left If he had divided all his estate fairly between them each would have got 10s. in the £. What was the value of the estate and how much did he owe each of his creditors?

IV. A bath can be filled by the cold water pipe in 9 minutes and by the hot water pipe in 11¼ minutes. A person leaves the bath room after turning on both pipes simultaneously and returns at the moment when the bath should be full. Finding, however, that the waste pipe has been open he now closes it. In 3¾ minutes after this the bath is full. In what time would the waste pipe empty it?

V. At a certain election the number of votes received by A was to that received by B as 19 : 16. At a subsequent contest between the same candidates the entire number of votes polled increased by 450, and the excess of A's votes over B's was 270 more than in the former contest. If at the second election the number of votes received by A was $\frac{14}{11}$ of that received by B, find the number of votes received by each candidate at each election.

VI. A man buys a house on condition that he should pay £ 625 now, £ 625 at the end of the first year and £ 625 at the end of the second year. Calculating compound interest at 5½ per cent per annum what value to the nearest shilling has been put on the house?

1927.

A

1. The population of a city is 155,625. There are 1,075 females to every 1,000 males. Among the males 12 per cent. are literate and among the females 4 per cent. are literate. Find what per cent. of the total population are literates.

2. At the last Secondary School-Leaving Certificate Public Examination 'a' candidates answered the English papers and the average per cent. of marks that they obtained in that subject was 'x'. Of these 'b' candidates were from the Andhra University area and the average that they obtained in that subject was 'y' per cent. Find z, the average per cent. obtained by the others.

If $a = 15{,}345$, $b = 7{,}375$, $x = 34$ and $y = 35$, find z.

3. If the difference between the simple and compound interest on a certain sum for 3 years at 5 per cent. is Rs. 7-10-0, find the sum.

4. In a club there are 200 members who pay a total annual subscription of Rs. 4,400. Of these some are active members who pay an annual subscription of Rs. 25 each, and the others are associate members who pay an annual subscription of Rs. 15 each. Find how many members there are in each class.

5. A, left Madras at 4 A.M. on a certain day and began walking westwards at 3 miles an hour. B left the same place at 6 A.M. on the same day cycling in the same direction at 6 miles an hour. Find graphically when and where B will overtake A.

6. Construct a quadrilateral ABCD of which two sides AB and BC 1·1 and 2·4 inches long respectively contain an angle of 130°, and the other two sides CD and DA are of lengths 2·5 and 3 inches respectively. Find the area of the figure.

B

7. A and B can do a piece of work in 15 days, while B and C do the same work in 18 days and C and A in 20 days. They are paid according to the amount of work that each of them turns out in a day. If C's daily wages are 14 annas, find how much A and how much B receives in a day.

8. A merchant mixes two kinds of coffee, which cost him Rs. 2 a viss and Rs. 2-12-0 a viss respectively, and sells the mixture at Rs. 3 a viss, thereby gaining 20 per cent. on his outlay. Find the proportion in which the two kinds have been mixed.

9. A field 4 acres in area has to be irrigated to a uniform depth of an inch. An 8 inch diameter pipe running full with a velocity of $2\frac{1}{2}$ feet per second carries water to the field from a channel. How long will it take to irrigate the field?

10. A man performs his journey partly by rail and partly by river the former at 3 pies per mile and the latter at 6 pies per mile; the fare amounts to Rs 3-12-0. But if the rate of the former be reduced by $\frac{1}{2}$ pie per mile and the latter increased by one pie per mile the fare is increased by 5 annas. Find the distances travelled by rail and river respectively.

11. A man has two plots of ground, one a square whose side is 3,564 yards long, and the other a rectangle 1,628 yards long and 891 yards broad. He exchanges them for a single square plot of the same area. Find the length of a side of the new plot and the cost of fencing it all round at 2 annas per yard.

12. A man 6 feet high standing due east of a street electric light and at a distances of 54 feet from it observes that his shadow is 18 feet long. Find from a plan drawn to scale the height of the light from the ground and verify your result by calculation. How much further east must he go so that his shadow may be 24 feet long?

1928.

A

1. Boys and girls of school-going age form 28 per cent. of the population of a taluk which is 355,000. There are 72 girls of the school going age for every 68 boys of that age. Of the boys 55 per cent and of the girls 35 per cent. are at school. Find what per cent. of the total population are at school.

2. A hostel consists of two blocks of buildings. There are *a* rooms in the western block of which two rooms are used as kitchen and dining rooms, the other rooms being occupied by *b* students. In the eastern block there are *c* rooms of which one room is used as a general reading room, the other rooms being occupied by *d* students. Each room in the western block is let at Rs 3 and each room in the eastern block, at Rs 4. If Rs. *e* is the average rent paid by a student in the hostel, connect *a*, *b*, *c* and *d* with *e* by an equation. From the equation find the value of *e*, if $a = 15$, $b = 18$, $c = 16$, $d = 15$.

3. What sum lent out at compound interest for 3 years at 5 per cent. will amount to Rs. 1,736-7-0?

4. A man performs a journey of 212 miles partly by river and partly by rail, the former at 2 annas 6 pies per mile and the latter at 1 anna 2 pies per mile. The total fare amounted to Rs. 21-2-0. Find the distance travelled by river.

5. On a base of 1·5 inches describe a regular hexagon and determine its area.

B

6. A certain piece of work can be done by 40 men in 60 days. Find the difference in cost if 10 men and 30 boys were employed instead of 40 men, if 3 boys can do as much as 2 men, and the wages of a man is 10 annas and of a boy 5 annas a day.

7. The length of a rectangular field is three times its breadth. The area of the field is 36·3 acres. Find the cost of fencing it all round at 2 annas a yard.

8. A merchant bought in Tinnevelly 160 bags of rice at Rs. 10 a bag of 140 lb. and sent them by rail to Chidambaram, a distance o 320 miles at 6 pies per mile per ton. During the transit 8 lb. of rice were stolen from each bag. He sold the remainder at 3 measures per rupee. Find his profit per cent. (1 measure = $3\frac{1}{7}$ lb.)

9. There is a huge cylindrical reservoir where water is stored for 7,700 persons for 3 days at 6 gallons per head per day. Find the height of water in the reservoir if its diameter is 42 feet. 1 cubic foot of water = $6\frac{1}{4}$ gallons.

10. A cyclist left A at 4 a.m and rode towards B a distance of 36 miles at the rate of 8 miles an hour. After riding for 2 hours he spent half an hour to do certain repairs and afterwards was able to ride at 5 miles an hour. Find graphically when he would reach B. Just as he reached B, his friend who had left A for the same place and was riding uniformly at 9 miles an hour also reached the place. Find graphically when the latter left A.

APPENDIX A.

Artifices and short cuts in Arithmetical operations.

> "*Quick and sure must be the motto of a Mathematician.*"
>
> DE MORGAN.

1. In order that one may be quick and sure in Mathematics one should have recourse to the shortest and consequently the easiest method of work. With a certain amount of practice one can easily acquire a knowledge of the art of shortening work in Mathematics. No treatment of the subject can be exhaustive. What is attempted here is only a synopsis. Much depends upon the student himself who, in applying the methods treated in this Chapter, would be led to discover for himself other principles which may at first sight seem to have been neglected.

2. *The Art of Computing.*—The student should acquire quickness in computation, which requires constant practice. The following set of exercises practised regularly would lead to the formation of good habits of computation.

(a) Count one, two, three, etc., as fast as you can speak, up to 100 and then go backwards; then count forward by two at a time, etc., up to 16 at a time, always going backwards also.

(b) Take a number, say 53; find out the number which must be added to it to get at the next number which ends in a given digit or 1 (say).—This form of exercise will be of much use in the Italian method of Subtraction.

(c) Learn the Multiplication Table up to 16 times 16 wording only the product.

(We always advise boys to learn the Multiplication Tables of fractions—வீசம், அரைக்கால், மூவீசம், கால், அரை, முக்கால் வாய்ப்பாடு.)

3. Addition.—(a) Accustom yourself to add numbers vertically as well as horizontally; to ensure accuracy perform the process twice. If you add vertically, add once from bottom to top and once from top to bottom. If horizontally, add once from right to left and then from left to right.

(b) You must be sparing in wording arithmetical operations. If you want to add 1987, 4568, and 3254, proceed thus:

$$\left.\begin{array}{llll}4 & 12, & 1 & 9\\ 6, & 12, & 2 & 0\\ 4, & 9, & 1 & 8\\ 4, & 8, & & 9\end{array}\right\}\quad \text{The total is 9809.}$$

4. *Subtraction* —The easiest method of subtraction is the method of complementary additions.

Subtract 79854 from 94673

$$\left.\begin{array}{rlllll} & 4 \text{ and } \mathbf{9} & \text{make} & 13 & \text{carry} & 1 \\ 1+5= & 6 \text{ and } \mathbf{1} & ,, & 7 & ,, & \text{nil} \\ & 8 \text{ and } \mathbf{8} & ,, & 16 & ,, & 1 \\ 1+9= & 10 \text{ and } \mathbf{4} & ,, & 14 & ,, & 1 \\ 1+7= & 8 \text{ and } \mathbf{1} & ,, & 9 & & \end{array}\right\} \ \mathbf{14819}\text{ Ans.}$$

In the course of working, the accentuation of 9 should be simultaneous with its being set down ; so also for 1, 8, 4 and 1.

5. Arithmetical Complements.—The arithmetical complement of a number is the number which added to the given number gives just the next power of ten. Thus the arithmetical complement of 537 is 463. The student will do well to practise finding the arithmetical complements of numbers chosen at random, always applying the method of subtraction in 4 *supra.*

6. Addition and subtraction combined.—Add together 123, 567, 408 and subtract the result from 5327.

The working should be done thus. Be sparing in wording.

$$\left\{\begin{array}{llrl} 8 & 15 & 18+\mathbf{9}=27 & \text{carry } 2 \\ 2 & 8 & 10+\mathbf{2}=12 & \quad ,, \quad 1 \\ 5, & 10 & 11+\mathbf{2}=13 & \quad ,, \quad 1 \\ & & 1+\mathbf{4}=5 & \\ & & & \\ & & \mathbf{4229}\text{ Ans.} & \end{array}\right.$$

7. Multiplication.—*Method of Arithmetical Complements—Multiplication and Subtraction Combined.*

Multiply 356789 by 3995. 3995 is (4000 − 5). Multiply the given number by 4000 and subtract from the result five times the given number.

$$\left.\begin{array}{l} 356789\times4000=1427,156000 \\ 356789\times\quad 5=\ldots\ldots\ldots\ldots\ldots \end{array}\right\}$$

Subtraction we have 1425372055

The working should be done thus:—

$$\left.\begin{array}{rl} 5\times9 \quad =45+\mathbf{5}=50 & \text{carry } 5 \\ 5\times8+5=45+\mathbf{5}=50 & \quad ,, \quad 5 \\ 5\times7+5=40+\mathbf{0}=40 & \quad ,, \quad 4 \\ 5\times6+4=34+\mathbf{2}=36 & \quad ,, \quad 3 \\ 5\times5+3=28+\mathbf{7}=35 & \quad ,, \quad 3 \\ 5\times3+3-18+\mathbf{3}=21 & \quad ,, \quad 2 \\ 2+\mathbf{5}=\ 7 & \end{array}\right\} \quad \textit{Ans.}\ 1425372055.$$

(b) If the multiplier happens to be a number a little in excess of a power of 10 or a power of 10 multiplied by 1, 2, 3, 16, the multiplication and addition may be done at the same time.

Multiply 42784 by 1007

$$\begin{aligned} 42784\times1007 &= 42784{}^\frown00+7\times42784 \\ &= 43083488 \text{ Ans.} \end{aligned}$$

(c) To multiply by powers of 5, 25, 125, etc., multiply by powers of 10 (which means affixing as many zeroes as the power indicates) and divide by the corresponding powers of 2.

To multiply by 125 affix 3 zeroes and divide the result by 8.

(d) With special multipliers the work of multiplication may be done easily,

Multiply 73462 by 14412.

We see that the multiplier may be split into two periods 144 and 12 and that one period is an exact (here 12) number of times the other.

$$\begin{array}{rl} 73462 & \\ 14412 & \\ \hline 881544 & \text{... 12 times multiplicand ... (A)} \\ 10578528 & \text{... 12 times A. } i.\ e.\text{, 144 times multiplicand, and since 144 is 144} \\ \hline 1058734344 & \text{hundreds. we put the first digit below the hundreds place.} \\ \hline \end{array}$$

8. *Some useful algebraical formulae and their applications.*

(a) The student will see by actual multiplication that $(a+b)(c+d)=ac+ad+bc+bd$. This may be used in multiplying 2 numbers.

Examples.
$$\begin{aligned} 23\times 37 &= (20+3)(30+7) \\ &= 600+140+90+21 \\ &= 851. \end{aligned}$$

This suggests the usual oral method of multiplication.

$20\times 30=600$, $20\times 7=140$, $600+140=740$

$3\times 30=90$, (830), $7\times 3=21$. Answer 851.

(d) If a and b are any two numbers
$$(a+b)(a-b)=a^2-b^2.$$

This may be proved as follows:—

$$(a+b)(a-b)=a(a-b)+b(a-b)=a^2-ab+ab-b^2=a^2-b^2.$$

Applications :—

i. Given the hypotenuse and one side of a right-angled triangle, the third side may be easily found out by applying this formula. If hypotenuse and one side be 125 ft. and 75 ft. respectively, the other side is given by $\sqrt{125^2-75^2}$.

$$\begin{aligned} &= \sqrt{(125+75)(125-75)} \\ &= \sqrt{200\times 50}=100 \text{ feet.} \end{aligned}$$

ii. Suppose you are required to extract the square root of 21, geometrically. If you can express 21 as the difference of 2 squares you can extract the square root by the construction of a right-angled triangle. See Note on the Theorem of Phythagoras.

iii. (*a*) Multiply 1002 by 998; $1002 \times 998 = 1000^2 - 4$.

(*b*) Find $1112^2 - 888^2 = (1112+888)(1112-888)$
$= 2000 \times 224 = 448000$.

(*c*)
$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a(a+b)+b(a+b)\\ &= a^2+ab+ab+b^2\\ &= a^2+2ab+b^2.\end{aligned}$$

Application.

Find the square of 85.

$$\begin{aligned}85^2 &= (80+5)^2\\ &= 80^2+2.80.5+25\\ &= 6400+800+25\\ &= 7225.\end{aligned}$$

(*d*)
$$\begin{aligned}(a-b)^2 &\\ &= (a-b)(a-b)\\ &= a(a-b)-b(a-b)\\ &= a^2-ab-ab+b^2\\ &= a^2-2ab+b^2.\end{aligned}$$

Application.

Find the square of 997.

$$\begin{aligned}997^2 &= (1000-3)^2\\ &= 1000^2-2.1000.3+3^2\\ &= 1000000-6000+9.\\ &= 994009.\end{aligned}$$

(*e*)
$$\begin{aligned}(a+b)^2+(a-b)^2 &\\ &= a^2+2ab+b^2+a^2-2ab+b^2\\ &= 2(a^2+b^2)\\ (a+b)^2-(a-b)^2 &\\ &= 4ab.\end{aligned}$$

These two formulæ will be found to be of much use in working out questions of the following type: (i) given the perimeter of a rectangle and the sum of the squares on the two sides, find the sides (ii) given the perimeter and area of a rectangle, find its sides.

10. *Factors and Tests of Divisibility.*

A number is divisible by :—

(1) 2, if the digit in the units place is divisible by 2 or ends in 0.

(2) 3, if the sum of the digits in the number is divisible by 3

(3) 4, if the number composed of the last two digits is divisible by 4.

(4) 5, if the last digit be either 0 or 5.

(5) 6, if the tests for divisibility by 2 and 3 apply.

(6) 8, if the number formed by taking the last 3 digits i divisible by 8.

(7) 9, if the sum of the digits is divisible by 9.

(8) 10, if the last digit is 0.

(9) 11, if the difference between the sum of its digits in the odd places and that of those in the even places is 0, or divisible by 11.

(10) 12, if the tests for 3 and 4 apply.

(11) 15, if the tests for 3 and 5 apply.

(12) 16, if the number formed by taking the last four digits together is divisible by 16.

(13) 18, if the tests for 2 and 9 apply.

(14) 20, if the tests for 4 and 5 apply.

(15) 22, if the tests for 2 and 11 apply.

(16) 24, if the tests for 3 and 8 apply.

(17) 25, if the no. ends in 25, 50, 75 or two zeroes.

Note.—(1) No test for divisibility by 7 or 13 is useful if the number is a small one. But if the number consists of several digits the following test will be found to be of some use. Divide the number into groups of 3 digits from the units place. Take the sum of the numbers formed by the odd groups and that formed by the even groups and find the difference. If this difference is divisible by 7 the number is divisible by 7; if the difference is divisible by 13 the number is divisible by 13; if it is divisible by 7 and 13 the number is divisible by 7 and 13 or 91; if the difference is 0, the number is divisible by $7 \times 13 \times 11 = 1001$.

(*c*) *Prime numbers and Composite numbers.*

Composite numbers are those that are produced by the multiplication of two or more integers greater than 1. Thus $18 = 3 \times 3 \times 2$ is a composite number. Prime numbers are those which are not divisible by any number save themselves and 1. Thus 2, 3, 7, 11, 61 etc, are prime numbers.

Note. It is evident from the test for divisibility for 2 and 5 that all prime numbers except 2 and 5 must end in 1, 3, 7, or 9.

(*d*) Multiplication of a given number or compound quantity by a composite number is best effected by using factors. Thus to find the value of 735 articles at Rs. 175-11-7 each, the sudent will find it easy to multiply Rs. 175-11-7 by 5, the product by 7, this product again by 7 and this product by 3. For $735 = 5 \times 7 \times 7 \times 3$. To multiply the same quantity by 733 which is prime, we may get the result for multiplication by 735, by the method indicated above and subtract from the product twice the given quantity. This often gives an elegant method in the working out of certain questions (See solution in the case of question 3 in the S. S. L. C. Examination Papers 1913).

(*e*) When a Compound quantity is to be multiplied by a no., choose that factor of the number which when multiplied with the quantity renders the last denomination zero. Thus to find (Rs. 10-12-4) × 15 multiply first by 3 and then by 5.

11. (a) To multiply two nos. each of two digits, the digit in the units' place in each being 5:—

Increase the product of the digits in the tens places by half their sum and prefix the result to 25.

Ex.—(i) $24\times45=1125$ Thus:— $(2\times4)+(1\frac{1}{2}$ of $6)$ to be prefixed to 25 i.e. 1125.

(ii) 85×45; $(8\times4)+(\frac{1}{2}$ of $12)=38$; The product is 3825

(iii) 75×65; $(7\times6)+(\frac{1}{2}$ of $13)=48\frac{1}{2}$

$=48|5$

$|25$

Product $=$ 4875

95×65; $(9\times6)+(\frac{1}{2}$ of $15)=61|5$

$|25$

Product $=$ 6175

(b) To multiply a number of two digits by 11.

Place the sum of the digits between the digits.

72×11; $(7+2)=9$ Product 792.

85×11; $(8+5)=13$ Product $\begin{cases}8\ \ 5\\ 13\end{cases}$

935

94×11; $(9+4)=13$ Product $\begin{cases}9\ \ 4\\ 13\end{cases}$

1034

(c) To Multiply any number by 11.

Put down the digit in the units place in the units place of product.

Add the digit in the units place to digit in the tens place for digit in the tens place of product (Take carry if any)

Add digit in the tens place to digit in hundreds place and add carry—put down the last figure for digit in hundreds place and take carry and thus proceed until all the figures are exhausted.

Multiply 74673 by 11

... 3

$3+7=10$ 0

$1+7+6=14$ 4

$1+6+4=11$ 1

$1+4+7=12$ 2

$1+7=8$ 8

Answer **821403.**

(*d*) To square any number of two digits ending with 5.
(say) 75: Take the next higher figure in the tens place 8.
Multiply 7 and 8 and prefix to 25.
5625 is the answer.
$95^2 = 10 \times 9$ to be prefixed to 25 i.e. 9025.

(*e*) To square a number consisting of nines only.
Ex. 9999 = 99980001

Write as many nines less 1 as there are in the no.
Affix one 8 (always)
Affix as many zeroes as there are nines you have put down
Affix *one.*
one 9 less, eight, zeroes, one – is the rule.

Division.

12. (*a*) The usual long method of division can be shortened by using the principle in 8 (*a*), i.e. of combining multiplication and subtraction by the Italian method of subtraction. This sort of division goes by the name—*Italian method of division.*

Example. Divide 1234567 by 5678.

$$\begin{array}{r@{\,}l} & 217 \\ 5678) & 1234567 \\ \hline & 9896 \\ \hline & 42187 \\ \hline & 2441 \\ \hline \end{array}$$

$$\text{Answer } 217\frac{2441}{5678}$$

(*b*) If the divisor is a composite number, the division can be done by using the factors of the divisor.

(*c*) To divide a number by 10 and powers of 10, you will have to move the decimal point to the right-hand side by a number of places equal to the number of zeroes in the divisor.

(*d*) To divide a number by 5, 15, 35, 45, 55 etc., multiply in each case by 2 and divide the product by 10, 30, 70, 90, 110, etc. respectively. To divide a number by 25, 75, 175, etc. multiply the number by 4 and divide the product by 100, 300, 700, etc. respectively.

(*e*) To divide by $12\frac{1}{2}$, $11\frac{1}{9}$, $3\frac{1}{3}$, $8\frac{1}{3}$, $33\frac{1}{3}$ etc. take advantage of the fact that the fractions are respectively equal to

$$\frac{100}{8},\ \frac{100}{9},\ \frac{10}{3},\ \frac{100}{12},\ \frac{100}{3}\ \text{etc.}$$

(*f*) To divide by a number consisting entirely of 9's.

Example.—(1) Divide 1674393 by 999.

Now, 1674393 divided by 1000 gives quotient 1674 and remainder 393; hence dividing by 999, we shall have the same quotient, and 393+1674 over. Similarly 1674 divided by 999 would give quotient and 674+1 over. Hence the division by 999 may b effected thus:

$$\begin{array}{r|r l}
\text{Q} & \text{R} & \\
1674 & 393 & \text{Quotient } 1676 \\
1 & 674 & \\
 & 1 & \text{Remr } \quad 69 \\
\hline
1676 & 068 & \\
 & 1 & \\
\hline
 & 69 &
\end{array}$$

Note.—1 is added to the remainder because, 1 carried to the quotient being 1000 yields another complementary 1 for the remainder. Divide 12345678987654321 by 9999.

$$\begin{array}{r|r}
\text{Quotient} & \text{Remainder} \\
1234567898765 & 4321 \\
123456789 & 8765 \\
12345 & 6789 \\
1 & 2345 \\
 & 1 \\
\hline
1234691367902 & 2221 \\
 & 1 \\
\hline
 & 2223
\end{array}$$

This rule may be extended to division of any number b another whose complement is such that its multiplication with anothe may be orally done.

Divide 678392 by 994
The complement is 6

$$\begin{array}{l r|r l}
(1) & \ldots\ldots\ldots 678 & 392 & \\
(2) & \ldots\ldots\ldots \quad 4 & 068 & \\
(3) & \ldots\ldots\ldots \quad & 24 & \text{The quotient is 682 and Rr. 484.} \\
\hline
 & 682 & 484 &
\end{array}$$

The first line is easily understood. We have cut off 3 digi by a line. The second line is obtained by multiplying the portion the left of the vertical line in (1) by 6, and (3) similarly derive from (2). Adding up we get the answer. If during the addition the is a carry from the right of the vertical to the left, add 6 times t carry to the remainder since the complement is 6,

Divide 7437846782 by 9996
The complement is 4

$$\begin{array}{r|r l}
743784 & 6782 & \\
297 & 5136 & \\
 & 1188 & \text{The quotient is 744082} \\
\hline
 & & \text{and the Rr. 3110.} \\
744082 & 3106 & \\
 & 4 & \\
\hline
 & 3110 &
\end{array}$$

This mode of division, called synthetic division, may be extended, and we leave it to the intelligent and painstaking student as exercise, because we do not want to trouble the student with too many rules.

(*g*) In cases where you have to divide several numbers by the same divisor, it will be found convenient to get the product of the divisor and each of the digits 2, 3, 4, 9 beforehand. This method of division goes by the name of the *Slip method.*

(*h*) The approximate division of a number by 73 can be effected by what is called the *3rd - tenth - and - tenth rule.* The student may easily verify that $\frac{1}{73} = \frac{1}{100}(1 + \frac{1}{3} + \frac{2}{30} + \frac{1}{300})$, very nearly. Hence to divide a number by 73, add to the number to be divided it $\frac{1}{3}$, $\frac{1}{10}$ of $\frac{1}{3}$ and $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{3}$ and divide by 100,

This rule is particularly useful in calculating interest on any given sum for a given number of days at a given rate % per annum. —See chapter on interest.

13. *Fractions.*

(a) *Comparison of fractions.* In comparing fractions students generally bring each of the fractions to the same denominator. Sometimes the work may be simplified as shown below.

Compare :—

$$\frac{72}{103},\ \frac{68}{411},\ \frac{41}{283},\ \frac{11}{116}$$

Out of the 1st 2 fractions $\frac{72}{103}$ is clearly greater, as the numerator is larger than 68 and the denominator less than 411. Again $\frac{68}{411}$ is larger than $\frac{41}{283}$ as $\frac{68}{411}$ is a little less than $\frac{1}{6}$ while $\frac{41}{283}$ is a little more than $\frac{1}{7}$ and $\frac{11}{116}$ is the least as it is clearly less than $\frac{1}{10}$. *Vide* Q. 4, S. S. L. C, 1911.

(b) The work of simplification of fractions may be simplified by multiplying the numerator and denominator by the same number.

$$\frac{\frac{1}{3} - \frac{1}{4} + \frac{2}{5}}{\frac{1}{9} - \frac{1}{3} + 1}$$

Multiply the numerator and the denominator by 180.

(c) In working out certain problems, especially in cases where an approximation of the result is required, it is better to work in decimals. The student will do well to make himself acquainted thoroughly with contracted methods of multiplication and division of decimals and decimalisation of money.

(d) In the multiplication of a compound quantity by a large number, always apply the method indicated in the paragraph on Factors.

(e) In multiplying a compound quantity by a fraction multiply the quantity by the numerator and then divide the product by the denominator, but if the denominator divides the nr. exactly begin with the division.

14. *Aliquot Parts:—*

The utility of taking aliquot parts, with the ordinary student, ends with working out questions on *Practice*. The student can apply the method in the working out of various questions as indicated below:—

(1) Find $\frac{17}{32}$ of Rs. 15-8-11.
Now $\frac{17}{32} = \frac{1}{2} + \frac{1}{16}$ of $\frac{1}{2}$.
Hence $\frac{17}{32}$ of Rs. 15-8-11
$= \frac{1}{2}(15\text{-}8\text{-}11) + \frac{1}{16}\{\frac{1}{2}(15\text{-}8\text{-}11\}$

which may be found out easily.

(2) Find $\frac{11}{28}$ of 3 tons. 4 cwts. 3 qrs. 14 ℔s.

$$\frac{13}{28} = \frac{14-1}{28} = \frac{1}{2} - \frac{1}{14} \text{ of } \frac{1}{2}$$

$$\text{or } \frac{13}{28} = \frac{12+1}{28} = 3 \times \frac{1}{7} + \frac{1}{4} \times \frac{1}{7}.$$

(3) Find 38 % of Rs. 73-11-2
$38\% = 25\% + 10\% + 2\frac{1}{2} + \%\frac{1}{2}\%$
$= \frac{1}{4} + \frac{1}{10} + \frac{1}{4}$ of $\frac{1}{4}$ of $\frac{1}{10} + {}_{5}$ of $\frac{1}{4}$ of $\frac{1}{10}$

(4) If 15% of a certain sum be Rs. 12-10-8, what will 37½% of the sum be?
Hint: $37\frac{1}{2}\% = 2 \times 15\% + \frac{1}{2}$ of 15%.

15. Note the following rules:—

(1) To find the value of a dozen things, the value of each being given in Indian money, reckon every pie in the given price as one anna.

(2) To find the value of any number of articles, find the value of the dozens included in it and add the value of the extra ones.

(3) To find the value of 16 articles in Indian money reckon each anna in the value as a rupee.

(4) To find the value of 108 articles (Indian money) reduce price to pies and for answer take as any half-rupees as there are pies and add 4 times the number of pies.

(5) To find the price of articles at as. 10-8 ps., as. 5-4 ps. each, note that they are the aliquot parts of a rupee.

(6) To find the interest on any sum of money for one month at 5% (Indian money), reckon 8 pies for every Rs. 10 in the given sum.

(7) To convert miles per hour to feet per second, multiply by $\frac{22}{15}$.

(8) The price of 240 things at 1 d. each is £ 1.
The rule may be developed.
Find the price of 456 things at £ 1-6·7½ d. each.
456 = (480 − 24)

Price of 480 things at	£ 1 each	£ 480
do.	„ 6 sh. „	144
do.	„ 7½ d, „	15
		£ 639

„ 24 things at £ 1-6-7½ d. each. How will you proceed?

16. If only the pupils are made to work by taking aliquot parts and to do multiplication and division by means of factors, much of that ugly scribbling called "Rough work" might be done away with.

The students should be trained from the lower forms not to rush to stray corners in the note-book for rough work. The right-hand side margin should be used for that purpose. Even here the teacher should take care to see that no unnecessary work is done. For example, if a big number such as 87643 has to be multiplied by 797; the work that should be in the margin is only this:—

The first line is multiplication of no. by 800 and the second line is 3 times 87643 subtracted from the result. The model solution must always be exhibited on the board and the students should copy it down and imitate it.

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69851471

The system of teaching short cuts as such to be made use of only in Mental Arithmetic as it is called (is it an end in itself?), is highly to be deprecated.

Note:—This book has become too bulky for us to introduce some model solutions employing artifices and short-cuts.

APPENDIX B.

Some useful Tables and References.

1. *The Metric System.*

The metric system of lengths, weights and measures is now in force in France, Belgium, Switzerland, Italy, Spain, Portugal, Germany, Holland, Sweden and Norway. Men of science employ this system because of its decimal character. The metre was intended to be ten-millionth part of the distance from the Pole to the Equator measured on a meridian of the earth and is equal to about 39·37 inches. The denominations are unlike the English system of length, etc., either multiplied by 10 and power of 10 or divided by them, so that a quantity consisting of several denominations may be immediately written in terms of any of them, a quantity expressed in one denomination may be immediately broken up into several denominations.

2. *Measures of length*:—

10 millimetres = 1 centimetre = ·39371 inches.
10 centimetres = 1 decimetre = 3·93708 „
10 decimetres = 1 metre = 39·3708 „
10 metres = 1 decametre, and 10 decametres = 1 hectometre, and 10 hectometres = 1 kilometre, so that 1000 metres = 1 kilometre.

3. *Measures of Surface*:—

The *are* is the unit of land measure being a square whose side is a *decametre*; 100 ares = Hectare.

4. *Measures of Capacity*:—

The capacity of a cubic decimetre is a litre = 61·02705 cub. in.

5. *Measures of Weight*:—

The weight of a cubic centimetre of distilled water at 4° Centigrade called the gramme = 15·43235 grs. is the unit of weight.

Note:—(1) The prefixes *milli, centi, deci,* denote $\frac{1}{1000}$, $\frac{1}{100}$, $\frac{1}{10}$ of the unit, while the prefixes *kilo, hecto, deca,* denote 1000, 100, 10 times the unit.

(2) The capacity of a cubic decimetre being a litre and the weight of a cubic centimetre being a gramme, a kilogram is the weight of a cubic decimetre and therefore a litre of water weighs a kilogramme. A *litre* is *a cubic decimetre* and weighs a *kilogramme.*

1 inch =2·54 centimetres.
1 foot =30·48 ,,
1 yard =·914
1 mile =1·609 kilometres. Note that 5 miles are nearly equal to 8 kilometres.
1 sq. ft. =9·29 square decimetres.
1 acre =·405 hectare.
1 pint =·568 litre.
1 quarter =1·136 litres.
1 gallon =4·546 litres.
1 bushel =36·348 litres.
1 quarter =2·908 hectolitres.
1 grain =·065 gramme.
1 oz. Avoir. =28·35 grammes.
1 lb. Troy =373 grammes.
1 ton =1015 kilogrammes.

Some relations.

(*a*) A chain=22 yds; 10 chains=1 furlong.
(*b*) 10 sq. chains=1 acre.
(*c*) 1 fathom=6 ft; 6080 feet=1 nautical mile.
A speed of 1 nautical mile per hour is a *knot*.
(*d*) 1 litre=1¾ pints nearly=·220 gallon.
(*e*) A gallon of water weighs 10 lb.

APPENDIX C.

Note on the Framing of Problems.

Introduction.

Exercises on the Framing of Problems, give the students an insight into the composition of arithmetical and algebraical problems. This is an analytic method of learning the subject. He who knows to make a complete machine can very easily control the machine and find what is wrong with the parts. Students are advised to form batches of five or six, sit for an hour or two and draw up, each a question paper on the portions done in the class. Generally, the teacher will draw up a model paper and exhibit it on the blackboard.

Much as we should like to give in detail the method that must be followed, we are sorry that there is no space in this book which has already become voluminous However, before we conclude, we believe we will have given a clear idea on how this refreshing subject must be dealt with in the class-room. Here there is a wide field for those who follow the Dalton Plan or the Montessori method. " To do, to create, to imitate and to assume the role of teachers " is the instinctive nature of the youth.

1. Draw up a list of the things that we want for everyday use in the household. Mark against each article the bazaar rate. This list must be improved day by day. The students can copy the list in the Co-operative Societies.

We take two articles salt and rice.

Salt is sold at ¾ measure for two annas.
Rice sells at 3 measures for a rupee.

If you buy 1½ measures of salt and 4½ measures of rice, you will have to pay Re. 1-12.

(i) See how many questions may be framed from the data :— 1½ measures of salt and 4½ measures of rice cost Rs. 1-12. The price of a measure of rice being 5 as. 4 ps., find the price of a measure of salt.

(ii) 1½ measures of salt and 4½ measures of rice cost Re. 1-12. If rice is twice as costly as salt find the price of a measure of each separately.

(iii) Problems on simultaneous equations may also be proposed. For Type of problem see Q. 92 P. 45.

2. Transformation of Problems:—The students may take a problem from the book and transform it in ever so many ways. For example we take the problem:—Paddy worth Rs. 5-8 as. a kalam is mixed with an inferior variety worth Rs. 4-12 as. a kalam and this mixture is sold at the Rs. 5-6 as a kalam and the merchant neither gains nor loses. Find the ratio in which the two sorts are mixed up.

(a) The ratio is 5 : 1.

(b) The various questions that may be framed are:—

(i) Paddy worth Rs. 5-8 as. a kalam is mixed with an inferior variety in the ratio 5 : 1. If the mixture be sold at Rs. 5-6 as. a kalam, find the cost of a kalam of the inferior variety.

(ii) Paddy worth Rs. 5-8 as. a kalam is mixed with an inferior variety worth Rs. 4-12 as. a kalam and the mixture is sold at Rs. 5-6 as. a kalam. (a) If 85 kalams of the former enter into the mixture find the quantity of the cheaper sort in the mixture. (b) If the total quantity be 102 kalams, find how many kalams of each variety enter in the mixture.

(iii) The price of the mixture *viz.* Rs. 5-6 as. is the price at which the merchant neither gains nor loses. This may be altered to Rs. 6-11-6 i.e. 25 per cent. gain and question (ii) above may be altered thus:—

Paddy worth Rs. 5-8 as. a kalam is mixed with an inferior variety worth Rs. 4-12 as. a kalam, and by selling the mixture at Rs. 6-11-6 ps. a kalam, the merchant gains 25 per cent. (a) Find the ratio of the mixture. (b) If 85 kalams of the superior sort be taken, how many kalams of the inferior sort must have been taken etc.

3. The students can very easily propose problems on Profit and Loss. The student may take that the cost of a bicycle to the manufacturer is Rs. 120. Supposing he sells it at a profit of 20 per cent. to the wholesale dealer, the cost of each bicycle is Rs. ... .. 144.

If the wholesale dealer sells it at a profit of 25 per cent. to the retailer the cost of a machine to the retailer is Rs. (144+36) = Rs. 180.

If the retailer sells it at a profit of 15 per cent. to the customer the cost to the customer is Rs. (180+27) = Rs. 207.

A number of questions may be proposed with these data. Anyone of the prices or of the rates of profit may be asked to be found out, the other prices and rates having been given.

4. Data for direct and inverse questions on averages may be had from the school Attendance Registers, Mark Registers, and Meteorological observations recorded in the school observatory or in the reports and in the newspapers.

The Equational form and the idea of Proportion, the free use of the annual Publications on statistics, a knowledge of the current rates of the various commodities supply the necessary data for arithmetical problems. It will be a very good exercise, we believe, to ask the students to propose questions of the types we have given in the examples, in everyone of which we have inserted a good number of examples bearing on the activities of everyday life. The so-called 'Think of a number' problems give a lot of useful form. For type see Pages 6 and 7, Examples 1 and 2. Now and then a talk with the students on the market rates on the speeds of trains and cycles and motors, on the capacities of the various bottles, cases or tins, will considerably enhance their knowledge of the world.

The kind of exercise detailed in this section enables a student 'to pierce a subject to its core and discover its elements' Mathematics must be taught 'to cultivate a taste for neatness and a love for the good, the beautiful and the true; and to become more like the greatest of mathematicians, the Mathematician of the Universe.' Good-bye then, my friends! Remember this and study mathematics to realise that, "Mathematics is the life supreme. The life of the gods is mathematics. All divine messengers are mathematics. Pure mathematics is religion. There is no prophet which preaches the superpersonal God more plainly than mathematics."

NOVALIS.

THE END.

ANSWERS.

EXAMPLES I.-(A).

EXERCISE I.

1. $4a$,

2. $a+b$.

3. $6x-y$.

4. $-m$.

5. $3(c-b)$.

6. $3x-5$.

7. $2a^2+3ab+b^2$.

8. $a+5b-3c$.

9. $2y^2+xy-3x^2$.

10. $8a-b$.

11. $2(3a-4b)$.

12. $-4a$.

13. $2(y^2-x^2-3z^2)$.

14. $4\frac{13}{15}$.

15. (1) 56. (2) 692.

16. $1\frac{21}{40}$.

17. 2.

18. (1) $3(a-2c)$. (2) $2a$. (3) $a+5b+c$.

EXERCISE II.

1. $a+7b$.

2. xy.

3. $a^2+2ab+5b^2$.

4. a^2+ab+b^2.

5. (1) $b-a$. (2) $c-d\,;-d\,;\,c-2d\,;\,0\,;\,c-d-f$.

6. $-2x^2$.

7. $ab+r$

8. $\frac{P}{R}$

9. (1) $N=3x+1$.

(2) $P=x(x+1)$.

(3) $P=(2x-1)(2x+1)=4x^2-1$.

(4) $P=2x(2x+2)=4x(x+1)$.

(5) $P=100(x+1)x$.

(6) $P=25(x+1)x$.

10. (1) x^2-9.

(2) $2a^2+3ab+b^2$.

(3) x^3-1.

(4) x^3+1.

(5) a^2-b^2. (6) a^3-b^3.
(7) a^2-b^2. (8) a^3+b^3.
(9) $x^3-3x(x-1)-1$. (10) $x^3+3x(x+1)+1$.
11. (1) $x-1$. (2) $a-b$.
(3) $x-1$. (4) $x+3$. (5) $x+3$.
12. (1) $x=3$. (2) $x=-3$.
(3) $x=5$. (4) $x=5$.
(5) $x=8$. (6) $x=17\frac{1}{2}$.
(7) $x=-15$. (8) $x=7$.
(9) $x=5$. (10) $x=-25$.
(11) $x=17\frac{2}{7}$. (12) $x=-9$.
(13) $x=-4$. (14) $x=-8$.
(15) $x=6$. (16) $x=4$.
(17) $x=7$. (18) $x=2$.
13. Rs. 65 and Rs. 25.
14. Father $32\frac{2}{3}$ yrs; son $13\frac{1}{3}$ yrs.
15. 60 non-brahmins.
16. The shorter is 2·8 cms. long.
17. 298 votes.
18. Breadth 80 ft.
19. 16 cm.
20. The numbers are 13; 20; 27; 34; 41.
21. The strength is 120.
22. Rs. 6–14–0.
23. 4 men.
24. $12\sqrt{2}$.
25. Father 36 years; sons 12 years and 6 years.
26. 8 years hence.
27. 18 miles.
28. 33 miles.
29. 1st part is 28; 2nd part is 36.
30. 9 inches.

EXAMPLES I.—(B).

1. (a) 7. (b) $b-a$ **2.** (a) 3. (b) $\frac{p-a}{2}$

3. (a) 12. (b) $m+a$ **4.** (a) 6. (b) $\frac{m+a}{2}$

5. (a) 4. (b) $\frac{q}{2}-a$ **6.** (a) 14. (b) $\frac{b}{2}+a$

7. (a) 4. (b) $2(b-a)$ **8.** (a) 16. (b) $2(r+a)$

9. (a) 16. (b) $2(b+\frac{a}{4})$ **10.** (a) 0. (b) $\frac{2}{3}(b-2a)$

11. (a) 9. (b) $\frac{1}{6}(5b+2a)$ **12.** (a) $b-7$; 20. (b) $\frac{b+2a}{2}$

13. 13 thousands **14.** $\frac{36a+b}{2}$ in. ; $\frac{36a-b}{2}$ in. **15.** $\frac{p-b}{a}$

16. $x=ma+nb+\frac{pc}{16}$ **17.** $a-\frac{b}{20}$ **18.** $53\frac{1}{3}$ **19.** $\frac{1760m}{(2a+b)}$

20. $b-p$ **21.** 22 **22.** $\frac{p}{q}$ **23.** (a) $33\frac{1}{3}$ & $16\frac{2}{3}$ (*b*) $\frac{a}{3}$

24. (a) $10\frac{2}{3}$ and $15\frac{1}{3}$ (b) $\frac{2a+b}{3}$ and $\frac{a-b}{3}$

25. $a+b=c-d-5$ **26.** (a) $x-y=2(p-q)+5$

27. $ap=bq-m$ (b) $x-y=b(p-q)+a$

28. $\frac{p+q}{m-n}=(lx-cd)+t$

29. $C=\frac{180}{1+b+ab}$ $B=\frac{180b}{1+b+ab}$ $A=\frac{180ab}{1+b+ab}$

30. 1. (i) $\frac{2}{15}$ (ii) $\frac{2}{15}$ (iii) $\frac{2}{2q+15}$ (iv) $\frac{1}{x+y}$

2. $x+\frac{1}{x}=2\left(y-\frac{1}{y}\right)$

31. (a) $a-3$ (b) $b-2$ (c) $(bx+ay-xy)$ sq. in.

32. $2y-3x=6$.

◆

EXAMPLES II.

1. $x=a+b$; 704 **2.** (i) $x=2(a+b)$ (ii) $x=\dfrac{3(a-b)}{2}$

(iii) $x=4a-b$ (iv) $x=2(2b+a)$ (v) $x=\frac{1}{4}\left(c+\dfrac{ab}{16}\right)$

3. $x=5(a-b+c)$ **4.** $x=\dfrac{2p}{m}-a$

5. $3x=y$ **6.** $\frac{2}{3}x+\frac{4}{3}b=a$ and $x+b=a+c$ **7.** $18x=10a$

8. $x=35m-20a-15b$ **9.** (a) $\dfrac{x}{x+2}$ (b) $\dfrac{7}{9}$

10. (i) $\dfrac{p+w}{2}$; (ii) $a(w-p)+p$; **11.** $d+s+\dfrac{h}{2}$

12. $\dfrac{p-b-w}{108}$ **13.** $x=16a+b$ **14.** $x=\dfrac{16b}{a}+c$

15. $x=\dfrac{16(5p-am-2b)}{16n+q}$ **16.** $40a=9p+3{\cdot}5q+2r+t$

17. $x=\dfrac{12c}{\frac{4}{3}b+a}$ or $\dfrac{36c}{4b+a}$ **18.** $x=\dfrac{ap+bq}{r}$

19. $x=\dfrac{4a}{17}$, $y=\dfrac{a}{10}$ **20.** (i) $\dfrac{4b-a}{3}$ (ii) $\dfrac{2(b-a)}{3}$

(iii) $\dfrac{2b-a}{2}$ (iv) $\dfrac{2pb-a}{p+1}$ **21.** $\dfrac{h}{5280}$ hrs.

22. $\dfrac{1}{\mathrm{S}}(36x+12y+z-72a)$ **23.** $\dfrac{12b-av-ct}{x}$

24. $\dfrac{px}{a}+\dfrac{vy}{b}$ **25.** $\dfrac{5280}{x-1}$; $\dfrac{3600(n-1)}{m(x-1)}$

26. $x+y=a$; $\dfrac{ab}{b+1}$, $\dfrac{a}{b+1}$

27. $x-a$; $y-a$; $x-a=2(y-a)$. **28.** $3b-2a$. **29.** $4a-b$

30. (i) $\dfrac{1760c}{2(a+b)}$ (ii) $\dfrac{2p(a+b)}{1760}$ (iii) (a) $\dfrac{a+b}{220}$

(b) (1) $\dfrac{220c}{a+b}$ hrs. (2) $\dfrac{p}{4}$ hrs.

31. $\dfrac{8a-38}{7}$. **32.** $ma-x(m+n)$ annas.

33. (*a*) $2a(x+y)$ in. (*b*) a^2xy sq. in. (*c*) $a\sqrt{x^2+y^2}$ in.

(*a*) 286 in. (*b*) 4991·25 sq. in. (*c*) 102·3 in.

34. $2v=x-(a+b-c)$; 125.

35. $x=\dfrac{P(a+b)}{a}$; $x=37$, $P=16{\cdot}875$.

36. $\dfrac{pq-ab+cr}{p-a+r}$ 37. $x=\dfrac{pr+t}{q+r}$; 28

38. $x=\dfrac{p+q}{b-a}$; 62 39. $60T=\dfrac{a}{p}+\dfrac{b}{q}+\dfrac{c}{r}$

40. $\dfrac{1}{x}+\dfrac{1}{y}=\dfrac{2}{z}$

41. $T=\dfrac{ax-(x-p)(a-b)}{P}$; 20·9

42. $\frac{1}{4}(x+y)(3a+60)-(ax+16by)$; Rs. 2-4 as.

43. $2a-(b+c+d)=x$.

44. Rs. $\frac{1}{192}(48fx+18sy+160tz.)$ 45. $m=3n$.

46. $P=\dfrac{x-a-b+2c}{4}$; $Q=\dfrac{x-a-b}{2}$;

$R=\dfrac{x-a-b-2c}{4}$

47. $12a(16-m)+b(12-m)=12x$; 9.

48. $x=0{\cdot}02c(a+b)^2$; 82 c. ft.

49. $\dfrac{ab(s-p-q)}{ab-bp-aq}$; $6\frac{6}{7}$ 50. (i) $\dfrac{q}{2}$. (ii) $\dfrac{q}{p}(p-a)$ from the original position. 51. $\dfrac{3s}{4p}$; Maximum volume of luggage allowed for each passenger = 27 c. ft. Substitution of the values given, gives a negative value, which means that no additional stowage room is necessary for 30 additional passengers.

EXAMPLES III.

1. (a) 161·875. (b) $l=\dfrac{A}{b}=40$. (c) $b=\dfrac{A}{l}=9$.

2. (a) 15, (b) 16·48, (c) 11, (d) 840. 3. $2h(l+b)+2lb$

4. $b=8$; $h=5$; $A=28$. 5. 5 in.; 4·6 in.; 12 in.

6. $h=\dfrac{2A}{a+b}$; $a=\dfrac{2A}{h}-b$; $b=15$.

7. $a=\dfrac{2\Delta}{b}=5$; $b=\dfrac{2\Delta}{a}=9$.

8. $a=5$ nearly, side $=10$ nearly. **9.** $a=\sqrt{\dfrac{2S}{3\sqrt{3}}}$
(i) $6\sqrt{3}$ sq. in. (ii) $54\sqrt{3}$ sq. in. **10.** $S=15{\cdot}08$ sq. cms. $a=1{\cdot}8$ cms.

11. $r=3$. $S=16\pi$. **12.** $S=\dfrac{\pi d^2}{4}=4\pi$.

13. $P=\dfrac{100I}{nr}$; $n=2$; $r=5$; $P=300$.
14. $A=$ Rs. 110-4 as. $P=1600$.
15. $S=15\pi$; $h=2\sqrt{15}$ $h=\sqrt{91}$
16. $V=36\pi$, $r=2$, $h=5$. **17.** $c=20\pi$, $r=3$, $h=6$.
18. $S\times 30\pi$; $h=12$. **19.** $V=96\pi$; $r=3$, $h=5$.

20. $w=\dfrac{W(S-1)}{5}=32{\cdot}21$

21. (1) 5904. (2) $\dfrac{s}{t}-\frac{1}{2}gt^2$ (3) $\dfrac{2(S-ut)}{t^2}$

22. $c=\dfrac{5F-160}{9}=30$

EXAMPLES IV.

5. (*a*) apple $=5$ as.; orange $=3$ as. (*b*) $x=3$, $y=5$
6. (*a*) apple $=5$ as.; orange $=3$ as. (*b*) $x=3$, $y=5$
7. (i) $x=3$, $y=2$, (ii) $x=2$, $y=7$,
(iii) $x=4$, $y=3$, (iv) $x=3\frac{1}{2}$, $y=2\frac{1}{4}$,
(v) $x=2\frac{1}{4}$, $y=1\frac{1}{4}$, (vi) $x=2\frac{1}{2}$, $y=-3$,
(vii) $x=-1\frac{1}{2}$, $y=2$, (viii) $x=\frac{3}{7}$, $y=\frac{2}{7}$,
(ix) $x=1{\cdot}2$, $y=1{\cdot}4$, (x) $x=2{\cdot}1$, $y=-{\cdot}7$
(xi) $x=1{\cdot}7$, $y=3{\cdot}1$, (xii) $x=-4{\cdot}2$, $y=3{\cdot}9$,
(xiii) $x=3$, $y=2\frac{1}{2}$, (xiv) $x=4$, $y=6$,
(xv) $x=9$, $y=-13$, (xvi) $x=\frac{10}{14}$, $y=\frac{10}{11}$,
(xvii) $x=\frac{2}{3}$, $y=\frac{4}{5}$, (xviii) $x=-\frac{2}{3}$, $y=\frac{2}{3}$,
(xix) $x=\frac{7}{3}$, $y=\frac{7}{2}$, (xx) $x=\frac{3}{5}$, $y=\frac{9}{11}$,
(xxi) $x=\frac{10}{21}$, $y=-\frac{10}{7}$. **8.** $a=3$, $b=-7$; **9.** 67.
10. $P=3$, $Q=-2$.

EXAMPLES V.

1. $45\frac{1}{2}$ and 47
2. 7 benches, 38 boys
3. $\frac{11}{17}$
4. $\frac{7}{12}$
5. 33
6. 54 and 18
7. 30 and 10
8. 100
9. 192
10. 13 and 27
11. Rs. 1-4 as.
12. 162 ollocks
13. 1 measure of water to 3 measures of milk
14. $2\frac{1}{2}$ miles
15. 315 and 189
16. 10 miles
17. 33 cycles ; 10 and 23
18. 23 years hence
19. 11 persons ; $x(x-2)=11\times 9$
20. 2 miles per hour
21. 16
22. $4\frac{1}{2}$, 4, $3\frac{3}{4}$
23. 17th mile
24. 105 ft.
25. 15 and 12 ft.
26. 120 and 400
27. 8 pieces
28. $3\frac{3}{4}$ miles, $1\frac{1}{4}$ miles per hour
29. On Rs. 1000
30. 400 and 1200
31. 12 miles, $3\frac{1}{3}$ hrs.
32. 55 sq. miles and 45 sq. miles
33. 37
34. Rs. 350
35. 60 ; 90
36. 30
37. 24
38. 9216
39. 40 ; Rs. 35
40. 8, 12, 5, 20
41. 16 acres ; Rs. 2040
42. 84 years
43. Rs. 11-10-8 ; Rs. 16-10-8
44. 40
45. 12 miles
46. Rates are 12m., 15m., 20 m. per hour, respectively. Distance is 30 miles.
47. Second class 6 ; third class 9
48. 8 miles, 5 miles
49. Cloth of each = Rs. 30, Boarding—Rs. 15 per month, cost Rs. 360.
50. 93
51. 255, 165 ; 420
52. 1040, 560
53. 17
54. 48
55. £ 550
56. 150 each
57. 18 miles
58. 11 seers
59. 200, 328
60. 18·6
61. 500 tickets ; Rs. 2150

62. (i) $\dfrac{PQR}{P+Q}$ miles
(ii) 9 miles
63. (i) 14 as. ; 3½ as.
(ii) Rs. 13-12-6 ps.
64. 9 gallons
65. 3540, 3600
66. 1996, 2000
67. Coffee seeds : 1½ viss
sugar : 2¼ viss
68. 1566 gallons
69. 2914, 2606
70. 9·3″, 7·3″
71. 45 years
72. 57
73. Rs. 12, Rs. 6
74. Rs. 22, Rs. 24
75. 72
76. 750
77. 40
78. £ 4680 ; £ 4720
79. 300, 700
80. 1½ mds.
81. 420 oz. ; 255 oz.
82. R. 200 ; 90 persons
83. 30 miles
84. 2110
85. (1) 32. (2) 5-8-0
86. $\frac{7}{12}$
87. $\frac{13}{21}$
88. 80 yds., 67 yds.
89. 180 ft., 100 ft.
90. 11½ as., 1½ as.
91. Rs. 4000, Rs. 2600
92. Re. 1
93. Rs. 925, Rs. 500
94. Length 10 yds., breadth 7 yds.
95. Rs. 506-4-0
96. $\angle A = 50^\circ$, $\angle B = 100^\circ$
$\angle C = 130^\circ$, $\triangle D = 80^\circ$
97. 173
98. Rs. 2-13-6 ps. ;
3 as. 6 ps. per quire
99. 5, 11
100. 100
101. 60 miles
102. 100°, 40°, 40°
103. (a) $(m-y)\,x$ pence is tax
$\dfrac{(m=y)\,x}{m}$ pence per £ is rate
(b) $x=21$; $y=1000$: £ 2333, £ 2100, £ 1909
104. 80°, 135°, 100°, 45°
105. 84, 63
106. 120
107. (1) Re. 1-2 as. ; 4½ as.
(2) Rs. 126-13-6
108. $\dfrac{60mt}{60(t-a)+mb}$
109. ¾, −5 ; 36½, 32
110. 1·8, −32
111. $a=20$, $p=8\frac{1}{2}$
cost Rs. 360
112. $y=3x-5$
113. $x=19$ years (nearly)
114. $a=6$, $b=\cdot 009$;
$R=17\cdot 025$
115. £ 16750.

EXAMPLES VI.

1. 107°. **2.** $180^\circ - x^\circ$. **3.** (*a*) 109°, 71°. (*b*) (i) OA, OD are in the same st. line. (ii) OB, OE are at rt. angles. **4.** $\angle PXR = \angle\ SXQ = 20^\circ$; $\angle PXS = \angle RXQ = 160^\circ$. **5.** $\angle PQB = 55^\circ$; $PQA = 125^\circ$, $XQY = 90^\circ$. **6.**

$$\left.\begin{array}{l}\angle PQB = QRD\\ = \angle CRS = AQR\end{array}\right\} = 65^\circ;\quad \left.\begin{array}{l}\angle PQA = QRC\\ = \angle SRD = \angle RQB\end{array}\right\} = 115^\circ$$

7. (*a*) $180^\circ - 5x + 2y$, (*b*) $y = 2x$. **8.** $\angle A = 25^\circ$, $\angle B = 60^\circ$, $\angle C = 95^\circ$. **9.** 11°. **10.** 58°. **11.** $\angle A = 50^\circ$, $\angle B = 60^\circ$, $\angle C = 70^\circ$. **12.** (*a*) $60^\circ, 72^\circ, 48^\circ$ (*b*) $A = (b - a)^\circ$, $B = a^\circ$, $C = (180 - b)^\circ$ (*c*) $A = 92$, $B = 46^\circ$, $C = 42^\circ$. **13.** (*a*) $\angle PXR = \angle QXS = 78^\circ$; $\angle PXS = \angle RXQ = 102^\circ$ (*b*) $\angle PXR = \angle QXS = (a - 180)^\circ$; $\angle PXS = RXQ = (360 - a)^\circ$.

14. $\angle PXR \times \angle QXS = (2a - b)^\circ$; $\angle PXS = \angle RXQ - 180^\circ - (2a - b)^\circ$.

15. (i) $180^\circ - \frac{1}{2}(y + z)^\circ$ (ii) $90^\circ + \frac{1}{2}x^\circ$. **16.** $\frac{1}{2}x^\circ$.
17. p, $p + m$, $p = 60^\circ$. **18.** A parallelogram.
19. $\angle B = 90^\circ - x^\circ$; $\angle DAC = 90^\circ - y^\circ$ $\angle^s ADB$, ADC are rt. angles.

20. $\angle BAD = y^\circ - x^\circ$; $\angle\ ACD = 180^\circ - y - z$, $\angle ACF = z^\circ$.

21. (*a*) Equal.
(*b*) $\angle ACB = x^\circ$; $\triangle\ BAC = 180 - 2x$
(*c*) $x = 59^\circ$; $\angle A = \angle\ ACE = 62^\circ$; $\angle B = \angle C = \angle\ ECD = 59^\circ$.
22. $x + y = 90^\circ$.
23. If $B = x^\circ$. $C = 90 - x^\circ$. $\angle CAD = x^\circ$ and $\angle BAD = 90 - x^\circ$.
24. $\angle BAX = \angle\ CAX$ and $BX = XC$.

25. $\angle ABD = 90 - \dfrac{a}{2}$ degrees;

$$\angle DBC = b + \frac{a}{2} - 90.$$

$\angle DCB = 180 - (a + b)$.
26. $\angle\ BDC = 19^\circ$ and $\angle BCD = 61^\circ$.

27. $DE + DF = AG \times \dfrac{BC}{AB}$ **29.** 20°.

30. (*a*) 70°. (*b*) $\dfrac{90a^\circ}{a+1}$

31. (1) $\frac{90(2a-1)}{a-1}$; (2) 45° (3) 210°.

32. $\angle ACB = 55^\circ$, $\angle CBO = 30^\circ$, $\angle COB = 120^\circ$, $\wedge AOD = 50^\circ$.

33. (*a*) 360°, (*b*) 8 : 16 rt. angles; 12 rt. angles.

34. 135° or 1½ rt. angles.

35. (*a*) n triangles.

(*b*) $2n-4$ rt. angles. (*c*) $\frac{2n-4}{n}$ rt. angles.

(*d*) $\frac{4}{n}$ of a rt. angle or $\frac{360}{n}$ degrees.

36. (*a*) 4 sides; square. (*b*) 6 sides.

37. (*a*) A regular hexagon.

(*b*) Starting from a point A on the circumference of the circle mark off points B, C, D, E and F along the ⊙ce so that $AB = BC = CD = DE = EF = FA =$ radius of the circle. Then $ABCDEF$ will be the hexagon required.

38. a regular octagon. **39.** a regular hexagon.
40. (a) 90° (b) 45° (c) Let AB be the given st. line; bisect it at C and draw CL the perpendicular. Cut off from CL, $CD = CA$, $DE = DA$. With E as centre, and radius EA, describe a circle. Mark off points L, M, N &c. along the ⊙ce so that lengths LM, MN &c. are each $= AB$. $ABLMNOPQ$ is a regular octagon.

EXAMPLES VII.

2. (4), (6) impossible. Sum of two sides being less than the third, the arcs will not cut to form the third vertex.

3. (4), (7) impossible; *two* angles together exceed 180°.

4. Draw $MN = 1{\cdot}1''$; produce it to C so that $NC = 4{\cdot}5''$. Bisect NC at A. Then $NA = c = 1{\cdot}7''$; $AC = b = 2{\cdot}8''$. With A and C as centres, and radii $= 1{\cdot}7''$ & $1{\cdot}9''$, draw arcs to cut at B.

5. Draw $AB = 3{\cdot}7''$. Bisect it at C. Draw CD at rt. angles to AB and equal to CA. Produce CD to E so that $DE = DA$. ABE is the required $\triangle$.

6. From any point O in AB draw OK and OL on the same side of AB so that $AOK = 98^\circ$ and $BOL = 114^\circ$. Then angles AOL, LOK and KOB will be respectively equal to P, Q and R. Since QR is given and angles Q and R are found out, the $\triangle\ PQR$ can be described.

7. At any point D in a st. line MN, erect a perp. $DA = 2{\cdot}9''$. Draw AB, AC on opposite sides of AD, making 40° with it, and cutting MN at B and C. ABC is the $\triangle$ reqd.

8. Proceed as in $Q.\ 7$: $DA = 3{\cdot}4$; $\angle^s\ DAB$, DAC each $= 30^\circ$.

9. From $MN = 9$ cms., cut off MA, NB each $= 3{\cdot}4$ cms. With A and B as centres, and with radii $= 3{\cdot}4$ cms., draw arcs to cut at C. ABC is the $\triangle$ reqd.

10. It is a rt.-angled triangle.

11. Draw $AB = 2{\cdot}6''$ and on it describe a semi-circle. Make at A, an $\angle = 30^\circ$ so that the arm cuts the semi-circle at C. Then ABC is the rt.-angled $\triangle$.

12. (a) Draw $AB = 3''$. Make at A and B, $\angle^s = 75^\circ$. Let the arms cut at C. Then ABC will be the triangle. (b) Draw CO perp. to AB. Draw AD to cut CO at D making with CA an angle equal to ACD. Then DAB will be the second triangle reqd.

13. Proceed as in $Q.\ 7$ with $DA = 2\frac{1}{4}''$ and with angles BAD and $CAD = 65^\circ$ *viz.* $(90 - 25)^\circ$ each.

14. (a) With base $AB = 4{\cdot}2$ cms., and angles A and $B = 70^\circ$ each, describe the $\triangle\ ABC$.

(b) Divide AB into 4 equal parts. Produce AB to D so that $AD = 5$ such parts. Draw DE parallel to BC to meet AC produced in E. ADE is the reqd. $\triangle$.

15. $10\frac{5}{7}$ feet.

16. (ii) 10 yds. and 60 yds. from B. 129·6 yds. and 103·9 yds. of pipe nearly.

17. $x = 2''$, $y = 3''$. An isosceles $\triangle$.

19. Two triangles. Draw $AB = 2{\cdot}7''$. At A make $\angle BAP = 30^\circ$. With centre B, and radius $= 2{\cdot}1''$ draw an arc. This cuts AP at two points, say at M, N. ABM, ABN are the 2 triangles. When side opposite $\angle 30^\circ = 3''$, AP is cut by the arc only at one point, so that only one triangle results.

21. The sum of the three angles of a triangle $=180^\circ$; so that Q is found by drawing BQ, making $\angle ABQ = 180^\circ - (55^\circ + 65^\circ)$. S is found by drawing BS, making $\angle ABS = 180^\circ - (55^\circ + 103^\circ)$.

22. 1·37 miles.

EXAMPLES VIII.

3. Draw AB and CD as in Q. 1. Bisect ACD by CO; $\angle$BCO = 135°. Bisect $\angle$OCD by CM. $\angle$BCM = 112½°. Bisect $\angle$MCD by CL. $\angle$BCL = 101¼°. Bisect $\angle$MCB by CR. $\angle$BCR = 56¼°. Bisect $\angle$BCD by CS, $\angle$BCS by CT, and $\angle$TCS by CP. Then BCP = 33¼°.

4. 30°, 30°, 30°.

5. Draw a rt. angle and trisect it, adopting the construction in Q. 4. By repeated bisection of 30°, get 15°, 7½° and 3¾° and hence construct the angles required.

7. (i) An isosceles △. Yes. ACD is another isos. triangle.

(ii) ED = 11·8 cms.; Perimeter = 11·8 cms. ED = perimeter.

(iii) $\angle^s$ AEB and EAB are each = ½ABC. Angles CAD and CDA are similarly each = ½ACB.

8. Draw ED = 13·7 cms. At E and D make angles = 37½° and 22½° resp. Let the arms meet at A. Draw AB, AC to meet ED at B and C, making with AE and AD angles 37½″ and 22½°. Then ABC is the required triangle.

10. Let OA and OB represent the roads and let OC bisect $\angle$ AOB. If OC = 100 ft., C is the well.

11. Let AB and XY represent the banks and LM the bridge. Let RK be drawn ∥ to LM at a distance of 2 furlongs from it, and ST midway between XY and AB and ∥ to them. The point of intersection W of these two lines, will be the position of the boat.

12. On the bisector of the angle.

(a) At the point of intersection of the bisectors of the two $\angle^s$ 100° and 60°. (b) 21½ yards nearly. (c) 1453 sq. yds. nearly.

13. At the point where the perpendicular bisectors of the three sides meet.

14. (i) Let MO, NO make at O an angle $=120^\circ$.

Draw BA, CA parallels to MO and NO respectively, distant 4 cms. A is the point.

(ii) Draw BA, 3 cms. distant, and CA, 4 cms. distant.

15. (i) P is the point where the bisector of $\angle BAC$ meets BC.

(ii) Draw a parallel to AB, distant say 2 in., cutting BC at D; also a parallel to AC, distant 1 in., cutting BC at E. Let the two meet at O. Bisect $\angle DOE$ by OQ meeting BC at Q. Q is then 1″ nearer to CA than AB.

16. Similar to 14 (ii).

17. At the point of intersection of the perpendicular bisector of AB and the railway track.

18. Draw the perpendicular bisector of PR and mark the point T in it, 7 furlongs from P. Draw a semicircle on TS and mark K on the $\odot^{ce}$, 1 mile from S. Then TK will be the Railway Track.

19. Draw RS at right angles to CD. Produce RS to T such that $ST=RS$. Join PT to cut CD in V; RV and PV make equal angles with the normal at V.

20. 118 ft. nearly.

21. 62·5 yds. from each steamer.

22. (*a*) School to be built within the area enclosed by the 2 arcs drawn with A and B as centres and with 6 furlongs as radii. (*b*) At the point of intersection of the two arcs. (*c*) 3 furlongs nearly.

23. At the point of intersection of the st. line drawn parallel to the longest side at a distance of 15 yds. from it and the bisector of the angle between the two smaller sides.

24. 475 yards nearly.

25. (*a*) The bangalow to be built at the circumcentre of the triangle formed by the office, recreation ground and the station. (*b*) 3·1 furlongs nearly.

EXAMPLES IX.

1. $b=1{\cdot}4$ cms. and $1{\cdot}9$ cms. resp.; $A=58°$ and $122°$ resp.

3. $A=72°$, $C=61°$; $c=1{\cdot}75''$.

4. $B=107°$; $c=2{\cdot}1$ cms.

5. $BC=2{\cdot}4$ cms. or 12 cms.

6. (1) When $C=28°$, $b={\cdot}7''$ or $2{\cdot}8''$.
(2) When $A=28°$, $b=3{\cdot}1''$.
(3) When $B=28°$, $b=1''$. Four triangles in all.

7. 2 Triangles; 7·1 cms. and 3·75 cms.

8. (*a*) One triangle only.
(*b*) One triangle only. The triangle is a rt.-angled triangle.
(*c*) 2·65 cms.

10. 60 yds. **11.** 6 ft. **12.** (*a*) 6⅔ miles. (*b*) 13¾ miles. **13.** 50 yds. **14.** 64 yds.; 3072 sq. yds. **15.** 259 yds. **16.** 16 ft. **17.** 46 feet (nearly).

18. 1800 sq. inches.

19. $AB=4$ cms.; $AD=3{\cdot}45$ cms.; $BD=2$ cms.; $\angle BAD=30°$.

20. 77 ft. **21.** 41 ft. **22.** 20 ft.; 1¼ ft.

23. (a) 22·9 feet. (*b*) 12·8 feet. (*c*) 1·34 feet. (*d*) 22·6 feet. (*e*) 24·7 feet. (*f*) 18·2 feet.

24. 13·2 ft. **25.** 297 yds. nearly.

26. (*a*) 11·3 inches (*b*) $1{\cdot}4''$, $2{\cdot}8''$, $4{\cdot}1''$, $5{\cdot}5''$, $6{\cdot}8''$, $8''$, $9{\cdot}1''$, $10{\cdot}3''$, $13{\cdot}9''$ and $15{\cdot}5''$.

27. (*a*) 17½ ft. (*b*) 10 ft. (*c*) 6½ ft.

28. (*a*) $30''$ (*b*) $30{\cdot}5''$.

29. 32 ft., 58·5 ft., 68 ft., 47; (i) 56 ft. or 27 ft. (ii) 43 ft. or 25 ft. (iii) 18 ft.

30. 2⅝ ft. **31.** 20 ft. **32.** 12 ft.
33. 60° **34.** 224 ft. **35.** 924 ft.
36. 50 seconds; 6928 ft. **37.** 1989 ft.
38. 761 ft. **39.** 346 ft. **40.** 27° nearly; 38 yards.

EXAMPLES X.

1. 9. **2.** 12·5. **3.** 24·25 and 11·75.

4. (i) 9¾ (ii) 52½ (iii) 23.

5. (i) b^2c (ii) a^2bc (iii) ab^2c^2.

6. $R = \frac{a^2}{16}$; Rs. 64.

7. $R = \frac{1}{16}(a^2 + 3b)$; 167½.

8. (i) $ab + ac$ (ii) $2xy + 4x$ (iii) $14xy + 7y^2$ (iv) $6a^2 + 3ab$ (v) $4ab - 2a^2$.

9. (i) $a^2 + 2ab + b^2$ (ii) $c^2 + 2cd + d^2$ (iii) $x^2 - y^2$ (iv) $a^2 - 2ab + b^2$ (v) $c^2 - d^2$ (vi) $a^2 - b^2$ (vii) $c^2 + d^2 - 2cd$.

10. (i) 1,1. (ii) 4,4. (iii) 16,16. (iv) $(2001 + 1999) \times (2001 - 1999) = 4000 \times 2 = 8000$. (v) $(1000 + 2) \times (1000 - 2) = 999996$ (vi) $(2000 - 2)^2 = 3992004$ (vii) $(2000 + 16)^2 = 4064256$.

11. (*a*) (i) 2. (ii) 3. (iii) 6. (iv) 12. (v) 16.
(*b*) ±12. (*c*) ±12.
(*d*) when $a + b = +7$, $a = 4½$, $b = 2½$.
,, ,, $= -7$, $a = -2½$, $b = -4½$.
(*e*) $x + y = \pm 11$, $x - y = \pm 3$; we have therefore *four* sets of values. (i) $x = 7$, $y = 4$ (ii) $x = -4$, $y = -7$ (iii) $x = 4$, $y = 7$. (iv) $x = -7$, $y = -4$.
(*g*) 90°. (*h*) Hyp. = 13, $13^2 = 5^2 + 12^2$.

16. (a) 1·3″. (b) 1·2″.

17. (a) $x^2 + 1$ inches.
(b) $x = 1$. Sides are 2, 0 and 2 (No triangle).
$x = 2$. ,, 4, 3 and 5.
$x = 3$. ,, 6, 8 and 10.
$x = 4$. ,, 8, 15 and 17.
$x = 5$. ,, 10, 24 and 26.
$x = 6$. ,, 12, 35 and 37.
$x = 7$. ,, 14, 48 and 50.
$x = 8$. ,, 16, 63 and 65.
$x = 9$. ,, 18, 80 and 82.
$x = 10$. ,, 20, 99 and 101.

18. (b) 4·8 nearly.

19. 5, 6·85, 6·25, 5·2 and 5·1.

20. (b) 3·9. (c) 6·9, 6·2, 8 and 6·1

22. (i) $\sqrt{2}''$ (ii) $\sqrt{3}''$ (iii) $\sqrt{4}''$ or 2″ (iv) Proceed similarly, and find points G, H, I etc. ; then $CG = \sqrt{5}$; and $CI = \sqrt{7}$.

23. (a) $\angle DBA = \angle DAB$ $\therefore DB = DA$. $\angle DAC = 90°$ $\angle DAB = 90° - \angle DBA = \angle DCA$. $\therefore DA = DC$ $\therefore DB = DA = DC$.

24. $\angle BAC$ is a rt. $\angle$ $\therefore BC^2 = (BA^2 + CA^2) = (BD^2 + CD^2 + 2DA^2)$
also $BC^2 = (BD^2 + CD^2 + 2BD.\ CD)$
$\therefore DA^2 = BD.\ CD = 4 \times 2 = 8$; $\therefore DA = \sqrt{8}$.

25. (i) $c = 3\sqrt{2}$ cms. (ii) $a = b = 3$ cms. $\angle C = 90°$.

26. (a) 30°. (ii) 90°. (iii) 2 cms. (*b*) $BD \angle \frac{a}{2}$ in.; $AB = a$ in.; and $AD = \frac{a}{2}\sqrt{3}$ in.

27. (a) (v) $a = 6·04$ cms. (vi) $a = 2·7$ in. (vii) $b = 3·9$ in.
(d) (i) 55·4 ft. (ii) 87·4 ft.

28. $\frac{p}{2}$; $\frac{p\sqrt{3}}{2}$; p.

29. $APB = \frac{3}{16}$; $BPC = \frac{3}{8}$; $CPD = \frac{5}{16}$; $DPA = \frac{1}{8}$.

30. 150 yds; $75\sqrt{3}$ yds. **31.** $14\frac{2}{7}$ miles per hour.

32. (1) $6\sqrt{3}$ ft. (2) It may emerge at any point on the ⊙ᵉᶜ of a circle which is the base of a right cone of vertical angle 120° and axis equal to the greatest depth $6\sqrt{3}$ ft; so that the distance will vary from 12 ft. behind, to 18 ft. in front of, the line joining the starting point with the centre of the circle. (3) It is clear that in any case the total distance travelled is $12(\sqrt{3}+1)$ ft.

33. 44·2 ft. **34.** (i) twice. (ii) 12·07 minutes.

35. (1) 43·3 yds. (2) 86·6 yds. (3) 120°.

36. (i) 56 ft. (ii) nearer by 17 ft.

37. $PQ = p - 2a$; $SR = p - 2b$, $PS = \sqrt{q^2 + (a-b)^2}$.
Hence perimeter $= 2\{(p - a - b) + \sqrt{q^2 + (a-b)^2}\}$.

38. $11\frac{1}{4}''$ nearly. **39.** $2\frac{1}{2}$ ft. **40.** (a) 3·1″ (b) 4″.

41. 173 ft. from the top; 173 ft. from the ground.

42. 41°; midway between A & B.

43. 416 ft. **44.** Height of the nearer : height of the other : : 3 : 2; distance between tops is 79·4 ft.

45. 1 mile; ·268 mile.

46. $\begin{cases} \text{Distance from A or B} = \dfrac{\sqrt{2}a}{2}; \\ \quad\text{,,} \qquad \text{from C or D} = \sqrt{b^2 + \frac{1}{2}a^2 - ab}. \end{cases}$

47. 677 ft. **48.** $AP = \sqrt{l^2 + a^2 + 2am}$
$BQ = \sqrt{l^2 + b^2 - 2bn}$
$PQ = \sqrt{l^2 + a^2 + b^2 - 2am - 2bn}$

EXAMPLES XI.

1. 443, 4042, 73462.

2. $7 \times 11 \times \sqrt{3} = 77\sqrt{3}$; $7 \times 11 \times 3\sqrt{3}$; i.e., 3 times the previous result.

3. (*a*) $11\sqrt{5} - 3\sqrt{5} + 5\sqrt{5} - 3\sqrt{5} = 10\sqrt{5} = 22{\cdot}361$.
(*b*) $2\sqrt{5} - 13\sqrt{5} + 14\sqrt{5} = 3\sqrt{5}$.

4. $\frac{\sqrt{51}}{4} =$ See tables for $\sqrt{51}$; $\frac{11}{3\sqrt{3}} = \frac{11\sqrt{3}}{9}$

5. $\frac{1}{4}(\sqrt{7} + \sqrt{3})^2 = \frac{1}{4}(10 + 2\sqrt{21})$; $\frac{(\sqrt{3} + \sqrt{2})(\sqrt{7} + \sqrt{3})}{1}$ i.e., $\sqrt{21} + 3 + \sqrt{14} + \sqrt{6} = 3 + \sqrt{21} + \sqrt{14} + \sqrt{6}$. Find $\sqrt{21}$, $\sqrt{14}$, $\sqrt{6}$ from tables. $\frac{\sqrt{6}(1 + \sqrt{2})}{\sqrt{6}(1 + \sqrt{4})} = \frac{1 + \sqrt{2}}{1 + 2}$
$= \frac{1}{3}(1 + \sqrt{2})$.

6. 5 ac. 1 ro. 33·41 poles nearly; 162 yards 1 ft. 7·5 in. nearly.

7. $\frac{\sqrt{15}(\sqrt{5} - \sqrt{3})}{2} = \frac{5\sqrt{3} - 3\sqrt{5}}{2}$

8. 67 yds.; 125. **9.** 460 yds.; 1 ft. 6 in. **10.** 108.

EXAMPLES XII.

6. ·672″; ·7″; 2·4 in.; 8·4 sq. in. **8.** (a) 3 cms. nearly. (b) 3 cms. **9.** (a) $a\sqrt{3}''$. (*b*) 30°, 60°, 90°.

10. (a) $2\sqrt{2}$ cms. (b) $\frac{x}{\sqrt{2}}$ in. **11.** $\frac{9}{4}\sqrt{3}$ sq. cms.

12. 8 sq. cms. **13.** 3·7 sq. cms. **14.** 2·65 cms. nearly. **15.** (i) 17·8 sq. cms.; (ii) 3·1 cms.

18. 3·4 cms.; 6 cms. **19.** (i) 65. (ii) 51. (iii) Proceed as in 5(*b*) on p. 95.

20. (i) Let $a = b$, then $S = b + \frac{c}{2}$ } and formula becomes $= \frac{c}{2}\sqrt{b^2 - \frac{c^2}{4}}$

(ii) $a = b = c$, $S = \frac{3a}{2}$,, $= \frac{a^2}{4}\sqrt{3}$.

(iii) Let c be the hyp. ,, $\frac{ab}{2}$

(iv) In (iii) let $a=b$ „ $\frac{a^2}{2}$ or $\frac{b^2}{2}$

(v) Let $a=b\sqrt{3}$ „ $\frac{b^2\sqrt{3}}{2}$

(vi) Let $a=b$; $\frac{c}{2}=\frac{b}{2}\sqrt{3}$ or $\frac{a}{2}\sqrt{3}$ in (i) above; „ $\frac{b^2}{4}\sqrt{3}$.

21. AE=48 ch.; DE=29; $\frac{25}{54}$ is left.
22. 8·5 in. **23.** 18.4 in.

EXAMPLES XIII.

1. (*b*) 5 cms. **2.** 5 cms.; 7·5 cms. **3.** 1·6 sq. in.; 9 sq. in.; 2·5 sq. in. **4.** 3325 sq. yds. **5.** 3150 sq. yds. **6.** 17350 sq. yds. **7.** 6·5 (∠CDA=90°). **8.** ·8384. **9.** 196 sq. ft. **10.** 94050 sq. yds.

11. (a) Construct △AKD, AK=(AB−CD)=5·9 cm. A=75°, ∠K=∠B=60°. Produce ∠AK to B so that AB=11·2 cms. Through B and D draw BC and DC ‖ to KD and KB resp.

(b) Construct △ABL, AB=11·2 cms., ∠A=75°, ∠B=60°. Divide BL at C so that BC : CL : : 5·9 : 5·3. Through C draw CD ‖ to AB cutting AL at D. ABCD is the reqd. quadl. (Similar △s).

12. 27·625 acres. **13.** 34·6 acres. **14.** 220 yards. **15.** 5 cms. **16.** 520 chs. **17.** 600 yds. **18.** 895 sq. ft. **19.** 28¾ acres. **20.** 95 yds. **21.** AB=3·3 cms., BC =4·5 cms., CD=6·9 cms., DA=3 cms. **22.** 41·5″.

23. To find R, describe a semi-circle on QS as diameter and with C as centre and 4·5″ as radius describe an arc to cut it. Note that there are two points of intersection; hence two quadrilaterals.

25. 10587·5 sq. yds. AD=500 links, CD=375; but AC=625 links. Hence AD and CD are at rt. angles. ∴ AD is the shortest distance.

26. 5 sq. in. (nearly). **27.** (*a*) 40 yds. (*b*) Bisect BC at O. Thro' O draw a ‖ to AD, meeting AB (produced) at E, and CD at F. AEFD is the rectangle. **28.** 8·8 sq. cms. **29.** 1·47 acres. **30.** 50 yds. **31.** 160 sq. in. **32.** 3″. **33.** 1697 links ; 1200 links.

34. Draw base triangle AKC with AK = (5 + 2·5) cms. and AC and KC equal to 6 cms. and 3·5 cms. respectively. Take B in AK so that AB = 5 cms. Draw CD and BD parallel resp. to BA and KC. Then ABCD is the reqd. trapzm. ; 2·8 cms. and 3·9 cms.

35. Draw AL, 5″ long. Find C, 3·4″ from A and 4·1″ from L. Draw CD in the direction of LA and parallel to it, equal to 2″. Take B in AL, 3″ from A. Then ABCD is the required trapezium. **36.** 2·2″ ; 2·6″.

37. Draw MO, 320 yds. long. At O make an angle MOR = 130°. With M as centre and with (300 + 180) yds. as radius draw an arc to cut OR at R. Now MOR is the base triangle. Find N in MR so that MN = 300 yds. and drawing NP and OP parallel resp. to RO and RM, complete the quadl. **38.** Draw two lines KL and MN, 4·5 cms. and 3·9 cms. and divide them at P and S in the ratio of 9 : 5. Draw AC and find O in it so that AO = KP. Make an angle of 130° at O by drawing the st. line BOD. Cut off OB and OD on opp. sides of AC equal to MS and SN resp. ABCD is the quadl. required. There will be two solutions according as ∠AOB or ∠AOD is equal to 130°. The parallel sides therefore will be 4·9 cms. and 2·7 cms. *or* 2·3 cms. & 1·3 cms. **39.** (*a*) Pentagon ; 7 measurements ; 9, 13, $1 + 2(n - 2)$ measurements. (*b*) 3500 sq. yds **40.** (a) The areas are equal. (c) Follow the construction in the previous sub-division.

EXAMPLES XIV.

4. (7·5, −8) or (−7·5, −8.) **5.** Parallel to Y axis. **7.** 3·9 sq. in. **9.** a rhombus. **10.** 960 sq. ft. **11.** (b) i. (7·5, 3·5). ii. (−30, 17). **12.** (15, 36).

EXAMPLES XV.

1. $x = 1$, $y = 2$. **2.** $x = \frac{5}{14}$; $y = \frac{1}{14}$; **3.** $x = 1\frac{1}{2}$, $y = 2$ **4.** $x = 3$, $y = 1\frac{1}{2}$. **5.** $x = 5$, $y = -7$. **6.** $x = -5$, $y = -1·5$.

EXAMPLES XVI

1. (i) $2x-y=7$. (ii) $2x-5y=11$. iii. $2y-9x=85$. (iv) $y+x=4$. (v) $23y+11x+69=0$. (vi) $y=x-23$. (vii) $y=x-5$. (viii) $7y=8x-77$.

2. (a) $x+2y+1=0$. (b) $x+y+3=0$, $x+y+3=0$, $7x+8y+29=0$. (c) $2y=9x-106$; $y+x=0$.

4. 4 hours. **5.** Rs. 130. **6.** 3400 ozs; 4·5 c. ft. **7.** £ 1–12–0 : Rs. 30. **8.** 3·93″, 8·89 cms. **9.** 10·9 lbs., 14·06 lbs., $31\frac{1}{4}$ lbs., 5·12 viss, 9·6 viss. **10.** $4\frac{3}{8}$ lbs. 20 lbs. **11.** 5,$3\frac{3}{4}$, $1\frac{7}{8}$. **12.** $29\frac{1}{3}$ ft., $26\frac{2}{5}$ ft., 45 miles. **13.** $5\frac{3}{4}$ hrs. **14.** 2 hrs. 10 mts. **15.** Rs. 125, Rs. 7–4–8.

EXAMPLES XVII.

1. 1 hr. 48 min. from the time of starting, 16·2 miles from P. **2.** In 59 min. *i.e.* at 8-59 nearly, 9-58; 11-10. **3.** 25 min. later. **4.** 9-40 A.M. at the 10th mile. **5.** 18 miles. **6.** 12 miles. **7.** $4\frac{1}{2}$ miles an hour. **8.** 16 miles. **9.** 10 miles an hour. **10.** 9 miles an hour. **11.** 90 miles. **12.** 6-30 P.M. $3\frac{13}{29}$ and $4\frac{1}{6}$ miles an hour. **13.** 1o'clock. **14.** 24 mls. **15.** 4. **16.** $3\frac{3}{4}$ hours. 75 min. 25 min. **17.** 31st mile, 2 hours. 4 min. from start. **18.** 14 miles along the slope. **19.** 27 yards at 8 as. **20.** 13 days. **21.** 46th mile. **22.** Rs. 240. **23.** (a) 2 : 3. (*b*) $6\frac{3}{4}$, $11\frac{1}{4}$. **24.** 6 years, 26 years. **25.** Rs. 5. Annual increment. **26.** Rs. 80, Rs. 4. **27.** y rev. mks, x original mks. $y=mx+c$. Then, $y=1\frac{1}{3}x+5$, 49, 85; 36, 48. **32.** $24\frac{4}{9}$ yards or $24\frac{1}{2}$ yards nearly. **33.** 90 yards. **34.** $21\frac{9}{11}$ min. $5\frac{5}{11}$ and $38\frac{2}{11}$, $54\frac{6}{11}$ min. past four. **35.** Free 12 lbs. $\frac{1}{2}$ a. per lb. **36.** 9 days. **37.** 1 anna. Re. 1-11 as. **38.** 6 fur. 88. yds. per min. **39.** 33 miles.

40. (i) Rs. 185 (ii) $x+20y \ldots 250$; $x+28y=276$ (iii) Rs. 263.

41. (1) 20″, 120° (2) $-4°$ on A, $2\frac{2}{9}°$ on B (3) 5°.

42. $a=2$, $b=13$ (i) he who gets 13 (ii) those who get below 13 (iii) 28.

43. At the beginning of the 6th year.

44. (b) 12 annas more. **45.** $x=45$; Total cost Rs. 540. **46.** Three thousands. **47.** 30. **48.** (1) 40 lbs. (2) 34 ℔s. **49.** 88 ℔s. **50.** (a) Rs. 700 (b) Rs. 64. **51.** (1) Rs. 280. (2) $\frac{19}{30}$ carat. **52.** 128°; 16 sides. **54 & 55.** Fastest is that whose graph makes the greatest angle with the time axis.

56. No 1. C.B.M. fastest ; No. 29 P. slowest.

57. Let $\frac{1}{10}''$ along x axis represent a mile and let ½″ along y axis represent an hour. Let OA along x axis denote 48 miles and OB along y axis 4 hrs. Complete rec. OACB. Draw the graphs of 4 and 15 miles an hr. Call them OP and OQ. From C draw a ∥ to OQ cutting OP at T; the co-ordinates of T will give the required answers.

(b) x hrs $+$ y hrs $=$ 4 hrs ; $4x$ miles $+$ $15y$ miles $=$ 48 miles. $1\frac{1}{11}$ hrs after start ; $4\frac{4}{11}$ miles from starting point. **58.** 13·8 ; 7·8. **59.** (i) 117·4 lbs. (ii) 93·2 seers. **60.** 42 sq. units. 30 sq. units. **61.** 28·8 years.

62. Scale along X axis $\frac{1}{10}''=10$ yds., along Y axis $\frac{1}{10}''=1$ in. Through P where OP is 5 ft. 4 in. draw a parallel to X axis to cut the curve in two points ; through Q where OQ = 6 ft. 2 in. draw the parallel. Through R where OR = 4 ft. 4 in. draw the parallel.

63. 35·7 lbs. ; 45 lbs. nearly. **64.** (i) 48° (ii) 18°.

65. Feb 1st : 9 hrs 10 minutes ; July 1st : 16 hrs 29 min. April 1st : 12 hrs 33 minutes. Oct. 1st : 11 hrs. 52 min.

67. 300 yds. and 845 yds from firing point.

68. Highest 10 P.M. ; Lowest 6 A.M.

69. 8 as. ; about 20th Feb.

70. (a) 16. (b) 12 as., 14 as., 16 as.

71. Rs. 53-14-0 ; Rs. 55-9-0 ; Rs. 57-6-0.

72. Rs. 2-6-0 ; Rs. 3-10-0.

EXAMPLES XVIII.

1. 50% ; 33⅓%; 60%; 70% ; 36% ; 72% ; 12½% ; 20% ; 52%. **2.** (i) 6¼%. (ii) 8⅓% (iii) 12½%. (iv) 1 anna. **3.** Rs. 4-4 as. **4.** Rs. 101½. **5.** 66⅔%. **6.** $77\frac{7}{9}$% ;

7. $56\frac{1}{4}\%$. **8.** $44\frac{4}{9}\%$. **9.** 5%. **10** $33\frac{1}{3}\%$.
11. Rs. 2100. **12.** 21,000. **13.** $97\frac{1}{12}\%$. **14.** Rs. 300.
15. $5\frac{5}{9}\%$. **16.** 96%. **17.** $\frac{100\,(b-a)}{a}\%$. **18.** Rs. 2–5–6.
19. Rs. 2600. **20.** 57 ; $40\frac{5}{7}\%$. **21.** 40%.
22. $3\frac{1}{3}\%$ gain. **23.** Total 560 ; C 56 ; N 84. **24.** 120.
25. 12 as. per viss, 5 as. 3 ps. per viss. **26.** $33\frac{1}{3}\%$.
27. (i) Rs. 13-2 as. (ii) Rs. 7-14 as. (iii) Rs. 3-8 as.
(iv) Rs. 14-14 as. **28.** (a) 3·77. (b) 34·3575*a*.
(c) 6930. (d) 3430. (e) Rs. 54-3 as.
(f) Rs. 4-2-8. (g) Rs. 8-12 as. (h) Rs. 8-12 as.
29. (i) 4. (ii) 20%. **30.** $\frac{500}{a}\%$ **31.** $\frac{200}{a}\%$
32. (a) $c=8(b-x)$. (b) (i) 4. (ii) 13. **33.** (a) $x+b$.
(b) $x+\frac{bx}{100}$. **34.** Rs. 96. **35.** (a) Rs. 225.
(b) Rs. 200. (c) Rs. 248.

EXAMPLES XIX.

1. 28·6% ; 633·3% ; 4·0% ; 90·9% ; 62·5% ; 30% ; 17% ;
32% ; 175%. **2.** 40%. **3.** $12\frac{1}{2}\%$. **4.** 50·7%. **5.** $28\frac{4}{7}\%$.
6. $\frac{100a}{a+b}$ **7.** $33\frac{1}{3}\%$. **8.** Rs. 174 ; loss = Rs. 21.
9. Rs. 264. **10.** (a) Rs. 97·8 as. Rs. 227-8 as.
(b) Rs. 32-4-11 ; Rs. 13-12 as. **11.** 17. **12.** Rs. 3-2 as. 10%
13. 21 miles 7 furlongs. **14.** Rs. 13570. **15.** $4\frac{1}{2}$ as.
16. $3\frac{1}{3}\%$ gain. **17.** Rs. 5000. **18.** $6\frac{1}{4}\%$. **19.** Rs. 210.
20. Rs. 2-1-9. **21.** Rs. 2-1-4. **22.** Rs. 275 ;
Rs. 300 ; 9·6%. **23.** 92% nearly. **24.** A = 50·6% ;
E = 19·9%. **25.** Rs. 118-12-10. **26.** $\frac{4}{5}\%$ increase.
27. $8\frac{1}{3}\%$. **28.** $11\frac{1}{9}\%$. **29.** Decrease by
Rs. 10736-4 as. **30.** Rs. 142-8 as. **31.** Rs. 5-7-6
per hundred. **32.** 26%. **33.** By 95·6% nearly.
34. Rs. 625. **35.** Rs. 5 ; $44\frac{4}{9}\%$. **36.** Rs. 95 gain.
37. 4·17 measures. **38.** 3 annas. **39.** the former
$1\frac{3}{4}\%$ more. **40.** £. 2-9s. nearly. **41.** gain 20% nearly
42. $7\frac{17}{32}\%$. **43.** $13\frac{1}{3}\%$. **44.** $\frac{1}{2}$ a measure

45. 2500. **46.** $83\frac{1}{3}$ acres. **47.** 20200 **48.** 72·6 lb. of gold.; 1540·2 lb. of other metal. **49.** Rs. 1600; Rs. 4800. **50.** $27\frac{3}{11}$%. **51.** 32·8% nearly. **52.** $45\frac{5}{7}$%. **53.** Rs. 26. **54.** $18\frac{2}{9}$%. **55.** 20% (a) $x=1·2y$ (b) Rs. 3; Rs. 2–3 as. **56.** Rs. 62-1-6. **57.** 24% nearly. Rs. 30-12 as. **58.** 12 lacs; Rs. 3-8 as. per thousand. **59.** 3 as. 6·2 ps. **60.** (a) Rs. 20. (b) the former. **61.** (i) 24 lb. (ii) 1 a. 3 ps. **62.** Rs. 4800. **63.** Publisher Rs. 1058; Author Rs. 5322; Bookseller; Rs. 4920. **64.** 206%; 21·9%. **65.** 30%, 25%, 40%, $12\frac{1}{2}$%, (a) 1·5%. **66.** Rs. 49-5-9. **67.** 91·5% nearly. **68.** Rs. 800. **69.** Rs. 2400. **70.** (i) 20% (ii) $6\frac{1}{4}$%. **71.** $\frac{t}{100+r}=\frac{P}{100-q}$ **72.** 160. **73.** $9\frac{3}{8}$%; 18·5%. **74.** $9\frac{11}{52}$%. **75.** (i) 6% (ii) 18% gain. **76.** (i) 15%. (ii) 10% of the *original adulterated* milk. **77.** 46 c. ft. **78.** (i) 12 : 5. (ii) 960 viss. **79.** 23·1%. **80.** 29·2%. **81.** $2ab=7c$; Rs. $1071\frac{3}{7}$. **82.** $6x=5(y-1)$; 7 as. 10 as. **83.** (1) Rs. 1450. (2) Rs. 1160. (3) Rs. 22-4 as. gain. **84.** $12\frac{1}{2}$%. **85.** Rs. 64-10 as.; 30·5%. **86.** 60%; 40%. **87.** 30·6%. **88** Cost: Rs. 4-12 as. Quoted price Rs. 5-15 as. **89.** 11 s. 7 d. nearly, **90.** Quadrupled the capital. **91.** 57%; Rs. $12\frac{1}{2}$; Rs. $62\frac{1}{2}$. **92.** 80300000 francs. **93.** 37·2%. **94.** Rs. $357\frac{1}{7}$. **95.** (i) Rs. 63. (ii) Rs. 4830; Rs. 4200. (iii) Rs. $4105\frac{1}{2}$ **96.** Rs. 620–13–4. **97.** Rs. 16-4-3. **98.** 93·35%. **99.** (a) at *B* Rs. $22\frac{1}{2}$; at *A* Rs. 27, at *C* Rs. 30. (b) *A* 35%; *B* $12\frac{1}{2}$%; *C* 50%. **100.** Rs. $337\frac{1}{2}$. **101.** (a) Rs. $1\frac{1}{2}$ (b) Rs. $262\frac{1}{2}$. **102** $45\frac{5}{11}$%. **103.** $37\frac{11}{12}$%. **104.** $y=\frac{4}{3}x$. Rs. 6; Rs. $2\frac{1}{2}$. **105.** Profits decrease, 30; 29. **106.** (i) 2 as. 1 pie; 3 as. 4 ps.; 3 as. 9 ps.; 8 as. 9 ps. (ii) 4 as. $9\frac{3}{5}$ ps.; 4 as. $4\frac{4}{5}$ps.; 2 as. $9\frac{3}{5}$ ps.; 1 a. $7\frac{1}{5}$ ps.

EXAMPLES XX.

1. 10. **2.** 17% **3.** (a) 1·2%. (b) 77·4%. **4.** 362. **5.** 7·9%; 10·8%; 12·2%; 8·6%; 22·9%. **6.** £ 4–5 s. **7.** 39·9%; 26·6%; 24·5%; 9%. **8.** 564·560. **9.** 506 to 509. **10.** 1s. 3d.; £ 1107726.

11. 62 lb. 8 oz. **12.** 453 gms. **13.** 5s. 5d. **14.** £1-4-10d.
15. 9s. **16.** 16827 francs. **17.** 567 leaves. **18.** Rs 1-7-2.
19. 19·3. **20.** 270·513 oz. **21.** 2·21 **22.** ·45.
23. $853\frac{1}{3}$ ft.; £23437½. **24.** £ 73081-14-4d;
£16800-11-10; 80·3%. **25.** *G.W.* 61·8: *L. & N. E.* 61·6;
S. 56·8; *LM & S* 56·1 **26.** ·99%; 22·69 crores.
27. 7912 miles. **28.** ·0195%. **29.** 19·5%; 20·4%;
9·7%; 12·0%; 38·4%. **30.** 1397866, 62799, 27911,
118621, 146532, 134902, 130250, 307019. **31.** (i) 0·24.
(ii) 160. **32.** (i) 35·4 lb. (ii) 19·3. **33.** 39·37.
34. 1903; £2-15 s.; 1904; £3-3 s.; 1905: £2-17 s.
9·4%; 18·7%. **35.** (a) 18·33. (b) 3.1417; $3\frac{17}{120}$.
36. 7 as. 6 ps. **37.** *C.* **38.** Bengal 50%; Bombay
24·8%; Madras 11·7%; Other Provinces 5·1%
Punjab 4·7%; Burmah 3·6% **39.** 103, 102, 23
40. 32·88. **41.** (i) Gross 64800, net 21800. (ii) 43000
(iii) 5·00%. **42.** 5·854%. **43.** 66·8, 7·7; 25·5
44. Re. ·01143; Re. ·01296; 13·4%. **45.** (b) 98·3%
99·2%; 98·6%; 100%. (c) 98·7% **46.** 48·0, 56·5, 52·5
51·3, 50·6. **47.** Rs. 99242 loss. **48.** (i) 367 millions
(ii) 38·8%. **49.** (i) 30 crores. (ii) 9 lacs, (iii) 180 lacs
50. (i) 62 (ii) Rs. 6055-7 as. (iii) Rs. 3365-12 as
(iv) Rs. 54-4 as. **51.** (i) 6·8″. (ii) 25·09 oz. (iii) 1·95″
(iv) 7400 miles. **52.** (i) 43939100. (ii) £43499710
53. 15·5. **54.** 123·2. **55.** £29170800; £83350000
56. (i) ·2179. (ii) 43 ft. **57.** 93,000,000; 400,00
58. ·003003. **59.** 76·2. **60.** 1117, 745, 378
61. (a) 179. (b) ·00446″. **62.** 231 kg. **63.** 3243 to 328
64. ·292; 14·543. **65.** (a) 8840 metres. (b) 14 s. 8
66. (i) 1068541: 554; 31377801. (ii) 2·9%. (iii) (a) 73
(b) 26·1. **67.** Rs. 2-4-3 ps.; Rs. 2-4-3 ps; Rs. 1-0-
68. ·980, 1·59, ·794, ·206. **69.** ·2408, ·6152, 1·88
11·86, 29·46. **70.** 3955.

EXAMPLES XXI.

1. 441·36. **2.** ·0003. **3.** 2·7. **4.** 4·2 cm
5. 40·5, 8·16. **6.** (a) $r=h=1·9''$ Proce
(b) $h=\frac{r}{2}=·95''$ Proceed. **7.** (i) 44·0 sq. ft. (ii) 164 gra

8. ·00495. **9.** 110·3 grams. **10.** 4177 ft. **11.** ·1708% ; 3·416 metres. **12.** 35. **13.** 208 c. in. **14.** 12.

EXAMPLES XXII.

1. Rs. 3920. **2.** Rs. 4322½. **3.** Rs. 11. **4.** Rs. 105. **5.** 12½%. **6.** 8 years. **7.** the latter ; Rs. 4½. **8.** Rs. 3733⅓. **9.** 10%. **10.** Rs. 500. **11.** (a) 12½%. (b) (i) 18¾%. (ii) $20\frac{5}{16}$%. (iii) $17\frac{7}{64}$%. (c) (i) the third (ii) $\frac{57}{64}$%.

EXAMPLES XXIII.

1. $I = \frac{Pnr}{100}$; $A = P\left(1 + \frac{nr}{100}\right)$. **2.** S.I. in each case is :—(a) Rs. 54. (b) Rs. 61-14 as. (c) Rs. 37-3 as. (d) Rs. 62-8 as. (e) Rs. 156-4 as. **3.** $P = \frac{100I}{nr}$ or $\frac{A}{1 + \frac{nr}{100}}$; $n = \frac{100I}{Pr}$ or $\frac{100(A-P)}{Pr}$; $r = \frac{100I}{Pn}$ or $\frac{100(A-P)}{Pn}$ **4.** (a) i. 8% ii. 7½% iii. $7\frac{49}{78}$% (b) i. Rs. 68-12 as. ii. Rs. 3000 iii. Rs. 532 (c) i. 5 years ii. 4 years. **5.** (a) Rs. 200 : 10% (b) $x = A - \frac{(B-A)m}{n-m}$: 400. **6.** $4\frac{13}{50}$%. **7.** $18\frac{3}{4}$%. **8.** Rs. $91\frac{1}{2}$. **9.** Rs. 10010-11-5. **10.** $7\frac{13}{21}$ years. **11.** Rs. 2655. **12.** Rs. 1430-10 as., Rs. 219-6 as. **13.** $21\frac{3}{7}$%. **14.** Pensioner by Rs. 1282½. **15.** Rs. 103-2-½. **16.** $4\frac{1}{6}$ years. **17.** Rs. 30 ; Rs. 3 ; Rs. 399—4—10. **18.** C. I. in each case is :—(i) Rs. 253-14-11½. (ii) Rs. 435-12 as. (iii) Rs. 347-1-2. **19.** (a) $A = P\left(1 + \frac{r}{1000}\right)^n$. (b) (i) 3½%. (ii) 2 years. (iii) 10000. **20.** Rs. 3121-12-8. **21.** Rs. 903. **22.** Yes. **23.** Rs. 2700. **24.** 8·3 % nearly. **25.** Rs. 959-7-6 nearly. **26.** Rs. 1947-13-5 ; Rs. 1546 nearly. **28.** Rs. 18662-2-4. **29.** Rs. 1210 ; Rs. 1000 and Rs. 1100. **30.** Rs. 92610. **31.** (b) $xy : mn + 1$ (c) $xy : mn$. **32.** Rs. 1131, Rs. 1664, Rs. 46013, Rs. 13089.

33. Rs. 24000. **34.** Rs. 685-7 as. (to the nearest anna). The former by Rs. 4-7 as per cawnie. **35.** Rs. 1927-4-8; 28½%. **36.** Rs. 189200. **37.** 5% nearly; Rs. 13395. **38.** 104·5%. **39.** 7 as. 10 ps. **40.** 8·9 %. **41.** Rs. 7266. **42.** (a) 12½%. (b) 599-11-8. **43.** 32·8% nearly. **44.** Rs. 304-6-3. **45.** Younger. Rs. 16200. **46.** Rs. 2759-12 as. **47.** Rs. 202-4 as. **48.** *B* by Rs. 3375. **49.** Former by Rs. 297½. **50.** The latter disadvantageous by 7·3%. **51.** The former by Rs. 668-12 as. **52.** The former by Rs. 34-14 as. **53.** The former by Rs. 1224-4 as. **54.** Rs. 39·9. **55.** Rs. 6-14-5. **56.** The former; Rs. 265 (to the nearest rupee), 11%. 9·6%. **57.** Rs. 8801-4 as. **58.** The former by Rs. 13-11-7 ps. **59.** Yes; Rs. 136-4 as.; 84⅜%. **60.** 12%; Rs. 120. **61.** 8%; Rs. 1924-12-3. **62.** 182 days. **63.** Rs. 53000; Rs. 56180. **64.** 21″ nearly. **65.** 72·8%; 579 hundreds. **66.** 1 : ·556. **67.** 18¾%. **68.** $x=b\frac{(a-c)}{c}$; 11 mths. **69.** 3·1′; 29·4′. **70.** 515944; 3·2% decrease. **71.** 4. **72.** Rs. 1241-4 as.; 14·5%. **73.** Rs. 660; Rs. 176; Rs. 82½. **74.** Rs. 99-12-3; Rs. 127-0-11; Rs. 93-2-10. **75.** Rs. 36. **76.** Rs. 529, Rs. 324, Rs. 144; 15⅓%, 12%, 8%. **77.** Rs. 300. Rs. 300. Rs. 320. **78.** A, Rs. 22-10-8; B, Rs. 23-5-4; C, Rs. 23-10-8. **79.** Rs. 9269-5-4. **80.** 7 as. (nearly). **81.** 9000; 30·4; 29·6. **82.** Rs. 4500; Rs. 5400; 12⅓%; $18\frac{1}{5}$%. **83.** Rs. 5000. **84.** 10 as. 3 ps. **85.** Either of the two can be the manager; thus there are two solutions :—

(a) Rs. $142\frac{6}{7}$; 7·7% and 6·8% nearly.

(b) Rs. $83\frac{1}{3}$; 4% and 4·3% nearly.

86. 10 as. 8 ps. **87.** 9 as. 6 ps. **88.** (i) Rs. 2-4-6. (ii) Rs. 2518-13-2. (iii) Re. 0-4-9.

EXAMPLES XXIV.

1. 7 as. 1 p.; 1 anna 5 pies. **2.** 12 years 10 months. **3.** 13 years. 4·1 months nearly. **4.** The *same* 14·25 gr. **5.** 12 years 6·6 months. **6.** $7\frac{31}{32}$ %. **7.** 10 %.

8. (1) $\frac{p+x}{p+y}$ miles per hour. (2) 16·5 miles per hour. (3) 13 miles per hr. (4) 8·6 miles per hr. (5) 8·6 miles per hr. (6) $\frac{v^2-n^2}{v}$ miles per hr. **9.** $\frac{ap+bq+cr}{a+b+c}$. **10.** 35·2%. **11.** 15 yrs. 5 mths. nearly. **12.** 224. **13.** (a) $10\frac{10}{17}$ miles per hour. (b) halt of 11 minutes. **14.** (i) 152. (ii) $96\frac{2}{7}$. **15.** $x=\frac{q(c-q)}{(p-q)}$; 45. **16.** 16·12 years. **17.** 52.

EXAMPLES XXV.

A

1. (a) $2\frac{1}{2}$ miles, $1\frac{3}{4}$ miles, a miles. (b) 3 hrs., $4\frac{1}{2}$ hrs., p hrs. **2.** (a) 6 miles, 15 miles, $6m$ miles. (b) $1\frac{1}{6}$ hours. $\frac{3}{4}$ hr., $\frac{p}{6}$ hrs. **3.** (a) 25 miles. (b) 5 miles. (c) 5 miles. (d) 3 hrs. (e) $\frac{x}{a+b}$ hrs. **4.** $\frac{p}{2x-a}$ hrs. **5.** 3 miles; 2 miles; 1 mile; $2\frac{1}{2}$ hrs. **6.** (a) $12\frac{1}{2}$ miles from P. (b) $6\frac{1}{2}$ miles. (c) $10(x+1\frac{1}{4})-16x$ miles. It means that B overtakes A. $2\frac{1}{12}$ hrs. $33\frac{1}{3}$ miles from P. **7.** $\frac{a}{b-c}$ seconds; 45 seconds. **8.** $13\frac{1}{3}$ minutes. $5\frac{3}{5}$ miles. **9.** (1) 10 minutes. (2) $6\frac{1}{2}$ miles. (3) $6\frac{2}{5}$ hrs., 4 hrs. **10.** (a) $\frac{60a}{t}$ miles per hr. (b) $\frac{60a}{t-2b}$ miles per hr. **11.** 14 miles per hr. **12.** $2\frac{2}{5}$ miles per hr. **13.** 20 minutes. **14.** 4 miles 1270 yds. **15.** $\frac{S(tk-h)}{k-s}$. **16.** No. 3 RE & No. 6 TE meet at 211th mile, at 19 hrs. 5 min. **17.** (i) At Nallur at 9-34 A.M. nearly. (ii) 11 A.M. (iii) 11-25 A.M. **18.** $7\frac{1}{2}$ miles. **19.** 4 miles; 9-20 A.M.; 6 miles per hr. **20.** 1 hour 42 minutes after the former had started. **21.** (1) 6 miles an hour. (2) $2\frac{2}{3}$ miles. (3) $1\frac{3}{4}$ hrs. after Inspector started. **22.** (1) 9 miles 10 A.M. (2) 11-45 A.M. **23.** (1) 1 hr. 15 mins. (2) 1 hr. **24.** $17\frac{1}{2}$ miles per hr.

25. (i) 165 ft. or $31\frac{3}{7}$ ft. per min. (ii) $272\frac{4}{5}$ ft. per min. **26.** $b\sqrt{2}$ yards per min. **27.** 2½ miles. **28.** 24. **29.** 4½ miles per hr. **30.** 7 miles. **31.** 20 miles per hr. **32.** $20\frac{8}{11}$ miles from the station wherefrom there is a rise. In 3 hrs. $51\frac{3}{11}$ minutes. **33.** $21\frac{5}{6}$ miles. **34.** 6½ mins.; **35.** 96 leaps. **36.** 3 min. 10 sec. nearly after I started. 279 yds. from M. school. **37.** (i) 36, 36, 24 miles per hr. (ii) Train from B at 1 P.M.; Train from A at 2 P.M.; third Train at 4-20 P.M. (iii) 36 miles; and 38·4 miles. **38.** $1\frac{1}{7}$ hrs.; 2 hrs. 4 min. **39.** 11-30 A.M.; 30 miles from starting pt. **40.** In 3 hrs.; at 36th mile. **41.** 19 fur. **42.** $3\frac{3}{5}$ miles per hr.; 6 miles per hr. **43.** $12\frac{14}{23}$ min. after the dog saw the hare; 689 ft. (nearly) from where the dog left the hunter. **44.** 4 miles per hr. *Hint*: the man has walked in $12\frac{27}{29}$ min. the distance which the report has travelled in $\frac{2}{29}$ min. **45.** *Note*: Distance made by the steamer in 9 min. 48 sec. = distance made by sound in 12 sec.; 15 min. 15·8 sec.; 15·7 miles per hr. **46.** 9 min.

B

47. (1) 22 yds. (2) 48·4 yds. (3) 193·6 yds. **48.** B; (1) 11 yds. (2) 44 yds. **49.** (a) 25 yds. (b) 100 yds. **50.** (a) y. (b) i. $\frac{x}{2}$ yds. ii. $\frac{y}{2}$ yds. iii. $\frac{880(y-x)}{1760-x}$. iv. $\frac{5}{11}x$. **51.** 120 sec.; 132 sec. **52.** 8 mts.; 8·2 mts.; 8·4 mts. **53.** 293⅓ yds.; 352 yds. **54.** $12\frac{2}{7}$ yds.

C

55. Ex. 1. (a) i. $16\frac{4}{11}$ minutes. ii. $32\frac{8}{11}$ minutes. iii. $49\frac{1}{11}$ minutes. (b) i. 2-$10\frac{10}{11}$ P.M. ii. 2-$27\frac{3}{11}$ P.M. iii. 3 P.M. iv. 2-$43\frac{7}{11}$ P.M.

Ex. 2. (a) 12; 12–$16\frac{4}{11}$ P. M.; 12–$32\frac{8}{11}$ P. M. (b) 2-$10\frac{10}{11}$ P.M.; 2-$27\frac{3}{11}$ P.M.; 2-$43\frac{7}{11}$ P.M.

Ex. 3. (a) i. 5–$27\frac{3}{11}$. ii. 5–$16\frac{4}{11}$ or 5–$38\frac{2}{11}$; iii. 5-$10\frac{10}{11}$ or 5-$43\frac{7}{11}$. iv. 6. (b) i. 12. ii. 11-$5\frac{5}{11}$ or 11-$49\frac{1}{11}$. iii. 11-$10\frac{10}{11}$ or 11-$43\frac{2}{11}$. iv. 11-$27\frac{3}{11}$.

56. (1) Ser^{t} & Cor^{l} at 8-40 P.M., $3\frac{1}{3}$ miles from starting pt. (clockwise).
Ser^{t} & Cor^{l} at 8-48 P.M., $2\frac{2}{5}$,, ,, ,,
Sol^{r} & Cor^{l} at 12 P.M. 4 ,, ,, ,,
(2) Sol^{r} & Cor^{l} at 4 A.M.; Ser^{t} & Sol^{r} at 4 A.M.; Ser^{t} and Cor^{l} at 4 A.M. (3) 4 A.M.

D

57. (a) 32 miles. (b) 64 miles. (c) $16x$ miles. (d) $16(a+b)$ miles.

58. (a) 24 miles. (b) 20 miles. (c) $13\frac{1}{3}$ miles. (d) $8x$ miles. (e) $8\left(a+\frac{b}{60}\right)$miles. **59.** $3\frac{3}{8}$ hrs. **60.** $3\frac{3}{4}$ hrs. **61.** $t(a+c)=b$. **62.** $2c(a+b)=bx(2a+b)$; $3\frac{3}{4}$ hrs. **63.** 45 min. **64.** *Hint:* boatman's speed in still water, rate of river-flow, and the resultant speed are represented respectively by the hyp. and the two other sides of a rt.-angled triangle; rate of river-flow $=\sqrt{5^2-(\frac{7}{6})^2}=4{\cdot}86$ miles per hour (nearly).

E

65. (1) 18 sec. (2) 29 sec. (3) $24\frac{6}{11}$ sec. (4) $2\frac{2}{5}$ mts. (5) $13\frac{1}{2}$ sec. (6) $28\frac{1}{5}$ sec. **66.** $4\frac{7}{8}$ mts. **67.** $12\frac{3}{10}$ mts. **68.** 5 mts. 23 sec. nearly. **69.** 30 miles; 15 miles (nearly). **70.** $45(l+x)=22mn$. 176 yds.; 45 miles per hour.

F

71. (1) $\frac{1}{2}x$ mts. (2) $4; 8; 4x$; (3) $\frac{1}{2}$; $\frac{1}{4}$. **72.** Let each pump out x ft. of water in 1 minute. Water rises $(96x+3)$ ft. in the first 8 mts.; in the next 6 mts. water rises $\frac{6}{8}(96x+3)$ ft., which is $(84x+2)$ ft. Hence $x=\frac{1}{48}$ ft. Supposing all hands at work for next y mts., we have $\frac{y}{8}(96+3)$ ft. $=(18yx+1)$ ft., where $x=\frac{1}{48}$ $\therefore y=4$ mts. Ans $=18$ mts. **73.** $\frac{3(yz+xz-xy)}{xyz}$; 6 P.M. **74.** 4 hrs. 10 mts. nearly. **75.** (a) $102{\cdot}3$ galls., (b) $323{\cdot}2$ gallons. **76.** 20 mts. **77.** 12 hrs.

G

78. 16 days 16 hours hence. 3-57$\frac{2}{9}$ A.M and 4-2$\frac{2}{9}$ A.M. **79.** May 30th, 6-1¼ P.M. **80.** 100 c. ft. **81.** 9 miles per hr. **82.** 1$\frac{19}{56}$ stones. **83.** 10-30 P.M **84.** 3½ hrs. **85.** Noon Dec. 31st 7-30. **86.** 16 days 16 hrs. hence. 3-57$\frac{2}{9}$ P.M. 4-4$\frac{2}{9}$ P.M. **87.** 30 weeks.

EXAMPLES XXVI.

1. 2 : 3 ; 600, 900. **2.** 1·6 cms. **3.** 3$\frac{9}{11}$ fur., 2$\frac{2}{11}$ fur. **4.** 3 : 2. **5.** 5, 7, 2. **6.** (1) 16 : 1. (2) 1 : $\sqrt{2}$ (3) $\sqrt{2}$: 1. (4) 2 : $\sqrt{3}$. (5) 44 : 7. **7.** 2240. **8.** Rs. 18400. **9.** 10$\frac{1}{2}$ as. **10.** 18 carats. **11.** 19·5 carats. **12.** 4 : 7. **13.** (i) 800 (ii) 16⅔%. **14.** 1 fur. 155 yds. **15.** Rs. 10. **16.** 187 : 192. **17.** 240,000. **18.** 1 hr. 7½ mts. past midnight. **19.** 6⅔%.

EXAMPLES XXVII.

1. (1) Rs. 7-11-4. (2) Rs.8-5-4. **2.** (1) 2 viss (2) 13$\frac{1}{3}$ viss. **3.** 3 : 1. **4.** 2 : 1. **5.** 3¾s. **6.** 7 : 1 (a) 7 : 1. (b) 56 and 8. (c) 84. **8.** 255·36 seers. **9.** 4½ oz. **10.** $\frac{17}{96}$ maund. **11.** 18 carats. **12.** 16 oz. **13.** 14 : 39. **14.** 343 : 169. **15.** (a) 10 : 7. (b) Rs. 731. **16.** 6 : 5. **17.** $\frac{a^2x}{(a+b)^2}$, $\frac{abx}{(a+b)^2}$, $\frac{abx}{(a+b)^2}$, $\frac{b^2x}{(a+b)^2}$, 19·2″, 14·4″, 14·4″, 10·8″, **18.** $\frac{a(c-b)}{c(a-b)}$; ·69 nearly. **19.** $\frac{b-c}{c-a}$, $\frac{4}{35}$ **20.** (b) i. 1½ sov. weight. ii. 13½ sover^{n} weight. **21.** 20·4 carats. **22.** 28. **23.** Rs. 312. **24.** 19 : 17. **25.** Rs.7 ; Rs. 4-6 as. ; Rs. 2-10 as. **26.** 13¼ oz. (nearly). **27.** $x = \frac{(a-b)(a-c)}{a}$; $\frac{1}{5}$ of the mixture. **28.** 28·76 grams of pure gold ; 11·5 carats fine. **29.** (a) 9. (b) 18%. **30.** Rs. 69-12 as., Rs. 116-4 as., Rs. 162-12 as., 353-15-7. **31.** Rs. 3$\frac{3}{89}$ per acre. **32.** Rs. 320,000. **33.** 6 : 5 in each case. **34.** 132. **35.** 37½, 80, 28. **36.** 60 lacs ; 3⅜ lacs.

37. 60%. **38.** Officer £ $\frac{4000}{173}$; midshipman £ $\frac{3000}{173}$; sailor £ $\frac{1100}{173}$. **39.** A's profit Rs. 2460 ; C's profit 3580. B's capital and profit Rs. (6000 + 1380). **40.** Rs. 85, Rs. 50. **41.** Headmaster Rs. 376-4-0 ; Senior L.T.'s Rs. 150 each ; Junior L.T.'s Rs. 95-10 as. each ; other L.T.'s Rs. 80 each ; Secondary Grade Rs. 33-12 as. each ; rest, Rs. 46-14 as. each. **42.** C has to give his dues Rs. 4-5 as., then, A gets Rs. 39-7 as., B gets Rs. 6-1 anna. B is to get Rs. 4-5 as. from C. **43.** 320, 256. **44.** Rs. 600, Rs. 150.

EXAMPLES XXVIII.

1. $\frac{27}{28}$ cms. ; $\frac{15}{14}$ cms. ; $3\frac{13}{14}$ cms. **2.** (i) 6 cms. 6½ cms. (ii) 7½ sq. cms. **3.** $12\frac{4}{21}$ ft. **4.** (a) 4500 ft. (b) $6\frac{9}{16}$ ft., 26¼ ft., 78¾ ft., $3\frac{15}{16}$ ft. **5.** (a) 112½ yds. (b) Faster 96 yds., slower 72 yds. **6.** (a) i. $a^2 : b^2$. ii. $p^2 : q^2$. iii. $x^2 : y^2$. iv. $p^2 : q^2$. v. $x^2 : y^2$. vi. $a^2 : b^2$. vii. $\sqrt{p} : \sqrt{q}$. viii. $p^2 : q^2$. (b) (i) 4 : 1. (ii) 8 : 1. **7.** 3 5 cwts. *Note* : Volumes of similar solids are proportional to the cubes of corresponding sides or lengths. **8.** Similar to 6 (b). **9.** 1 : 8. **10.** (i) 3 : 4, (ii) $(3+2\sqrt{2}) : (4+2\sqrt{2})$. **11.** 72 ft. **12.** In $\triangle^s$ APO, ACB $\angle$A is common, $\angle$AOP = rt $\angle$ = ACB. Hence $\triangle^s$ are similar. $\frac{AC}{AB} = \frac{AO}{AP}$ $\therefore$ $AC = \frac{AO}{AP} \times AB = \frac{x}{\sqrt{x^2+y^2}}$. $2x = \frac{2x^2}{\sqrt{x^2+y^2}}$. Similarly $BC = \frac{2xy}{\sqrt{x^2+y^2}}$. Area = ½ AC. $CB = \frac{2x^3y}{x^2+y^2}$. **13.** 43·5 ft. **14.** 4 yds. **15.** 100 yds. **16.** 7½ miles per hour. **17.** OX and OY two st. lines at 60° ; along OX, take OA = 8 units ; along OY take OB = 5 units ; join AB. Now along OY take OC = 8 units, and through C draw parallel to AB to meet OX at D ; OD gives the distance in the first case. Proceed similarly for the second case. **18.** (i) ·28 in. (ii) 2·13 in. **19.** 4 : 3. 16 : 9. **20.** 11¼ ft. **21.** 48,000 sq. ft., 12000 tons. **22.** (b) Proceed as in examples X. Q. 22 (iv) (a). (c) with

$\sqrt{5}$ AB as one side, describe a rt. $\angle^{d}$ $\triangle$ with one of its angles $= 30°$. The other side will be $\frac{\sqrt{5}}{\sqrt{3}}$AB. **23.** Proceed as in 6 and 7 on pp. 282-83. **24.** 15 sq. in. **25.** 0·17. **26.** $24\sqrt{6}$ in. × $16\sqrt{6}$ in. **27.** (a) 45 miles 3 fur. 140 yds. (b) 26155 sq. miles. **28.** 64 ft. nearly; 4·3″ per sec. **29.** 29400000 sq. miles. **30.** 7439 sq. ft. **31.** $30\frac{5}{9}$%. **32.** 9400 sq. metres. **33.** (i) 15·78 miles. (ii) 100845 sq. miles. **34.** (i) 78·914. (ii) 18682 sq. miles.

EXAMPLES XXIX.

1. (a) $18\frac{6}{7}''$, $25\frac{1}{7}''$, 22″, 44″, 66 cms., $12\frac{4}{7}$ ft., $15\frac{5}{7}$ ft.

(b) $1\frac{3}{4}''$, $3\frac{2}{11}''$, $5\frac{1}{4}$ cms., $6\frac{1}{8}$ cms., $1\frac{1}{6}$ ft., $2\frac{5}{8}$ ft.

2. $14\frac{2}{3}$ ft. 360 revolutions, $1\frac{9}{19}''$ less. **3.** 7911 miles. **4.** $D = \pi nxd$; 19250 ft. **5.** 4·34 seconds nearly; 339 times. **6.** $4\frac{1}{3}$ minutes. **7.** 5 miles. **8.** (a) $3\frac{1}{7}$ sq. in.; $78\frac{4}{7}$ sq. in.; 154 sq. in.; $19\frac{9}{14}$ sq. in.; $38\frac{1}{2}$ sq. yds.; (b) $8\frac{46}{63}$ sq. in. (c) $7\frac{1}{2}$ sq. in. (d) $r = 12\frac{3}{5}''$ ○$^{ce} = 79\frac{1}{5}''$; area of sector $= 69·3$ sq. in. **9.** ·4 sq. in.; (1) ·6 sq. in.; (2) ·8 sq. in. **10.** (a) 1·4 in.; (b) 25 in. nearly. (c) $9\frac{3}{4}''$, 8″, 1·1″, 2″, 2·5″. **11.** Rs. 1-10-10. **12.** (a) $(x^2 - y^2)$ sq. ft.; (b) $\pi b\ (24a + b)$ sq. in. **13.** (a) 3850 sq. yds.; (b) Rs. 591-15 as. (c) Rs. 115-8 as. **14.** 11. **15.** 3·2″ nearly; $\frac{7}{16}$. **16.** 7·08 sq. in. **17.** Rs. 10-8 as.; Rs. 124-7-3. **18.** 33·8 sq. ft.; Rs. 25-5-8.

		Percentage.	*Angle at Centre.*
19.	Arctic	2·8	10°
	Atlantic	17·8	64°
	Pacific	34·5	124°
	Indian	12·7	46°
	Land	26·4	95°

20. 132·4; 141·3; 41·0; 19·5; 20·3; 5·8. **21.** 123·3 ft. **23.** 8·54″. **24.** 3·1 sq. in. or ·24 sq. in. **25.** $2\sqrt{a^2 - b^2}$ in. **26.** (a) $p + \sqrt{p^2 - q^2}$ in.; $p - \sqrt{p^2 - q^2}$ in. (b) 2·9 cms.; 6·5 cms. and 7 cms. **27.** $\sqrt{5}$ in. (a) $2\sqrt{a^2 - b^2}$ in. **28.** (a) 2 cms. (b) 1·95 cms. **29.** (a) $\sqrt{5}$ in. (b) 2·3 in. (c) ·2 in. **30.** 2·1″ **31.** 28 : 3. **32.** 6·3 miles.

33. 606 ft. ; Rs. 2970. **35.** 0·12a, 0·14a. **36.** rope = 22 yds. ; area $806\frac{6}{7}$ sq. yds. **37.** 23880 trees ; 7598 yds. **38.** $40\sqrt{3}$; $40\sqrt{6}$. **39.** 1·2″.

40. (a) (i) $2a + \dfrac{22\,(b-60)}{7}$ yds. (ii) $2a + \dfrac{22b}{7}$ yds. (b) $227\frac{1}{7}$ ft. in advance.

41. (a) 9·06 sq. cms. ; 305·1 sq. cms. (b) 3·14 sq. cms. **42.** area = ¾ of circle of radius 60 ft. + twice of fourth of a circle of radius 20 ft. **43.** $\frac{4}{5}$; $\frac{1}{5}$. **44.** (i) total area = area of enclosure + area of two semi-circles with 8 ft. radius + area of four quadrants of radius 2 ft. (ii) 88 ft. **45.** 1612·5 sq. ft. **46.** 141·4 yds. **47.** $(\pi r^2 - 6\pi\frac{1}{9}r^2)$ i.e., 12π sq. in. **48.** 74536 sq. yds. **49.** $535\frac{5}{7}$ sq. ft. **50.** 3 ft. 4·6 in. **51.** EC = ED = 4·7 cms. ; EN = 4·6 cms. **52.** $2r^2\left(\dfrac{\pi}{3} - \dfrac{\sqrt{3}}{4}\right)$ sq. units. **53.** 4·5 sq. in. **54.** 27 mls. **55.** 82·1 sq. in. **56.** 192π. **57.** $(1+\sqrt{8})$ i.e., 3·8 ft. **58.** 9·5″. **59.** 16 : 1. **60.** $144\,(\sqrt{3}+\pi)$. **61.** similar to Q. 24 Ex. X. **62.** (b) split the number into 2 factors x and y ; draw AB = x units (greater factor). Describe a semicircle on AB ; from AB cut off AM = y units. Through M draw MN perp. to AB, to meet the $\bigcirc^{ce}$ at N. AN gives the required root. **63.** 120°. **64.** 6½ ft. **65.** 1·4 ft. **66.** 3⅜ cms., 3·8 cms., 1·5 cms. **67.** 20½″. **68.** (b) 3 cms. (c) 4·3 cms **70.** 72°, 4·3 cms. **71.** 23°, 4·9 ft., 6·1 ft. **72.** 19″. **73.** Angles all equal, and half of central angle ∠BAM = ∠ACB. **74.** (a) perp. bisector of AB : perp^{r} from A to AM ; the pt. of intersection of these two loci is the centre. (c) 3 cms., 2 cms., 2·4 cms., 2·1 cms. ; ·5″, 1″ ; ·3″, 1·1″.

75. (c) 1. a = 3·1 cms. ; A = 81°, B = 54°
2. b = 1·3 cms., a = 1·8 cms. ; A = 68°, B = 42°
3. b = 1·7 cms., a = 3·6 cms. ; B = 29°, A = 96°

76. (c) 2 : 1 **78.** 28·1 in.

79. (a) With centre O, draw radii OP, OB making ∠ of 60°. Produce OB to D so that OD = 6 cms. DP will be a tangent of $3\sqrt{3}$ in.

(b) Area of OPD = $\frac{9}{2}\sqrt{3}$ sq. cms. ∴ area of OPDQ = $9\sqrt{3}$ sq. cms. Area of sector POQ = $\frac{1}{3} \times 9 \times \frac{22}{7}$ sq. cms. ; ∴ *Ans.* = sq. cms.

80. 1 ft. **81.** $d \times r = 28m$. **83.** $9\frac{17}{32}''$ $(= 59 \times \frac{5}{32} + \frac{10}{32})''$. **84.** $\sqrt{3}\,;\,(1+\sqrt{2})\,;\,(1+\sqrt{2}) : \sqrt{3}$.

85. ABCD the square, O its mid-point. With A as centre and radius AO cut off AB at M; with B as centre and same radius cut off AB at N. MN is a side of the Octagon; $1{\cdot}69\pi$ sq. in.

86. 40°, 80°, 50°. **87.** 78°, 82°, 102°, 98°.

EXAMPLES XXXI.

1. (a) 64. (b) 60; $V = lbh$. **2.** (b) $l = \frac{v}{bh}$; $b = \frac{v}{lh}$ $A = \frac{v}{h}$; $h = \frac{v}{lb}$ or $\frac{v}{A}$. (c) 60 c. inches; $4\frac{4}{5}$ ft.; 12 ft.; $1\frac{1}{8}$ ft.; 13″. **3.** 786 cwts. **4.** 25 c. ft. **5.** Rs. 10-11-0 **6.** Rs. 18—6—11. **7.** $2\frac{1}{2}$ inches. **8.** 334—8—8 **9.** 1 ft. 9 in. nearly. **10.** $\frac{1}{244800}$ of an inch. **11.** 6·8 mms. **12.** 10 d. **13.** 2·7 times leaves. **14.** 50 hrs.; 6 hrs. 40 min. **15.** $a(x-2a)(y-2a)$ c. inches; $302\frac{1}{2}$ c. inches; cube; a^3 c. inches. **16.** 28 lb. **17.** (1) xyz. c. inches; (2) $\frac{xyz}{abc}$ where a, b & c are factors of x, y & z. resp. (3) $\sqrt{x^2+y^2+z^2}$; (4) $(x+2a)\,(y+2a)\,(z+2a) - xyz$ c. in.; (5) $\frac{m}{72}(xy+yz+zx)$ annas. (6) $\frac{n}{72}\{(x+2a)\,(y+2a) + (y+2a)(z+2a) + (z+2a)(x+2a)\}$ pies. **18.** (1) $\frac{\sqrt{3}}{4} a^2h$; $3ah + \frac{\sqrt{3}}{2}a^2$; 55 c. in.; 110 s. in. (2) $6 \times \frac{\sqrt{3}}{4} b^2h$; $3\sqrt{3}b^2 + 6bh$; 187 c. in.; 191 sq. in. (3) $2c^2h\,(1+\sqrt{2})$ c. in.; $2c^2(1+\sqrt{2}) + 8ch$; 154·5 c. in.; 167 s. in. **19.** (a) 1·264 s. in. (b) 2·528 c. ft.; 8·728 s. ft. **20.** 29·25 gallons. **21.** (a) 4·68 s. in. (b) 147 students. **22.** Cylinder πr^2h c. inches; $2\pi rh$ square inches. **23.** (i) $V = 100\frac{4}{7}$ c. in.: $S = 100\frac{4}{7}$ sq. inches. (ii) 7 ft. (iii) 49·19 inches. **24.** $62\frac{6}{7}$ c. cms. 3·8 cms. nearly. **25.** 13 : 16. **26.** 5 hundredths. **27.** £ 47—[illegible]—6. **28.** 6 lbs. nearly. **29.** $\frac{1}{3}$ of an inch. (5 ft. 4 in. ÷ 192); 6 in. nearly (Note: circumference of the pillar $= \frac{300}{192}$ ft.) **30.** ·018 cms. **31.** $31\frac{3}{7}$ yds.; 8·9 c. in. nearly.

32. ·00022. **33.** 588 lbs. nearly. **34.** 1·245 in. nearly. (If the external radius be r'', $\pi(r^2-5^2) \times 61{\cdot}6 \times 7 = \pi({\cdot}2^2-5^2)\ (49 \times 11)$. **35.** $\pi(h-a)\ (x-a)^2$ c. in.; $\pi\{(x^2h-(h-a)\ (x-a^2)\}$ c. in. **36.** Rs. 99—7—1. **37.** 57 min. 36 sec. **38.** 120 ft., 12 ft. **39.** 7 ft. 7 in, **40.** i. lbh c. yds.; 192. ii. $2(lh+bh+lb)$ sq. yds.; 208. iii. $\sqrt{l^2+b^2+h^2}$ yds.; 10·8.

$$v=1/8\ V;\ a=1/4\ A;\ d=1/2D.$$

41. ·00000417 in. **42.** Rs. 115—8 As.

43. $\dfrac{28 \times 10^6 abc}{22x^2y}$; 55 nearly. **44.** 1514 leaves.

45. The length of the beam serves no purpose. The wt. of the pillar: the wt. of the beam = The area of the octagon; the area of the square $= 2(\sqrt{2}-1) : 1$. Length of a side of the octagon $12(\sqrt{2}-1)$ in. **46.** For finding the volume consider the bath as a prism 25 ft. high on a trapezium base whose parallel sides are 7 ft. 6 in. and 3ft. 6 in., and distance between 50 ft. 42968¾ gallons. The sides consist of 2 trapeziums of the dimensions given above, a rectangle 3 ft. 6 in. by 25 ft. another rectangle 7 ft. 6 in. by 25 ft. and a third rectangle the bottom 22 ft. by 50·16 ft. being the hyp. of the rt. angled △, sides being 50 and 4 ft; Rs. 519—12 as. **47.** 3·03 ft.; 99990 c. ft.; 1 hr. 23 min. **48.** 7·2 c. cms.

49. $t = \dfrac{144\ cxy}{(12x+3)\ \ (12\ y+4)}$; C = 0·2625.

50. 18·4 ozs. (*Note*; bottom, hid from view, need not be plated). **52.** (a) 2·3″ nearly; (b) 30. **53.** (a) Note:—The section of the strip of sand will be a rt. ∠d triangle with the base = 4′ and the inclination of the hyp. to the base 30°; 4·62 c. ft. nearly. (b) Rs. 5—13—10. **54.** Rs. 94—8—6 nearly. **55.** (a) ·6 nearly. (b) 9·7 cwt. nearly. **56.** 16·18″. **57.** 4·43 kilometers. **58.** (a) 71·04 sq. cms. (b) 13·7 c. cms. nearly 1·93 mm. (c) 1·18 cms.; 1·53 cms. **59.** (a) 12·2 sq. cms. (b) 124·8 sq. ft.; 122·1 c. ft. **60.** 1·95 lbs. **61.** 10·3 miles per hr. **62.** (i) 8 in. (ii) 0·61. **63.** All measurements are in the interior; 192½ sq. in. The piece to be purchased must be 29 inches long. A length of 22 inches, to line the curved surface and a circle of diameter 7 in.

to be cut off from the sq. part remaining; 10½ sq. in. will be wasted. **64.** 7 as. 10 ps. **65.** ·00889 cms. **66.** 2·34″. **67.** $9\frac{9}{14}$ c. in. **68.** 1 hr. 50 minutes. **69.** 37 lbs. **70.** 6·6 in. **71** 932 sq. in. **72.** 860 lbs. **73.** $\frac{7}{22}$.″ **74.** Rs. 129—4 as. **75.** (a) 29⅓ sq. ft. (b) 720. **76.** (a) ·005.″ (b) 3 ft. 2 in. **77.** 208 million cubic yds. ·043 million of pounds. **78.** 39·32 ft. per minute. **79.** 211·75 c. in.; 27·5 lbs. per sq. in. **80.** 10·4″; ·782″. **81.** 10·7 in. **82.** 25 ft. **83.** (a) 2 lb. 0·1 oz. (b) 3·84″ × 2.88″ × 2·25.″ 206. **85.** $\frac{15}{32}$ lb. **86.** 9231·6. **87.** Rs. 235–13 as. **88.** 253 lbs. **89.** 2·65.″ **90·** 2·5 oz. **91.** 43 min. (nearly). **92.** $\frac{3\pi d^2 w}{8960}$; $\frac{v}{15\pi d^2}$; 6·84 tons; 0·424 p. s. **93.** 142 c. in. (If r be the radius of the sector, circumference of the base of the con $= \frac{1}{3}\pi r$; hence radius of the base $= \frac{1}{6}\,r$. Hence $h = \frac{r}{6}\sqrt{35} = 1·4$ ft., $\therefore\ r = \frac{8·4}{\sqrt{35}}$ $\therefore$ radius of the base of the cone $= \frac{1·4}{\sqrt{35}}$ ft.) **94.** $4\pi - 3\sqrt{3} : 4\pi + 6\sqrt{3} : 4\pi - 3\sqrt{3}$. *Note*:—Volume being proportional to the base area, the volumes will be in the ratio of the area of the segments and the central zone. Area of the segment $= \frac{1}{3}\pi - \frac{\sqrt{3}}{4}$ and zone $= \frac{1}{3}\pi + \frac{\sqrt{3}}{2}$ c. ft since $r = 1$ ft. **95.** 24 gallons. (*Note.*–1 gallon = 277½ c. inches) $a\sqrt{2}$ inches. **96.** 412 sq. inches. **97.** 3·46 inches ·9 of a c. in. nearly. **98.** $2\frac{6}{47}$%. **99.** 4·9 kalams nearly 3·4 ft. nearly. **100.** (a) $4 + 3\pi$ S. in. (b) $4\,(4 + 3\pi)$ c. ft (Note – area of the fig. = area of the square + area o 3 circles of radius 1 inch.) **101.** 1·9 sq. in. nearly (b) 342 c. ft. nearly; 205 tins. **102.** (i) 20416 sq. ft (ii) 641 men. **103.** heights are 5·6√6″; 8·4″ **104.** 4·3″ nearly. **105.** 4%. **106.** $\frac{22}{63}$. **107.** 39 **108.** heights are 2″. 1·8 in. **109.** 0·7 kgm. **110.** 3″ ollocks; 79·4 sq. in. **111.** (i) (a) 8·4 in. (b) 4·4 in (c) 18·48 sq. in. (ii) 8·3 in.; 4·26 c. in. **112.** 3·7″ **113.** 18·56 tons. **114.** 3·3 in. **115.** 452·5 sq. ft. nearly **116.** (a) Two (b) They are equal. (c) 3·7 in. nearly The lengths of segments into which the sides of a △ ar divided by the contact of the in. circle are known. Assume radius $= y$ in. The part to the vertex of the cone from the incentre $= x$. Then $x + y = 13{\cdot}2$. Also $x^2 - y^2 = (8{\cdot}8)^2$. Proceed. **117.** 4607·4 c. in.

REVISION PAPERS.

I SERIES.

I. (1) 160 ft. (2) $11\frac{1}{9}$%. (3) 10,240 ft. (4) $\frac{105}{22(a-b)}$ mts. $\frac{140a}{a-b}$ yds. (5) 41 yds. (6) 3221625.

II. (1) (i) $22\frac{11}{12}$ miles. (ii) 209·4 sq. miles nearly. (2) Rs. 33-10-8. (3) 11 and 6. (4) $x = \frac{pq - m - n}{p - 2}$ and $13\frac{1}{9}$. (5) (i) 65 yds. (ii) AT = 130 yds : BT = 75 yds. (6) Passenger is clearly the faster.

III. (1) 476934. (2) Rs. 1733·4375. and $r = 5$%. (3) at 29th mile after 2 hours 25 minutes. (4) 6 as. 3 ps. (5) Rs. 787-8-0. (6) Between Madras Egmore and Guindy, 2·24 miles nearly from Madras Egmore.

IV. (1) 6 as. 1·3 ps. (2) $I = \frac{Pnr}{100}$, $P = \frac{100I}{nr}$, $n = \frac{100\,I}{Pr}$, $r = \frac{100\,I}{Pn}$ and $7\frac{1}{2}$%. (3) Rs. 4783-5-4. (4) Rs. 10173·6. (5) Min. 60, Max 150. (6) $P = 39$ lbs. ; $W = 361$ lbs.

V. (1) $20p = f + 3b + 6c + 14r$, 168. (2) 484042 grains nearly. (3) 140 yds. (4) $36\frac{1}{2}$ and 37·4 nearly. (5) 3·168 measures of water. Gain Rs. 3·984. (6) 57·2 sq. ft.

VI. (1) 2·86 cm. nearly, 7 students should cut off a piece of 20 cm. and then make a mutual division, geometrically dividing the 20 cm. into 7 equal parts. (2) $\frac{75}{22(b-a)}$ minutes. $\frac{88c\,(b-a)}{3}$ yds. apart. (3) A = Rs. 28·14 nearly. B = Rs. 52·49. (4) 49·3″ nearly. (5) 2·2 lbs. nearly. (6) $4\frac{6}{11}$ miles.

VII. (1) $\{h-u-v\}$ mls. $\{h-du-dv\}$ mls. (2) Rs. 60-12-0 less. (3) Rs. 12000. (4) Second sort is the cheapest, 5 yds. 2 ft. nearly. (5) 8·7 miles and 8·2 miles. (6) 20 minutes after the 1st car passed the milestone. and 4 miles from the milestone.

VIII. (1) 28 lbs. (2) 683·18 tons wt. nearly. (3) 41%, and 59%, $\frac{sp}{q-p}$ hours after starting. (5) 9 bags, and 4 bags. (6) 10%.

IX. (1) 1·43 metres (2) 33 miles. (3) $a(3b+1)$ miles. (4) $2\frac{4}{5}$ measures per Re. (5) 4·55 litres (6) (a) $\frac{x-a}{3}$. (b) 39°.

X. (1) 31%. (2) 19 lbs. nearly. (3) 54 c. mm. nearly. (4) Rs. 50. (5) $1\frac{1}{6}$ ft. (6) 10·2 miles ; 6 miles ; 5 miles...

II SERIES.

I. (1) ·0150. (2) 51 and 74 (3) 150 sq. cms (4) 53 nearly. (5) 38% of Copper, 26% of Zinc and 36% of Tin. (6) $9\frac{7}{17}$ ft. (7) (i) $8y-11x$. (ii) Rs. 16-8 as. ; Rs. 10.

II. (1) 1·2, 77.4, 22·6. (2) $\frac{13}{15}abc$ inches. (3) 15. (4) 3619·6 tons. (5) 10 ft. (6) 12 miles an hour.

III. (1) 33·1, 4·7, 236·2, 425·2. (2) $\frac{25c}{2ad-bc}$; Rs. 8-5-4. (3) 5 min. 13 sec. after 1 P.M. (4) 24°. (5) 12·6 cms. 55·44 sq. cms (6) The former by Rs. 7-9-7 nearly.

IV. (1) (a) $2^2 \times 11^2 \times 9$ i.e. 4356 is the number. (b) 66 yds. (2) (i) 13, 12, 5. (ii) rt. $\angle$; $6\frac{1}{2}$ in. (3) 20 lbs. (4) 9 ft. (5) $\frac{29}{24}x$. (6) 4·8, 3·2.

V. (1) 149·6 ; 153·2. (2) 17·5 lbs. (3) 4 as. (4) $5·165x$ ft. ; 103·3 lbs. (5) 1150 yds. (6) $\frac{c-bh}{a-b^2}=\frac{ap-c}{a-b}$.

VI. (1) 2·953 in. ; 6 ft. 3 in. ; 16 lbs. ; 1634 ft. per sec. ; 3351 lbs. (2) (a) $(a+b)$ in. ; (b) $\frac{2(l^2+S)}{l}$ in. ; $\frac{1}{l}\sqrt{l^4+S^2}$ in.

(3) 3·2 ft., 287. (4) 4000 sq. ft. nearly. (5) (i) $66\frac{3}{7}$ p. c. (ii) 446·4 tons. (6) (a) 16000. (b) 90 and 87.

VII. (1) (a) $\frac{5}{2}$, $\frac{10}{3}$, 5, $7\frac{1}{2}$, 10 miles. (c) $8\frac{3}{4}$ miles. (2) Rs. 20; Rs. 333-5-4. (3) Re. 1-9 as. (4) 2·44a ft. 2928 Copies. (5) **1** : 1·75 exactly. (6) 407 acres; $15\frac{1}{4}$ sq. miles.

VIII. (1) 3·2, 6 and 6 8 cms. Median 3·4 cms. (2) 30% (3) $\frac{15}{22}\left(\frac{x}{a}+\frac{b}{2y}\right)$: $6\frac{3}{8}$ miles. (4) 22° to E to W line. (5) 27 5%. (6) 5 miles an hour; 45 miles.

IX. (1) 6·25%. (2) 5; Analogous to Q. 6 paper (V). (3) (i) Rs. 39-13-6. (ii) Rs. 42-8-0. (iii) 3·58%. (4) 1 : 0·955. The geometrical construction of bisecting the sides and drawing a parallel to base may be taught, if boys work numerically and find height of rectangle. (5) Rs. 1819. (6) (i) $4\frac{1}{2}$ miles. (ii) 430 miles.

X. (1) The true remainder is 27 and in the latter case if x and y be the successive remainders $7y + x = 27$ and x must be less than 7; Hence, by trial, since x and y are integers, $x=6$ and $y=3$. (2) (a) $\frac{x}{y}$ seconds. (b) $A=24x+2y+3z$. (3) In the time the winner covered $4\frac{27}{50}$ miles, the loser covered $5\frac{1}{4}$ miles; 62·4 miles per hr. (4) Rs. 3500. (5) $77\frac{1}{7}$%. (6) In 1 hr. 52 min. from the time of starting. ii. 25 min. and 1 hr. 37 min. from the time of starting.

III SERIES.

I

(1) Timber ash ·784, Cork ·240, Clay 2·00. (2) 6 in. nearly; 9 75 in. Note that the second triangle is rt. angled. (3) Rs. 520. (4) Rs. 3900. (5) 15 lbs. and 9 ps. per. lb. (6) 22·1 in. nearly.

II

(1) Perimeter 91·39 fur. Area 495 sq. fur. (2) $\frac{84a+7b(ph-m)}{192}$; Rs. 7. (3) Rs. 55-10 as.

(4) *A*, 3250. *B*, 3000. (5) End of the 2nd season; 12544. (6) 1 : 1·102 nearly.

III

(1) (i) $685^2 - 684^2 = (685 + 684) = 37^2$. (ii) Cost of 30 viss at Rs. 1-15-6. i.e., (Rs. 60 – Rs. 0-15-0) = Rs. 59-1 a. (iii) Int. on Rs. 100 for 1 month = 10 as. Hence, for Rs. 400 for 6½ months = Rs. 16-4; for 375 = Rs. 15-3-9. (subtract $\frac{1}{16}$) (2) (a) 7 : 12. (b) Divide a st. line 3·8 in. long internally in the ratio 7 : 12. 140, 240. (3) Rs. 1500; Rs. 1125. (4) 169·4 litres nearly. Note that evaporating water is pure water. (5) 23·863 yds. nearly. (6) 1·5 in.

IV

(1) 13 from Madras, 6 from Bombay and 15 from Bengal. (2) $(a+b)+p(q-1)$ gr., a c. c. weighs a gramme. (3) Rs. 936-7-8. (4) 1140 ft. per second; The man is 2850 ft. from Target or soldier. (5) 1 c. ft. 73 c. in. (6) 8·34 sq. in.

V

(1) 4316000; 14·6. (2) 3476 ft. (3) 17·7%. (4) 285·6%. (5) 3758 c. cms.; 37580 sq. cms. (6) 30 miles per hr.

VI

(1) (i) 13. (ii) 30 sq. units. (iii) 6·5. (2) 375,000 (3) (i) $y = a + bx$. (ii) Rs. 10, Rs. 7. (iii) Rs. 73 (4) Rs. 5004. (5) $l\,(\pi r + 2h)$ sq. ft. (6) 32%.

VII

(1) (a) $\dfrac{2(30p \pm q)}{11}$. (b) $38\frac{2}{11}$ min. $5\frac{5}{11}$ min. afte 4 o'clock and $54\frac{6}{11}$ min. after 4 o'clock. (2) Ht = 20·8 f nearly, $BC = 17·5$ ft. nearly. (3) 4·9 acres. (4) (a) 25% (b) 96 cans. (5) 3½ measures of black gram, or 4½ measures of doll per rupee; 12 as. per bundle of hay (6) 39·3 in. nearly.

VIII

(1) 4·41 sq. in. (2) Rs. 280. (3) 8·7 sq. cm (4) 2·9 c. in. (5) $\dfrac{a+b}{2}$ gms. (6) Rs. 99.

IX

(1) 205·9 c. c. of water and 794·1 c. c. of the other liquid. (2) $\frac{1}{2}(a+100bc)$. (3) £ 303–15 sh. (4) (i) r^2. (ii) $r^2(\pi-2)$. (iii) $r^2(\pi-1)$. (5) 118¾lbs. (6) 880.

X

(1) $\frac{m(a-2)+n(b-1)}{5}=p$; 56. (2) Yes; by a majority of 40; 560. (3) 16·2 miles nearly. (4) $\frac{\pi r^2}{4}$, 33 : 25; Four. (5) Rs. 960. (6) A trapezium $ABCD$ whose parallel sides AD and BC are 10″ and 30″ long; vertical $AB=10$ ft. ∠s, A and B rt. angles; Rs. 234-6 as.

XI

(1) 1·7 cwts. nearly. (2) $PD=150$ yds; $DQ=247$ yds. $BQ=25$ yds. (3) $\frac{s}{60}\left(T+t\right)$, $\frac{s}{60}\left(T-t\right)$, 45 miles, 35 miles. (4) 16·8 kg. (5) Rs. 28-12-0; 19·7%. (6) (a) $\frac{2}{3}\pi r^2h$, c. in. (b) $\frac{5}{6}$ of the volume of the cylinder $=277$ cub. in. nearly.

XII

(1) $480f-(a+2b+10c+480d)$ grains; $29\frac{1}{8}$. (2) (a) Rs. 25. (b) 33⅓%. (3) Rs. 13791. (4) $\frac{3}{28}a^2$. (5) 18 miles (6) 1·85 ft.

XIII

1. 0·00029. 2. $AC=11·5$ inches; area $=58·36$ sq. in.; weight $=2·276$ lbs. 3. 9 cartloads $=4$ wagon-loads; 24 carts and 8 wagons carry 42 cartloads; 12 carts and 12 wagons carry 39 cartloads; ∴ 3 cartloads $=2$ tons; 42 cartloads $=$ *28 tons*, total quantity, 4. 12,480. 5. 6⅔% gain.

XIV

1. 3½ inches. 2. 40170. 3. 6 miles 4. 1134 ft. per second; 82°. 5. 51·1 ft. and 57·7 ft.

XV

1. Draw the perpendicular bisector of AB to meet CD at E; join AE; At A make angle $EAW = AEW$; W being in the perpendicular bisector. W is the well; $WA = WB = WE = 243{\cdot}75$ ft. If $WE = x$ ft.; then $x^2 - (480 - x)^2 = 3600$; Hence $480(2x - 480) = 3600$. $\therefore$ $2x = 487\frac{1}{2}$ and $x = 243{\cdot}75$. **2.** Exaggerated length...x'', Real length...y''. Then $\frac{x}{2} = 2y + 10$ and $y + 4 = \frac{1}{5}(x + 4)$; Hence $x - 4y = 20$ and $x - 5y = 16$. $y = 4$ and $x = 36$.

3. Total time 5 hrs. 48′; Rate required 8 miles an hour.

4. Agent got Rs. 75 by commission; Balance Rs. 19-8 from trader, his master. Total profits must be Rs. 351; This works to trader a profit of 11·7%; but the actual capital invested by agent is only Rs 2925; Profit made is 12%.

5. To the nearest sq. in.; area of square is 14; of circle is 18, and of equilateral triangle is 11; circle, square, equilateral triangle.

XVI

1. (i) Two angles and the corresponding side ... Δ^s congruent
(ii) Three sides ... Do
(iii) Two sides and the contained angle Do
(iv) Two sides and a corresponding angle ... Not necessarily congruent
(v) Three angles Do
(vi) Rt ∠ d△s, hypotenuses equal and a side equal to a corresponding side } ... congruent.

2. 20 trips to and fro. **3.** 2846, 3457, 3697 respectively.

4. $b = \frac{l - t - 2p}{a - b}$; 38. **5.** Rs. 21-10 as.

XVII

1. We copy the sum inserting a, b, c, etc. in the blank spaces.

$$\begin{array}{rrrr} & 3 & 8 & 7 \\ & & 2 & 4 \\ \hline & 7 & 7 & 4 \\ 1 & 5 & 4 & 8 \\ \hline 9 & 2 & 8 & 8 \end{array} \qquad \begin{array}{rrrr} & 3 & a & b \\ & & c & 4 \\ \hline d & e & 4 & \\ f & 5 & g & h \\ \hline 9 & i & 8 & j \end{array}$$

Evidently $h = j$; $\therefore g$ must be 4; $4 \times 3 = 12$; $\therefore f$ must be 1 (3 may be carry, not (13); a must be 8 (dividing 154 by 4). Dividing 154 by 4; R^r is 2 which being carry arising from multiplication of 4 and b, b must be 7; Hence c must be 2, d and e are 7 and 7. All the letters may then be found out.

2. $\frac{ag}{b+g} + 12p + m$. **3.** radius 393·2 ft.; arc portion is 1853·6 ft.; Area inclosed 11·9 acres nearly. **4.** 31¼%; 27½%. **5.** If x and y denote speeds $\frac{150}{x+y} = 5$ and $\frac{150}{x-y} = 25$; Hence their speeds are 18 ft. and 12 ft. per second.

XVIII

1. 232·5 lbs. **2.** (*a*) $24\sqrt{3} - 21\sqrt{2}$. (*b*) Rationalise the denominator; 1·98. **3.** One-fourth of the area is removed: Similar figures are to one another as squares on corresponding sides. **4.** He has 60 chairs and can make 42 per week. **5.** $30\frac{10}{11}$%.

XIX

1. Rs. 1500. **2.** 3¾ hrs. **3.** Trichy passenger each, Rs. 2-12-0; Kiranur passenger, each Rs. 1-2-8 ps.; Ammachutram passenger, each 14 as. 8 ps. **4.** 6 as. 3 ps. The new boy is 1¼ times as good a workman as the old boy. **5.** 4 cms.; 61·2 sq. cms.

XX

1. (i) $R^2 - r^2 = (R + r)(R - r) = 7 \times 2·74$; Taking $\pi = \frac{22}{7}$, area $= 22 \times 2·74$ sq cms. $= (54·8 + 5·48)$ sq. cms. $= 60$ sq. cms. nearly. (ii) of thrice the number $= 22539$. (iii) 64% $= 736$; $(64 + 32 + 4) = 100$; $736 + 368 + 46 = 1150$ is the strength. **2.** Let it be denoted by x (in seconds).

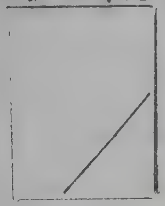
Distance travelled by sound in x seconds + Distance travelled by train $= 6 \times 1760 \times 3$ ft.; 26·2 seconds. *Ans.* **3.** One boy secured 7 and the other 4. **4.** Total surface, 688 sq. in. (add surface of circular hole); 510 cub. in. **5.** 2·344 miles.

XXI

1. Two sides and angle are given. In general two $\triangle$s are possible. $32\sqrt{3}$ and $16\sqrt{3}$. In the former case the $\angle$ is rt. angled. $\angle B$ being the rt. angle. **2.** 800,000. **3.** 976 inches. **4.** 63 children. **5.** 7623 yds.

XXII

1. $\frac{1}{n-1}\left(\frac{2s}{n} - 2a\right) = d$; 0·9. **2,** Four days; 56 inches. Draw graphs of $y = 72 - 4x$ and $y = 36 + 5x$. **3.** By $28\frac{4}{7}\%$ of 1921 price; 12%. **4.** 560 ft.; 440 ft. **5.**



If side be $2a$, volumes are as 7 : 1 1344 cub. in.

XXIII

1. $S = \frac{31}{32} R(1 - {\cdot}01\, p) - 12E$. **2.** Find areas of segments outside the rectangle. The arc outside the rectangle subtends 120° at the centre. 6889 sq. ft. of the pasture. **3.** 50%, Rs. 1800. **4.** Let PQ and RS be trees, P and R on the ground. T a point in PQ. 10 ft. below Q. Join TS. Where perpendicular bisector of TS cuts RS, is the point of breaking. By calculation the stump is 15 ft. **5.** 1·6 in. from the side DA; 2 in. from A.

XXIV

1. Find lengths of altitudes from D on BE, from E on FC and from F on DA. Subtract areas of $\triangle$s BDE, CEF and AFD from area of the equilateral $\triangle$. $\frac{31}{72}$ of the $\triangle$. **2.** Increase 6989; 0·2%. **3.** Rs. 30685. **4.** Position geometrically is where perpendicular bisector of line joining towns cuts the river. 7¾ and 2¼ miles; 8·3 miles of piping. **5.** $\frac{pq}{qt-p}$ miles per hour.

XXV

1. Inside of biggest $5\frac{1}{2} \times 5 \times 4$ in.; 15 in. 2. 273·8 tons. 3. 12 days. 4. Radius of circumcircle 12 cms. $3{\cdot}1416 \times 144$ sq. cms. $= 452\frac{4}{7}$ sq. cms.; angle at centre subtended by one leg of isosceles △ is 60°; Length of arc 20π or 4π.

5.

Cost Price.	*Cash Price.*	*6 Months' Credit.*			*12 Months' Credit.*		
Rs.	Rs.	Rs.	A.	P.	Rs.	A.	P.
200	240	254	6	0	269	11	0
180	216	228	15	0	242	11	0
150	180	190	13	0	202	4	0

XXVI

1. 18·1 per thousand. 2. Rs. 3110. 3. 61·9. 4. The car has travelled 6270 ft. in. 3 min. 45 sec.; 19 miles per hour. 5. D, (2, 6); 2·6 in. and 1 in.

XXVII

2. 15 ft. 3. 6·7 c. ft. per minute. 4. $100^2xy = xy(100 - m)(100 + n)$; hence $\frac{1}{m} - \frac{1}{n} = \frac{1}{100}$ on the supposition that original army was $100x$ and supplies lasted $100y$ days. 5. 0·42% in each case; 22·078 sq. in. and 22·263 sq. in.

XXVIII

1. 230; 340. 2. $\frac{w}{n^2b}\sqrt{a^2 + b^2}$ 3. $16\frac{2}{3}$%. 4. If radius of circle be r, side of inscribed octagon $r\sqrt{2 - \sqrt{2}}$ i.e. $0{\cdot}7654r$; Hence $6{\cdot}1232r$ is the perimeter and the circumference $= 6{\cdot}2832r$; Diff. $= {\cdot}16r$ i.e. 1·6 cms. 5. 23·98 and 27·48 knots per hour; By 3·5 knots per hr.

XXIX

1. $t = 190 + 0{\cdot}05\ (x - 12000)$; \$ 277·5; \$ 12800. 2. 1, $\frac{4}{3}$, $\frac{7}{9}$, $\frac{4}{5}$; 0·99. 3. Similar △ˢ. Take ratios. 36 ft. 4. 1·88% loss. 5. The portion cut out is a cylinder whose length is 18 in. and the radius of whose base is 3 in.; 0·26 of the weight.

XXX

1. (i) $a = \sqrt{\frac{1}{3x}\left(\frac{6v}{\pi} - x^3\right)}$; $a = 1·2''$. **2.** 75 oz. and 25 oz. **3.** 87·46 c. in.; 26·24 ℔. **4.** Rs. 127 nearly. **5.** The visible portion of the compound in each case is that part of the circumference between the two tangents that may be drawn from each position to the circle. Describe a △ ABC; AB = 30 yds.; ∠A = 30°, ∠B = 120°; Produce AC. The radius of the circle touching AC produced and BC, is the radius of the compound. May also be calculated; 35·5 yds.

XXXI

1. 28·8, 14·4, 9·6, 7·2, 24 and 16. **2.** Rs. 200; 19·7%. **3.** 44303·8 km. **4.** 6557 ft.; 1637 ft. **5.** When $x = 0$, $y = 4$. Join points (0, 4) and (2, − 3); one line is obtained Similarly when $y = 0$, $x = 3$. Join (3, 0) and (2, − 3). The other line is obtained.

XXXII

1. $2(x + 1) + 3(y + 1) = 4x + 6y$ and $(3x + 2y)2 = 2x + 3y$; $x = -\frac{1}{2}$, $y = 2$. **2.** 6th mile; 30 miles. **3.** 13½ ft. × 13½ ft. **4.** 730 yds. is distance of equidistant point from either village; 617 yds. nearly. **5.** 1046 c. ft.

XXXIII

1. Each portion is ⅓ of (big circle − the portion within the equilateral △ between the arcs). 1·2 sq. in. The portion within equilateral △ = ·04 sq. in. i.e., Area of equilateral △ minus ½ the area of a circle whose radius is ½″. **2.** $\frac{2}{15}$ of the original stock. **3.** Rs. 130 and 260. **4.** 11·5 ft. nearly; 609,664 c. ft. **5.** B will overtake C in 3½ hrs. from the time of starting of B or A or C.

XXXIV

1. *See* Q. I paper V. **2.** 0·1956 in.; 7·79 in.
3. Loss $\frac{5}{9}$%. He gives $40\frac{2}{9}$ viss for the price of 40 viss.
4. 10, 12, $2\sqrt{61}$; $\sqrt{61}$ = half the hypotenuse = 7·8″.
5. 4. 6, 12, and 16 as. respectively.

XXXV

1. $1{\cdot}37a^2$; Each part is ⅓ of $1{\cdot}37a^2$. 2. £ 5640. 3. 12 miles per hour. 4. Volumes as 11 : 18. 5. 28·9 oz. silver and 35·6 oz. gold.

XXXVI

1. $\frac{5}{9}$ of the original circle remains. 2. $\frac{x-2}{100}=36$ or $x=60$: Rs. 9; Rs. 10½. 3. 0·3%. 4. The former, Re. 1-12-6. 5. 1 : 0·97.

MISCELLANEOUS EXAMPLES.

1. $32\frac{8}{11}$%. 2. 195 yds. 3. 4%. 4. $y=(4x+8)$ as.; (a) 4 hrs. (b) Re. 1–12–0. 5. Rs. 81–2–0; Rs. 93. 6. $T=\frac{9}{5}t+30$ (a) 25 min. (b) 49·2 min. 7. PQ Vertical denote the man, foot being Q. Let $PQ=5$ cms. QR a horizontal $=7$ cms.; $\angle PRQ$ is inclination of sun s rays. Again with centre Q and radius $=5\frac{1}{2}$ cms., cut PR at S. QS is the road. 8. 45·7 miles. 9. (i) 40 ft. (ii) 17·3 ft. 10. 3 hrs 9 min. Suppose the distance first travelled is x miles. Take a certain length to represent this distance. Farther from second position is $2x$ and the nearer from first position is $\frac{x}{\sqrt{3}}$ Distance is $\frac{2}{3}x\sqrt{3}$. 11. 23⅓, 35, $17\frac{2}{19}$, 22. $31\frac{9}{11}$, $28\frac{18}{19}$, $41\frac{1}{9}$ respectively. 12. 2. 13. $BC=1{\cdot}4''$, $CD={\cdot}4''$, $DA=2{\cdot}1''$; Area $=1{\cdot}98$ sq. in. 14. 2·34″. 15. (i) 9 miles, (ii) 33 miles, (iii) 10·40 and 11 A.M. 16. Physics. 17. On AB describe a semi-circle and where perpendicular bisector of BC, cuts this semi-circle is X. Note that it cuts at two points. AX = 16 miles, BX = CX = 9 miles. 18. 270 ft. 19. $ABCD$ be the square and O the point, $OA=3''$, $OB=5''$ and $AB=6''$ so that altitude OE from O on $AB=\frac{2\sqrt{14}}{3}$ $AE=\frac{5}{3}''$; $AE=$ the alt. from O on AD; $EB=4\frac{1}{3}''=$ the alt. from O on BC; and the alt. from O on $CD=6-\frac{2\sqrt{14}}{3}$ $AOB=$ Rs. 187, $AOD=$ Rs. 125; $BOC=$ Rs. 325; $DOC=$ Rs. 263. 20. 61 ft. 21. (a) 24026 sq. ft. (b) 136 ft.

22. 3 miles 6 fur. **23.** (a) (29·4 − 26·3)″ (b) (26·3 − 22·3)″ **24.** 5·85 yds. nearly. **25.** Let the board be $ABCD$ with $AB = 12'$ & $BC = 2'\ 4''$ keeping BC and DA as the ∥ sides of the trapezoid. The trapezoid can be got by cutting the board along LM, where $DL = 4''$ and $CM = 10''$. The ∥ sides are 2′ and 1½′ and non-parallel sides 12′ and 12·01′. This question can admit of many solutions. **26.** Let $ABCD$ be the trapezium, $AB = 8$ and $CD = 14$. Let EF and GH be the equidistant parallel lines. Draw AL ∥ to BC cutting EF, GH and DC at P, Q and L respectively. Now $AE = EG = GD$ since they are intercepts made by equidistant ∥ st. lines. Hence $GQ : DL = AG : AD = 2 : 3$. $DL = DC - LC = 14 - 8 = 6$. $\therefore\ QG = 4$ and similarly $EP = 2$. $\therefore\ EF = 8 + 2 = 10$ inches and $HG = 8 + 4 = 12$ inches. **28.** 1810 sq. yds. nearly. **29.** (a) 2 as. 6 ps. (b) 10 months. **31.** 1 : 2·4. **32.** 322 ft. **33.** 86623 c. ft.; Rs. 9-0-7. **34.** (i) 15 p. c. of a certain amount = £ 94-3-6; £ 627-16-8. (ii) $47 \times 49 - 23 \times 95$ i.e., $49 + 23(98 - 95) = 118$. (iii) of 3 acres is Rs. 4; of 48 Rs. 64, of ·75 acres or ¾ is Re. 1. (iv) 36 $\frac{4}{11}$ p. c. is $\frac{4}{11}$; 1 a. 4 ps. is $\frac{1}{12}$ of 1 Re. $\frac{4}{11}$ of $\frac{11}{12} = \frac{1}{3}$. Rs. 17. (v) 16 p. c. Re. 1—0—8. **35.** £ 615000. **36.** 48, 57. **37.** ii. Apply $a^2 - b^2 = (a + b)(a - b)$. 2999080. iii. Find the price at £ 1 and note that 48 is $\frac{1}{50}$ of 2400. £ 2442-18-0. **38.** 1325610. **39.** $mx + fy = y(m + J)$. **41.** 125·4 ft.; 2·83 sec. **42.** $\dfrac{abc}{ab + ac + bc}$. **43.** $a = 45$, $b = 22$; 8⅓; $40\frac{20}{37}$. **44.** 8. **46.** 20·4%. **47.** 5%. **48.** 16·38%. **49.** 3·2%, Rs. 25. **50.** 25%, 50%. **51.** (a) 32, 51, 40, 47, 35, 40; (b) B and D; (c) 35, 55, 44, 51, 39, 44; (d) 2 more *viz.* C and F. **52.** If the length of the one be x cms., that of the other should be $x + 50$ cms.; for even at zero degrees Centigrade, the difference between their lengths should be 50 cms. The increases should be equal. $\therefore \dfrac{x}{53200} = \dfrac{x + 50}{113100}$ $\therefore x = 44{\cdot}4$ cms. and the other 94·4 cms. **53.** 53·8%, 20·1%, 26·1%, 9·8%. **54.** Using scale and compasses, divide a length of 13·9 cms. into 3 equal parts for (a), and 5 equal parts for (b).

55. $r(100 - p) = c(100 + q)$; 60. **56.** Rs. 162½, Rs. 280, Rs. 168¾. Rs. 2½, Re. 1, 6 as. **57.** A = 36⅔%(7)

B - 35% (9). C - $39\frac{1}{6}$% (4). D - $51\frac{1}{4}$% (2). E - $21\frac{1}{19}$% (11). F - $63\frac{3}{19}$% (1). G - $48\frac{1}{3}$% (3). H - 35% (9). L - $35\frac{5}{8}$% (8). M - $39\frac{1}{6}$% (4). N - $37\frac{1}{7}$% (6). **58.** $P = \left(15 + \frac{Q}{2}\right)$, 3 hours; Rs. 17–12 as., **59.** $(2a - b)(2b - a)$. **60.** (i) 1 s. 8 d. = £ $\frac{1}{12}$; £ 84127 × $\frac{1}{12}$ = £ 7010–11–8. (ii) = $1\frac{1}{8}$ (55·23 + 8·77) = $1\frac{1}{8}$ × 64 = 72. (iii) 6 s. 8 d. = £$\frac{1}{3}$; £ 2·56523 = £ 2–11–4 d. **61.** 617 or 167. **62.** (i) Subtract $\frac{1}{8}$ of the amount. (ii) Note that 3 + 5 + 8 + 9 = 25. Short division by 25. (iii) 16 cub. ft. **63.** 10 lbs. **64.** $26\frac{7}{8}$ miles. **65.** $52\frac{8}{11}$%. **66.** 0·96. **67.** Rs. $38\frac{1}{4}$. **68.** 75350. **69.** (a) $p\sqrt{a^2 + b^2}$. (b) i. 3 miles 2 fur. 58 yds. ii. 16 min. **70.** 87·3 ft. Draw a vertical line AB, 6 cms. long, B being the foot. Draw a horizontal BD of indefinite length; cut off BE from BD = 6·4 cms. Draw a vertical at E. Cut this at H by an arc whose centre is A and radius 8 cms. From BA cut off BG = 4 cms. and draw a parallel to BD. With centre H and radius 3·5 cms. cut this parallel at J. JK is the other column. **71.** (1) $\frac{8}{9}$ of 39·3708 = 35 yds. very nearly. (2) $6\frac{1}{4}$ gallons or 50 pints go to a cub. ft. or 1728 cub. in. ∴ In a pint there must be 17·28 × 2 cub. in. (3) ·75 of ·000806 or 3 times ·0002015 multiplication by 125. (4) 29 × 3 × 9 bricks. **73.** Describe the △ ABC, and draw arcs with radii given. The common segment is the area exposed to the three guns. **74.** (i) There can be no pence in the result as the multiplier is 240. (ii) There can be no exact sq. root as there are only three decimal places. (iii) $\frac{1}{16}$ of 7348 is Rs. 459–4–0. Income-tax has also been paid. The result must be less. (iv) (2151 - 87) one of the factors = 28, exactly divisible by 7. There can be no remainder. (v) The number ending in 7 cannot heve a square root. **75.** 28·9 sq. ft. **76.** Rs. 1–14 as. per month, each. **77.** Divisor 79. Remʳ. 51 **78.** 4·17, 3·125, 5·2 cms.; 6·51 sq. cms., $r = 2·6$ cms.

81. $T = \frac{pm + wq + cr,}{16}$, (i) $5\frac{15}{16}$, (ii) 19. **82.** $\frac{ap}{x} : \frac{bq}{y}$.

83. (i) $2a + 2b + \sqrt{x^2 + y^2} - x - y$. (ii) $\frac{1}{2}$ $(2ab - xy)$. (a) 39·6″; 95·7 sq. in. (b) 8. **84.** An equilateral hexagon; for the figure to be regular, the angles also should be equal, must be 120°; ∠BAX must be 30°. Hence a must be equal to $b\sqrt{3}$.

Find $\sqrt{3}$ geometrically and double it, for $b = 2\sqrt{3}$ cms.

85. The question admits of two solutions. The piece may be measured by the ruler starting from either end. 123·3″ or 123·52″.

86. Let A be his original position, A′ corresponding point in the nearer bank and T the tree; C his second position and C′ the corresponding point on the nearer bank. TA′A and AC are at rt. angles; AC = 60 yds. From C′ drop CD perpr. on AC; AD = 20 yds. = A′C′ From similar △ TA′ = 15 yds.

87. (a) $\left.\begin{aligned} 4242 &= PQ + R \\ 2903 &= Pq + R \end{aligned}\right\}$ $\therefore 1339 = P(Q - q)$ i.e. $13 \times 103 = P(Q - q)$

$\therefore$ P = 103 and R = 19.

(b) $\cdot375416 = \cdot375 + \cdot000416 = \cdot375 + \dfrac{\cdot0416}{100} = \frac{3}{8} + \dfrac{26}{62500}$

88. (*a*) 5″ (*b*) 239·1 sq. in. **89.** (a) Note terms 1, 3 and 5 can be evaluated easily by finding the root of the first alone; and terms 2 and 4 by finding the root of the second alone. 46·36322.

(b) 8. **91.** $5\frac{5}{19}\%$; 5%. **92.** 1·5 ft. **93.** $\dfrac{100a\,(q-r)}{(q-p)}$ $\dfrac{100a(r-p)}{(q-p)}$; 650. 150. **94.** 93. **95.** Rs. 49—5—9. **96.** $x = 7, y = 5$. **97.** $p = 4v(l - v)$; 872·45. **98.** (1) 16 hours 35 min. (2) 7 hours 46 min. **99.** $\dfrac{3ax + by}{3a + b}$; 12%. **100.** Rs. 120. **101.** 36 maunds. **102.** 41, 59. **103.** ·88; 8″ × 4½″. **104.** 259 sq. in. **105.** Water ·052, Flint-glass ·166, Copper ·462. **106.** $P = \dfrac{ay + bx}{16}$; Rs. 2½ per day; 4 as. per mile. **107.** Latter by Rs. 7690. **108.** Red = Green = $1157\frac{1}{7}$ sq. in.; White = $514\frac{2}{7}$ sq. in. **109.** 0·7 mm. **110.** Draw sectors of central ∠s 288, 11·2, 14·4, 14·4, 16 and 16 degrees respectively. **111.** (i) $\dfrac{ac}{a-b}$ miles from where the faster started; (ii) $\dfrac{100a}{100-a}\%$: 900%.

112. 21·34%. **113.** 500 men and 100 boys. **114.** $ 20,850,000. **115.** $a = 4·28$, $b = ·000916$; £ 5867. **116.** (i) 5400 c. ft. ; (ii) 2·99 in. nearly. **117.** 18·735, 24·98, 15 miles. **118.** 1073864 c. ft. **119.** 1000, 2250. **120.** 15 miles per hour ; 10 A. M. **121.** Note that the area of the interior hexagon = the sum of the areas of the six equilateral triangles. Area of circle − 2 times the area of the int. hexagon is the area of white glass required ; white glass required 3·17 sq. in. A of green glass = A of red glass = 1·95 sq. in. **122.** Rs. 66, Rs. 98-6, Rs. 88-13, Rs. 166-13 respectively. **123.** $\frac{3a^2c}{174240b}$ ft. ; 0·23 in. **124.** Rs. 1186-15 to the nearest anna. **125.** Rs. 358-5-4. **126.** 60 miles ; 7-30 A.M., 3-30 P.M. **127.** 302 c. in. **128.** Rs. 2-14 and Rs. 3-8 respectively. **129.** 11·3 sq. in. **130.** Rs. 630, Rs 630, Rs. 540 ; Rs. 852, Rs. 948. Rs 600. **131.** (*a*) Five. (*b*) 4·62 cms. ; 36·96 sq. cms. **132.** 4400 sq. ft. Taking $\pi = \frac{22}{7}$. **133.** About 2800.

S. S. L. C. PAPERS.

1911.—**3.** Rs. 3295-11-0. **4.** 63·7 and 18·8. **5.** Rs. 90 more than the man who bought the house. **6.** 878. **7.** 490 yds. **8.** The real length is either 40·124 or 40·118 as the line may be measured from either end of the ruler. **9.** $x = 8$, $y = 5$. **10.** No. 1 is 100 miles, No. 2, 305 miles and No. 3, 275 miles from Madras. (*b*) No. 1 passes No. 2 at 2-15 a.m. Tuesday and No. 1 passes No. 3 at 4 a.m. Tuesday. **12.** Area of △ = 0·8 sq. in. nearly. **13.** 27·6″. **14.** 35 ft. nearly. **15.** Rs. 165-0-7 to the nearest pie.

1912.—**1.** 70, 33⅓, 17⅓, 96⅔, 60, 25⅓, 14, 87⅓, 53, 30, 38⅔, 76 – Percentages respectively ; 4. , 11, 1, 5, 10, 12, 2, 6, 9, 7, 3 – Rank respectively. **2.** Rs. 80-12 as., Rs. 303-3 as., Rs. 192-11 as., Rs. 202-9 as., Rs. 260-11 as., Rs. 354-3 as., Rs. 51 respectively. **3.** i. Rs. 274-8 as. nearly. **4.** 60′ 4·5″ ; 141·43 correct to 5 significant figures. **6.** 1571 sq. ft. nearly. **8.** Midnight on Monday 103·4° nearly and at 8 – 30′ on Tuesday morning 100·35°

nearly. **9.** 4466·2 tolas. **12.** ·489x grammes per sq. cm. $= x$ lbs. per sq. ft. **13.** Rs. 4. **14.** 45 cms. nearly. **15.** 8 as. to the nearest pie.

1913.—1. (a) (1) 919,366; (2) 266,526; 67099; 61571; 40673; 31321; (b) 3·24. **2.** (1) £ 49-6-3, (2) Rs. 236-11-5. **3.** 3035 ft. **4.** $x=4$; $y=3$. **5.** $x/36$ furlongs where x is the no. of revolutions of the pedal; 13 fur. 195 yds. 1 ft. 8 in. **6.** II : I : : 2 : 1; $H_1 : H_2 :: 4 : 1$. **7.** 2 in. **8.** 90°, 53° and 37°; 7·07 sq. in. **10.** Rs. 307-8. **11.** Rs. 2-10-7. **12.** 3,103·788 kg. **13.** 6·9%.

1914.—1. + 35·1 %; − 7 %; + 356·5%; + 38·7 %; + 17·3% 698,667; 683,348; − 2·2%. **3.** $(mx-ny)\div(m-n)$ 10 nearly. **4.** $W : M :: 5 : 16$. 5·10000 $r\div(100+p)$; $(100\div q)$; Rs. 400. **6.** 26·5 ft. **7.** 504 sq. yds. **8.** 285 sq. in. **9.** 3 mm. **10.** 585,426.

1915.—1. 7 s. 6 d. **2.** 8 s. 1 d. **3.** 42% **3.** 18s. 3d. 3·5 oz. nearly. **4.** $T=(3g+w+2p+4y)\div 4$; 320. **5.** $a=b(f-e)$; $c=d\ (f+e)$: $e=2$; f 10; 851 tons nearly. **9.** Rs. 5-10; Rs. 3-12. **10.** Rs. 23283. **12.** 88 lbs.

1916.—1. (1) − 12%; − 17%. (2) + 11%; + 14%. (3) − 67%; − 75%. (4) − 26%; − 27%. (5) + 2%; 15%. **2.** Rs. 9-4-6. **4.** 8. **5.** 4 as. **7.** 1·64 in. **9.** $\frac{1}{6}(p+q+r)\times(4d+a)-(ap+bq+cr)$ as.; $d=20$. **10.** $x=6\frac{6}{7}$; $y=-\frac{4}{7}$. **12.** $H : E :: 9 : 8$.

1917.—1. 77. **2.** Rs. 61,100. Rs. 52,975-10-8. 13·3%. **3.** Rs. 525. **4.** Rs. 4472-14 as.; 6·4%. **5.** 205 ft. nearly. **8.** $x=a-7/2$ where x is the no. thought of and a the answer. **9.** Rs. 70; Rs. 130. **10.** 5 cm.

1918.—1. Rs. 3-13-3; 16·7%. **2.** Gain = Rs. 200 **3.** 13 − 37; 12 − 37; 16 − 45; I & II at the 29th Mile at 13 − 8; I & III at the 66th mile at 14-52. **4.** 1,800,000 sq. mls.; 1 in. = 120 mls. **5.** 4 %; 4·8 %; 5·8 % **6.** Rs. 1232. **7.** 19·3 sq. cm. **8.** $(mx+ny)\div(x+y)$ 36·4. **9.** 29 miles. **10.** (2) ¼. **11.** 30 miles. **12.** 17 in.

1919.—1. 38·6; C (highest) 41·4; D (lowest) 34·5. **2.** Rs. 1600; 32·5% **3.** 12·7 acres. **4.** 1. A reaches Q at 2-30 p.m.; B at 1-5 p.m.; 2. At 37½ miles from P at

11-15 a.m. ; 312½ miles. **5.** Rs. 112. **6.** 2. 5·85 sq. in. **7.** 2 ft. 8 in. **8.** 24 men and 20 boys. **9.** 1. 7·2 in. 2. 18 men. **10.** Cone ; 14 in. **11.** $(am+bn)$ $(100+c) \div 100$ $(m+n)$; Re. 1-9. **12.** 2,964·5 sq. ft.

1919.—Re-Ex. **1.** A 57 ; B 37 ; A 1st, B 67th. **2.** 16 lbs. **3.** 13 yds. **4.** 171·5 cub. in., 128. **5.** To get books from London by Rs. 42-8. **6.** 30 s. ; 5 s. ; 10 s. **8.** Rs. 583-5-4. **10.** Gain 20%. **11.** 881¼ yds. **12.** (1) $60/m-6/n$. (2) 3 $(m-n)$ min. (3) 15 min.

1920.—1. (1) 66·9%. (2) Rs. 1,265,368. **2.** 11,610 cub. yds. ; 7½ min. **3.** 175 ft. **4.** Rs. 115-3. **5.** Rs. 240 and Rs. 360 ; **9.** mnp/q ; 37½ (37 men and 1 boy). **10.** 3 in. **11.** 3000 c. cm. **12.** 6·8% ; £ 2,516-16 s.

1921.—1. 6,076 ft. **2.** Of the 3 measurements, at least *one* should be that of a side ; $a=2·1''$, $b=1·9''$, $c=1·5''$, A = 79°, B = 60° and C = 41°. **3.** 14·96 sq. in. **4.** (1) 3·9% (2) 7·7%. (3) 8 as. **5.** Rs. 45-4-10. **6.** 75%. **7.** 34·7. **8.** 14½ days. **9.** 12,667 sq. ft. **10.** Rs. $a\ (r+1)-rs$; Re. 1. **11.** BA : 1 : : 8 : 13 ; 294.

1923.—1. (1) 8047462 ; 8189960. (2) 3·6. (3) 52·5. **2.** (1) 15,574 gms. (2) 37·20″ ; 11 and 10. **3.** 17·8%. **4.** 4a. 3p. **5.** Rs. 192. **6.** 26. **8.** $(v^2-u^2)/v$, *m. p. h.* ; 59·6 *m. p. h.* **9.** 13 ft. 4 in. **10.** 46·6 and 41·6 ft. **11.** (a) 1 : 4. (b) $5b^2$, 1/4 ; 25,000. **12.** 21·3″ ; 1810·3 c. in. ; 655 sq. in.

1924.—1. (1) 1798, 2047, 2324, 1901, 1725, 1901, 10382, 667, 647. (2) 7·8, 5·6, 3·3, 5·7 ; Guntur. **2.** 998·8 oz. Rs. 7-5-4. **3.** Rs. 2000 : 13 of wine and 19 of water. **4.** Rs. 41-8-0 ; Rs. 3-7-4. **5.** $54\frac{6}{11}$ p. c. **6.** 98. **8.** 28 yds. **10.** $a=0·3$, $b=0·8$, $c=2·125$. **11.** 120 sq. ft. ; 7920000 gallons. **12.** $12\frac{5}{6}$ c. ft. : 44 sq. ft. ; 10·6 cub. ft. ; 19 p. c.

1925.—1. (1) + 2·7, + 2·2, − 1·4, − 3·1, + 5·7. (2) + 0·88. **2.** Rs. 15-12 as., Rs. 28-14 as., Rs. 20-4 as. **3.** £ 28-16-0. **4.** 40 years. **5.** Rs. 958-13-0. **6.** 5·7 miles. **7.** £ 24-13-4. **8.** 40 cms. nearly. **9.** 53°. **10.** 2 ft. below the surface ; 30°. **11.** 135 and 30. **12.** 4·9 acres nearly. **13.** 2·5 gallons. Rs. 0-9-5 ps.

PART A.

1926.—1. (1) 6506·8, (2) United Kingdom − 1·9% ; Belgium − 12·1% ; Germany + 223·8%. **2.** 18 miles per hr. **3.** 999·9 ounces. **4.** (1) Rs. 5¼ ; (2) 160 lbs. (3) 40 lbs. **5.** AC = 650 links, and CDA is a right angle ; area = 1·76 acres. **6.** 745·7 c. cms.

PART B.

1. 7% ; A received £ 1050 ; B received £ 872-10 s. ; sum left over = £ 52-10 s. **2.** Rs. 6-1-2$\frac{2}{5}$ ps. **3.** £ 150 value of estate ; £ 100 and £ 200—debts. **4.** 6⅔ minutes. **5.** First election A 3420, B 2880 ; Second election A 3780, B 2970. **6.** P. W. must be calculated on 1980 due two years hence at 5½% C. I. £ 1778-19 s.

1927 A.—1. 7·9 %. **2.** 33·1% ; $z = \dfrac{ax - by}{a - b}$. **3.** Rs. 1000. **4.** Regular members 140, Associates 60. **5.** B will overtake A at 8 a.m., 12th mile.

B.—1. A Re. 1-6, B Re. 1-10. **2.** 1 : 2. **3.** 4 hrs. 37 mins. 12 secs. **4.** 60 miles by rail and 90 miles by river. **5.** 3762 yds.

1928.—1. 12·5. **2.** $3a + 4c - 10 = e\,(b + d)$; 3. **3.** Rs. 1500. **4.** 68 miles. **5.** 5·8 sq. in. **6.** Rs. 250. **7.** Rs. 242. **8.** 31·8. **9.** 16 ft. **10.** 10-30 a.m., 6-30 a.m.